1. Circle T if the corresponding statement is True or F if it is False.

   - GCD(1,0) = 0.
   - For every recursive algorithm, there is an equivalent sequential algorithm.
   - In general, sequential algorithms use memory less efficiently than their equivalent recursive representations.
   - If $a$ and $b$ are positive integers, then $a = b(a \text{ div } b) + (a \mod b)$.
   - $1 + 2 + 4 + 8 + 16 + \ldots + 2^{24} = 2^{25} + 1$.
   - Algorithms whose order is $O(n \log n)$ are less efficient than those of order $O(n^2)$.
   - Given two algorithms, $f(n)$ and $g(n)$, $O(f + g) = \max(O(f), O(g))$.
   - GCD($p,q$) = GCD($q,p \mod q$).

2. Let \{a_n\} and \{b_n\} be the sequences defined, for $n \geq 0$, by:

   \[ a_n = (n + 1), \text{ and } b_n = (n + 2). \]

   Find $c_1, c_2, c_3$, and $c_4$ when $c_n = (a_n)(b_n) + n$.

3. Write out the Division Algorithm and trace its steps to calculate (54 MOD 12).

4. (a) Give a Recursive Definition for the even Natural numbers.

(b) Find the Big-Oh of the algorithm with complexity: $(n^3 + 3)(n^6 + 6) + [3n^3 + 2n^2(\log n)]$.

5. (a) Given $a = 2^3 3^0 5^2 7^4 11^0 13^1 17^3 19^0 23^0 29^9 31^2$.

   and $b = 2^1 3^2 5^2 7^0 11^1 13^3 17^0 19^1 23^2 29^1 31^4$, then

   \[ \text{GCD}(a,b) = \quad \text{and LCM}(a,b) = \quad \]

(b) List out the search intervals in applying the Binary Search algorithm to find 14 in the list:

   0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

   Pass 1:
   Pass 2:
   Pass 3:
   Pass 4:

6. Prove ONE of the TWO Theorems below using Mathematical Induction.

   **Theorem 1**: For all integers $n \geq 1$,

   \[ \sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6}. \]

   **Theorem 2**: If $a_0 = 5, a_1 = 10$, and $a_2 = 15$, then $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ is a multiple of 5, for all $n \geq 3$.

7. Prove ONE of the TWO Theorems below:

   **Theorem 1**: If $a$, $b$, and $m$ are positive integers, with $(a \mod m) = (b \mod m)$, then $a \equiv b \mod m$.

   **Theorem 2**: The difference of the squares of successive positive integers is odd.

8. Prove ONE of the TWO Theorems below by Contradiction or Contraposition.

   **Theorem 1**: There is no largest Natural number.

   **Theorem 2**: If $n$ is an integer and $n^2$ is odd, then $n$ is odd.