Fall 2007 - Discrete Structures - CMSC 203 - FINAL EXAM

1. Fill in the blank(s).

(a) $\dfrac{(39 \text{ div } 6)[\text{GCD}(1260,324)]}{39 \bmod 6}$ = ______. (Hint: Do problem 2 first!)

(b) $(\{1,3,4,7,9\} - \{2,3,4,5\}) \cup (\{4,5,6,7\} \cap \{1,2,3,4,5,6\})$ = ____________________

(c) H(1110001,1001001) = _________ (H is the Hamming Distance Function)

(d) If $\Sigma = \{x,y,z\}$, then $|\Sigma^4|$ = _________.

(e) DeMorgan's Law states that for any sets A and B, $(A \cap B)^c$ = ____________________________________.

(f) For finite sets A and B, the Inclusion/Exclusion Rule says:

$|A \cup B|$ = _________ + _________ – ______________.

(g) If $\Sigma = \{0,1\}$, then $\Sigma \times \Sigma$ = ____.

(h) If $\Sigma = \{0,1\}$, then Σ^2 = ____.

(i) Let $F:\{1,2,3,4\} \to \{a,b,c,d\}$ be a one-to-one correspondence. Then $F = \{(1,b),(2,d),(3,a),$_______$\}$

(j) If $G:\{x,y,z\} \to B$ is onto, and $G = \{(x,1),(y,2),(z,0)\}$, then B = _____________.

(k) If $s_n = 3s_{n-1} + 4s_{n-2}$ with $s_0 = 0$ and $s_1 = 1$, then s_4 = _________.

(l) If $f:\mathbf{R} \to \mathbf{R}$ is defined by $f(x) = (2x - 7)/3$, then $f^{-1}(x)$ = ________________.

2. Use the Euclidean Algorithm to find GCD(1260, 324).

3. Find the truth table of the statement $[p \vee (q \wedge \sim r)] \to \sim q$.

4. Find the Disjunctive Normal Form of the Boolean Polynomial, $f(x,y,z) = xy'$.

5. Find the negation of: ***For all n ∈ Z, if n is prime, then n is not divisible by 2***

6. Circle the validity of the following arguments:

(a) All dogs like swimming and Rover likes swimming, therefore Rover is a dog. VALID INVALID

(b) All dogs like swimming and Rover is a dog, therefore Rover likes swimming. VALID INVALID

(c) All dogs like swimming and Rover does not swim, therefore Rover is not a dog. VALID INVALID

7. Determine whether the relation $R = \{(x, y) \mid x, y$ are Integers and x divides $y\}$ is REFLEXIVE, SYMMETRIC, TRANSITIVE, or none of these.

8. A youth group is made up of 22 boys and 30 girls. How many different ways can:

(a) they form a line?
(b) they form a circle?

(c) they pick 5 boys and 7 girls to race each other if a certain pair of boys cannot be picked together?

(d) I have a large collection of $1 coins, and I want to distribute them into 10 piles so that each pile has at least 3 coins, and the total value of the piles is $50. How many ways can I do this?

(e) How many orderings are there of the letters of the word *ORGANIZATION* ?

9. In a class with 24 children, what is the probability that a child does not play soccer and is not on the honor roll if 14 children play soccer, 12 children are on the honor roll and 6 children both play soccer and are on the honor roll?

10. Find the best polynomial order for the algorithm with complexity $(n^3 \log n + 3n^2 + 1)(n + \log n)$

11. Prove: If a and b are *distinct* integers then there is a rational number between them.

12. Prove: The product of a rational number and an irrational number is irrational.

13. Prove using Mathematical Induction: $\displaystyle\sum_{i=0}^{n} 7^i = \frac{7^{n+1} - 1}{6}$