Discrete Structures - Examination 2 - Fall 2001

Name _____ Show All Work!

1. Circle **T** of the corresponding statement is True and **F** if it is False:

T **F** $1 + 2 + 3 + 4 + ... + 1{,}000 = 500{,}500$

T **F** If A is a non-empty set, and R is an Equivalence Relation on A, and $a,b \in$ A, then $(a,b) \in$ R.

T **F** If f:X $\rightarrow$ Y is a function and f(X) = Y, then f is an ONTO function.

T **F** If f:X $\rightarrow$ Y is a ONE-TO-ONE function then | X | = | f(X) |.

T **F** If f:X $\rightarrow$ Y is a function and X has 10 elements, then f has 10 elements.

T **F** If A is a set with 10 elements, then the *smallest* Equivalence Relation on A has 10 elements.

T **F** If f and g are functions with f:X $\rightarrow$ Y and g:Y $\rightarrow$ Z, then $f \circ g = g \circ f$.

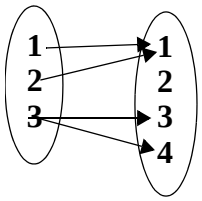
T **F** If f:X $\rightarrow$ Y is a function and i_Y:Y $\rightarrow$ Y is the identity function on Y, then $(i_Y \circ f) = f$.

T **F** $1 + 10 + 100 + 1000 + ...+10^{1000} = 10^{1001} - 1$.

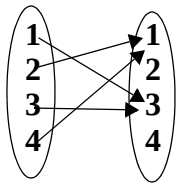
T **F** If R is the Equivalence Relation on the Integers, Congurence Mod 7, then [23] = [1423].

2. Circle **F** for function, **I** for one-to-one, **S** for onto, and **B** for one-to-one correspondence as the properties apply to the relations below. Circle the BEST choice for each relation, i.e. only 1 answer for each.

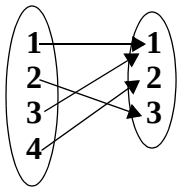
a. F I S B



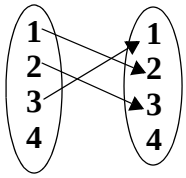
b. F I S B



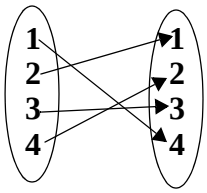
c. F I S B



d. F I S B



e. F I S B



3. Write $\frac{9^6}{21} + \frac{10^7}{24} + \frac{11^8}{27} + \ldots + \frac{40^{37}}{114}$:

a. as a summation in ***i*** ranging from ***i*** = 7 to 38 **b.** as a summation in ***j*** ranging from ***j*** = 9 to 40.

4. Let $\Sigma = \{0,1\}$ and let H be the Hamming distance function on binary strings. Consider the relation:

$$R = \{(s,t) \mid s,t \in \Sigma^4 \text{ and } H(s,1010) = H(t,1010)\}.$$

a. Prove that R is an Equivalence Relation. **b.** What partition of Σ^4 does the relation R induce?

5. Let f:**R** $\rightarrow$ **R** be the function $f(x) = y = 11x + 25$. **a.** Show that that f is 1-1 and onto. **b.** Find $f^{-1}(x)$.

6. Let $f = \{(1,b),(2,a),(3,d),(4,e),(5,c)\}$ and let $g = \{(a,8),(b,6),(c,4),(d,2),(e,0)\}$.
Show $(g \circ f)^{-1} = (f^{-1} \circ g^{-1})$.

7. Prove 1 of the following 2 statements:

a. There exists a 1-1 correspondence between A = $\{y \mid y = 3x + 2$, for $x \in$ **N**$\}$ and B = $\{y \mid y = 2x + 3$, for $x \in$ **N**$\}$.

b. Using Weak Induction, show that $1 + 2 + 2^2 + 2^3 +...+ 2^n = 2^{n+1} - 1$.