CMSC 203 - Fall 2005 - Exam 2

1. Circle **T** if the corresponding statement is True or **F** if it is False.

T **F** The sequence {1, 1, 2, 3, 5, 8, 13, 21,...} satisfies the Fibonacci relation.

T **F** The First (Weak) Principle of Mathematical Induction implies the Second (Strong) Principles of Mathematical Induction.

T **F** A recursive algorithm is one which never terminates.

T **F** 2 + 4 + 6 + 8 + ... + 200 = 10100.

T **F** Functions that are Logarithmic order grow faster than functions that are Linear order.

T **F** In costing algorithms, the sum of the orders equals the maximum order.

T **F** In costing algorithms, the product of the orders equals the order of the product.

T **F** In general, Linear Search algorithms are just as fast Binary Search algorithms.

2. Let $\{a_n\}$ and $\{b_n\}$ be the sequences defined, for $n \geq 0$, by:

$a_n = n^2 - 2$, and $b_n = n^3 + 2$. Find c_0, c_1, c_2, and c_3 when $c_n = (a_n)(b_n)$.

3. Given positive integer inputs, A and B, write out in pseudocode the algorithm to calculate (A MOD B) and (A DIV B). You may assume B < A.

4. Rank from 1 (least complex) to 10 (most complex) the complexity of algorithms with the following orders:

Order:	2^n	$\log n$	$n!$	n^2	1	n	n^n	$n\log n$	n^{10}	10^n
Rank:										

(b) Find the Big-Oh of the algorithm with complexity: $(n^4 + 1)(2n + n^2) + (3n^2 + 4n^4 + 5)(n^2)$.

5. Use the Euclidean Algorithm to find GCD(1100, 35).

6. Prove one of the two Theorems below using Mathematical Induction.

<u>Theorem 1</u>: For all integers $n \geq 0$ and $a \neq 0,1$, $\sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a - 1}$.

<u>Theorem 2</u>: If $a_0 = 0$ and $a_1 = 10$, then $a_n = a_{n-1} + a_{n-2}$ is divisible by 10.

7. Prove one of the two Theorems below:

<u>Theorem 1:</u> If an integer divides two other integers, then it divides their sum.

<u>Theorem 2:</u> If a and b are odd, then $(a^2 + b^2)$ is even.

8. Prove one of the two Theorems below by Contradiction.

<u>Theorem 1:</u> If a prime divides an integer, then it cannot divide the integer's successor.

<u>Theorem 2:</u> There does not exist a largest integer.