# Discrete Structures - CMSC 203 - FINAL EXAM - Fall 2008

**1.** Fill in the blank(s).

**(a)** $\dfrac{(38 \text{ div } 5)[\text{GCD}(976, 320)]}{(38 \text{ mod } 5)}$ = ______. (Hint: Do problem 2 first!)

**(b)** $(\{1, 3, 4, 7, 9\} - \{2, 3, 9\}) \cup (\{4, 5, 7, 8\} \cap \{1, 2, 4, 5, 6\})$ = ____________________

**(c)** H(1100011, 1001001) = _________ (H is the Hamming Distance Function)

**(d)** If $\Sigma = \{w, x, y, z\}$, then $|\Sigma^4|$ = _________.

**(e)** Applying DeMorgan's Law to sets A, B and C, $(A \cap B \cap C)^c$ = ________________________________.

**(f)** For finite sets A and B, the Inclusion/Exclusion Rule says: $|A \cup B|$ = ______ +______ – ___________.

**(g)** If $\Sigma = \{0, 1\}$, then $\Sigma \times \Sigma$ = ________________________________________________________________.

**(h)** If $\Sigma = \{a, b\}$, then $\Sigma^3$ = ____________________________________________________________________.

**(i)** Let F:{1, 2, 3, 4} → {$a$, $b$, $c$, $d$} be a one-to-one correspondence. Then F = {(1,___),(2,___),(3,___),(4,___)}.

**(j)** If G:{$w$, $x$, $y$, $z$} → B is onto, and G = { ($w$, 3), ($x$, 1), ($y$, 4), ($z$, 0) }, then B = ________________________.

**(k)** If $s_n = 2s_{n-1} + 3s_{n-2}$ with $s_0 = 0$ and $s_1 = 1$, then $s_4$ = _________.

**(l)** If $f$:**R** → **R** is defined by $f(x) = (5x - 3)/4$, then $f^{-1}(x)$ = ________________.

**2.** Use the Euclidean Algorithm to find GCD(976, 320).

**3.** Find the truth table of the statement $[p \vee \neg(q \wedge r)] \rightarrow \neg q$.

**4.** Find the Disjunctive Normal Form of the Boolean Polynomial, $F(x, y, z) = x'y$.

**5.** Find the negation of: ***For all n ∈ Z, if n is even, then n is divisible by 2.***

**6.** Circle the validity of the following arguments:

**(a)** All dogs like swimming and Rover likes swimming, therefore Rover is a dog. VALID INVALID

**(b)** All dogs like swimming and Rover is a dog, therefore Rover likes swimming. VALID INVALID

**(c)** All dogs like swimming and Rover does not swim, therefore Rover is not a dog. VALID INVALID

**7.** Determine whether the relation R = {($x$, $y$) | $x$ and $y$ are Integers and ( $y/x$) is an Integer} is REFLEXIVE, SYMMETRIC, TRANSITIVE, or none of these.

**8.** A youth group is made up of 12 boys and 10 girls. How many different ways can:

**(a)** they form a line? **(b)** they form a circle?

**(c)** they choose 5 boys and 7 girls if a certain pair of girls cannot be picked together?

**(d)** I have a large collection of pennies, and I want to distribute them into 25 piles so that each pile has at least 3 pennies, and the total value of the piles is $1. How many ways can I do this?

**(e)** How many orderings are there of the letters of the word *SEVENELEVEN* ?

**9.** In a class with 24 children, what is the probability that a child neither plays soccer nor is on the honor roll if 10 children play soccer, 6 children are on the honor roll and 4 children both play soccer and are on the honor roll?

**10.** Find the best polynomial order for the algorithm with complexity $(n^4 \log n + 2n^4)(n^4 + 2n^5)$.

**11.** Prove one of the two Theorems using either direct or indirect arguments.

Theorem 1: If $a$ and $b$ are odd integers, then $(a^2 + b^2)$ is even.

Theorem 2: If $n^2$ being even implies $n$ is even, then $\sqrt{2}$ is irrational

**12.** Prove one of the two Theorems below using Mathematical Induction.

Theorem 1: For all integers $n \geq 0$ and $a \neq 0,1$, $\displaystyle\sum_{i=0}^{n} 5^i = \frac{5^{n+1} - 1}{4}$.

Theorem 2: If $a_0 = 5$, $a_1 = 10$, and $a_2 = 15$, then $a_n = a_{n-1} + a_{n-2}$ is divisible by 5, for all integers $n \geq 3$.