SAMPLE Exam 2 - Fall 2006 - Discrete Structures

1. Circle **T** if the corresponding statement is True or **F** if it is False.

T **F** The sequence {1, –1, 1, –1, 1, –1,...} is an example of an alternating sequence.

T **F** The Principle of Mathematical Induction can be used to discover new theorems.

T **F** A recursive algorithm is generally more efficient in memory than its equivalent iterative version.

T **F** 3 + 6 + 9 + 12 + ... + 300 = 15150.

T **F** Algorithms that are Exponential order grow faster than ones that are Polynomial order.

T **F** In costing algorithms, the sum of the orders equals the maximum order.

T **F** The Fibonacci sequence only requires one initial condition.

T **F** In general, Linear Search algorithms are less efficient than Binary Search algorithms.

2. Let $\{a_n\}$ and $\{b_n\}$ be the sequences defined, for $n \geq 0$, by:

$a_n = n - 2$, and $b_n = 3n + 1$. Find c_0, c_1, c_2, and c_3 when $c_n = (a_n + b_n)$.

3. Given positive integer inputs, A and B, write out in pseudocode the algorithm to calculate (A MOD B) and (A DIV B). You may assume B < A.

4. (a) Rank from 1 (least complex) to 5 (most complex) the following orders:

___ 10^n ___ n^2 ___ n ___ n^n ___ n^{10}

(b) Find the Big-Oh of the algorithm with complexity: $(n^6 + 1)(2n^2 + 3n) + (n^3 + 5n^2 + 2)(n^3)$.

5. Use the Euclidean Algorithm to find GCD(100, 22).

6. Prove one of the two Theorems below using Mathematical Induction.

Theorem 1: For all integers $n \geq 0$ and $a \neq 0,1$, $\sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a - 1}$.

Theorem 2: If $a_0 = 0$ and $a_1 = 100$, then $a_n = a_{n-1} + a_{n-2}$ is divisible by 100.

7. Prove one of the two Theorems below:

Theorem 1: The product of odd integers is odd.

Theorem 2: If (a MOD p) = (b MOD p), then p divides $(a - b)$.

8. Prove one of the two Theorems below by Contradiction.

Theorem 1: $\sqrt{2}$ is irrational.

Theorem 2: There does not exist a largest integer.