

## SAMPLE Final Exam - Fall 2006 - Discrete Structures

1. (a) Use the Laws of Logic to show the POSITIVE and CONTRAPOSITIVE forms of a conditional statement are equivalent.

(b) Find the inverse of the following Universal Conditional:

**All people who like Mathematics get good jobs.**

(c) Use the rules of inference to show the following is a valid argument:

I am rich implies I am happy.

My job is fun or I am not happy.

I am rich.

I ride a bicycle implies my job is not fun.

$\therefore$  I do not ride a bicycle

2. (a) Find  $A \times B$  for the sets  $A = \{1,11\}$  and  $B = \{111, 1111, 11111\}$

(b) Using the Properties of Sets, show for any sets A, B and C,  $(A \cap B) - C = (A - C) \cap (B - C)$ .

3. (a) For  $F = \{(3,1), (2,4), (7,3), (4,7), (5,5)\}$ , what Domain and Image make F a bijection?

(b) What is the inverse function of F in part (a)?

(c) Find  $F(G(x))$  for the following Real-valued functions:  $G(x) = 2^{(x+2)}$  and  $F(x) = x - 2$ .

4. (a) Use the Euclidean Algorithm to find  $\text{GCD}(255, 75)$ .

(b) Find Big-O of the algorithm whose complexity is  $F(n) = (3n^5)(n^6 + 5n^7)(2n^4 + 3n + 2)$ .

5. Prove 1 of the 2 Theorems below:

**Theorem 1:** If the cube of an integer is odd, then the integer is odd.

**Theorem 2:** The square root of an irrational is irrational.

6. (a) Find the next 3 terms of the sequence  $s_n = (s_{n-1})(s_{n-2}) - (n^2)$  when  $s_0 = 1$  and  $s_1 = 1$ .

(b) Evaluate the series:  $\sum_{i=0}^{10000} 4^i - 2i$ .

7. Prove 1 of the 2 Theorems that follow by Mathematical Induction.

**Theorem 1:** A set with  $n$  elements has  $2^n$  subsets.

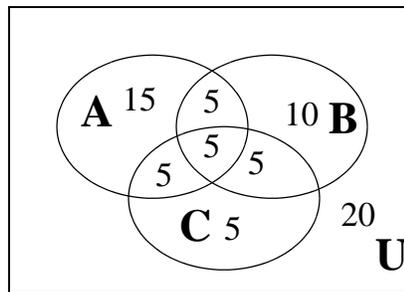
**Theorem 2:** Is  $s_n = s_{n-1} + s_{n-2} + s_{n-3} + s_{n-4}$  when  $s_0 = s_1 = s_2 = s_3 = 7$  then 7 divides  $s_n$ , for all  $n \geq 4$ .

8. (a) If a state issues license plates using the 36 characters  $\{A, B, C, \dots, Z, 0, 1, \dots, 9\}$ , how many distinct plates of 7 letters can they create that begin with C or end with G3?

(b) How many plates can they create if no plate can contain the string LOST?

(c) How many distinct piles of 700 coins (pennies, nickels, dimes, quarters, half-dollars, and dollars) can I create from a vast quantity of coins, if I must have at least 80 of each coin in every pile?

9. Consider the following sets with corresponding number of elements indicated in each region:



(a) Find  $P(B \cap C)$

(b) Find  $P(A | (B \cup C))$

10. If UMBC creates a database of information using the fields:

Student ID Number, Name, Address, Phone, Drivers License Number, Major, GPA  
which would likely be Primary Keys?

11. (a) Let  $R = \{(a, b) \mid a, b \in \{1, 2, 3, 4, 5, 6\} \text{ and } a \equiv b \pmod{3}\}$ . List the ordered pairs in R.

(b) Let  $\Sigma$  be the alphabet  $\{0, 1\}$ , and define the relation R on  $\Sigma^3$  to be such that string,  $s$ , relates to string,  $t$ , if their densities have the same parity. Show R is an Equivalence Relation.

(Note:  $\text{Parity}(s) = d(s) \pmod{2}$ )

(c) What partition of the  $\Sigma^3$  does R induce?

12. (a) Find the Disjunctive Normal Form of the polynomial Boolean Polynomial

$$F(x, y, z) = z' + x'yz$$

(b) Find the truth table of the polynomial in (a).