**CMSC 203 Fall 2004 Final Examination**

**1.** Use the Laws of Logic to show: $p \rightarrow \neg(q \wedge \neg r) \equiv (p \wedge q) \rightarrow r$

**2**. Negate: **Some integers are perfect squares and odd.**.

**3**. Find the Contrapositive form of: **All dogs that run fast have long legs**.

**4.** Find the Big-O of the algorithm with complexity:

$$(3x^6 + 4x^3 + 1)(x^4 + 2x) + (3x^7 + 7x^3 + 2)(2x^3).$$

**5**. What is the probability that a binary string of length 10 will have no more than three 1's?

**6**. How many ways can I fill a cooler with cans of soda if the cooler holds 60 cans, I have 8 different types of soda, and I want at least 4 of each type in the cooler?

**7**. Graph the relation R = $\{(a,b) \mid a,b \in \{0, 1, 2, 3, 4, 5, 6, 7\}$ and $b = [(a^2 - 2) \bmod 5]\}$.

**8**. Find the matrix ($M_R$ o $M_R$) of the relation on {1, 2, 3}:

$$R = \{(1,1), (1,3), (2,1), (2,3), (3,1), (3,2), (3,3)\}$$

**9**. Let $f = \{(1,3), (2,2), (3,4), (4,5), (5,1)\}$, $g = \{(1,4), (2,5), (3,1), (4,2), (5,3)\}$, and $h = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$. Find $h \circ g \circ f$.

**10.** Find the Disjunctive Normal Form of the Boolean Expression that describes a 3-way light switch controlling a lightbulb that INCLUDES the case of being ON when the first two switches are ON and the third is OFF.

**11**. Find the next 5 terms of $s_n = 2s_{n-1} - 3s_{n-2}$ when $s_0 = (-1)$ and $s_1 = 1$.

**12**. How many ways can I line up 5 Pennies, 2 Nickels, 4 Dimes, 3 Quarters, and 6 Half-dollars, if all the coins are from the same year? (e.g. HQHDNDHQHDNQDHH)

**13**. Let E and F represent event sets within the sample space S. If there is a 50% probability of E occurring within F and there is a 50% probability of F occurring within S, show the E is independent of F.

**14**. Graph an examples of:
**(a)** an Onto function that is NOT One-To-One. **(b)** a One-To-One function that is NOT Onto.

**15**. Prove one of the following theorems by Induction:

Theorem: For all Integers $n \geq 0$ and $a \neq 0,1$, $\sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a - 1}$.

Theorem: If $a_1, a_2, a_3, \ldots$ is the sequence: $a_0 = 3$, $a_1 = 5$, $a_2 = 7$ with $a_n = a_{n-1} + a_{n-2} + a_{n-3}$, then $a_n$ is odd for all $n \geq 3$.

**16**. Prove one of the following theorems by Contradiction:

Theorem: $\sqrt{2}$ is irrational.

Theorem: If every integer is divisable by a prime, then the set of primes is infinite.

**17**. Prove one of the following theorems:

Theorem: If $f(x) = 4x + 8$, then $R = \{(x,y) \mid x,y \in \mathbf{R}$ and $f(x) = f(y)\}$ is an Equivalence Relation.

Theorem: If $f(x) = 4x + 8$, then $f$ is a Bijection from $\mathbf{R}$ to $\mathbf{R}$.