Exam 1             CMSC 203            Fall 2004

1. Circle T for True or F for False as they apply to the following statements:
   T  F  In logic, all compound statements are either a tautology or a contradiction.
   T  F  The Rational numbers are all infinite decimal expansions.
   T  F  A set with 10 elements has 1024 subsets.
   T  F  Sets that are disjoint have no elements in common.
   T  F  H(010011000111, 110110110110) = d(100101110001).
   T  F  The Rationals are closed under addition and multiplication.
   T  F  The Irrationals are closed under addition and multiplication.
   T  F  If there is an ONTO function mapping set A to set B, then B is a subset of A.
   T  F  Algorithms of polynomial order complexity are better than those of exponential
        order complexity.
   T  F  That all dogs like bones and Spot likes bones implies Spot is a dog.

2. Use the Laws of Logic to show:  \((p \land r) \rightarrow q \equiv p \rightarrow (r \rightarrow q)\)

3. Find the inverse for the Universal Conditional Statement:
   Every integer that is even has a square that is even.

4. Let \(f\) be function defined as: \(f = \{(0, 3), (1, 2), (2, 4), (3, 1), (4, 0)\}\). Find \(f \circ f \circ f\).

5. Let \(A = \{1, 2, 4\}\) and \(B = \{0, 1, 2, 3, 4\}\) and \(U = \{0, 1, 2, 3, 4, 5, 6\}\)
   (a) Find \(A \times (B^C)\)  (b) Find the Power Set of A  (c) Find \((A \cup B)^C \cap (A^C - B)\)

6. Find the Big-O for the algorithm with complexity: \(n^4(3n^3 + 5) + n^5(2n^2 + 8n)\)

7. Show that the function \(f: \mathbb{R} - \mathbb{R}\) given by \(f(x) = 7x + 4\) is a bijection.

8. Use Valid Reasoning to obtain the given conclusion:
   \[q \lor \neg r \lor t\]
   \[s \rightarrow p\]
   \[r \land s\]
   \[q \rightarrow \neg p\]
   \[
   \begin{align*}
   \therefore t
   \end{align*}
   \]

9. Prove TWO of the THREE theorems below:
   **Theorem 1:** (by Contradiction) There is no greatest integer.
   **Theorem 2:** The product of 3 consecutive integers is an even integer.
   **Theorem 3:** (by Contaposition) For all integers, \(n\), if \(n^2\) is odd, then \(n\) is odd.