1. (20 pts.) Circle T for True or F for False as they apply to the following statements:

T (F) In logic, all compound statements are either a tautology or a contradiction.

T (F) The Rational numbers are all infinite decimal expansions.

T (F) A set with 10 elements has 1024 subsets.

T (F) Sets that are disjoint have no elements in common.

T (F) \( H(010011001111, 110110110110) = d(100101110001) \).

T (F) The Rationals are closed under addition and multiplication.

T (F) The Irrationals are closed under addition and multiplication.

T (F) If there is an ONTO function mapping set A to set B, then B is a subset of A.

T (F) Algorithms of polynomial order complexity are better than those of exponential order complexity.

T (F) That all dogs like bones and Spot likes bones implies Spot is a dog.

2. (10 pts.) Use the Laws of Logic to show: \((p \land r) \rightarrow q \equiv p \rightarrow (r \rightarrow q)\)

\[
(p \land r) \rightarrow q \equiv \neg (p \land r) \lor q
\equiv \neg p \lor \neg r \lor q
\equiv \neg p \lor (\neg r \lor q)
\equiv \neg p \lor (r \rightarrow q)
\equiv p \rightarrow (r \rightarrow q).
\]
3. (5 pts.) Find the inverse for the Universal Conditional Statement:
Every integer that is even has a square that is even.
\[ \forall x \in \mathbb{Z}, \text{even} \rightarrow x^2 \text{ even} \]
\[
\implies \forall x \in \mathbb{Z}, x^2 \text{ even} \rightarrow \text{not even.}
\]
OR
\[
\forall x \in \mathbb{Z}, \text{odd} \rightarrow x^2 \text{ odd.}
\]

4. (5 pts.) Let \( f \) be a function defined as: \( f = \{(0, 3), (1, 2), (2, 4), (3, 1), (4, 0)\} \). Find \( f \circ f \circ f \).

5. (15 pts.) Let \( A = \{1, 2, 4\} \) and \( B = \{0, 1, 2, 3, 4\} \) and \( U = \{0, 1, 2, 3, 4, 5, 6\} \)
(a) Find \( A \times (B^c) \)
\[
\{1,2,4\} \times \{0,1,2,3,4\}^c = \{1,2,4\} \times \{5,6\}
\]
\[
= \{(1,5), (1,6), (2,5), (2,6), (4,5), (4,6)\}
\]
(b) Find the Power Set of \( A \)
\[
\mathcal{P}(\{1,2,4\}) = \{\emptyset, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{1,2,4\}\}
\]
(c) Find \( (A \cup B)^c \cap (A^c - B) \)
\[
= \left( \{0,1,2,3,4\} \cup \{0,1,2,3,4\} \right)^c \cap \left( \{1,2,4\}^c - \{0,1,2,3,4\} \right)
\]
\[
= \{0,1,2,3,4\}^c \cap \{0,3,5,6\} - \{0,1,2,3,4\}
\]
\[
= \{5,6\} \cap \{5,6\}
\]
\[
= \{5,6\}
\]
6. (5 pts.) Find the Big-O for the algorithm with complexity: \( n^4(3n^3 + 5) + n^2(2n^2 + 8n) \)
\[
= n^4 \left( 3n^3 + 5 + 2n^2 + 8n \right)
\]
\[
= 3n^7 + 5n^4 + 2n^7 + 8n^6
\]
\[
= (3+2)n^7 + 8n^6 + 5n^4 \leq 5n^7 + 8n^6 + 5n^4 = 18n^7.
\]
Thus, the algorithm is \( O(n^7) \).

7. (10 pts.) Show that the function \( f : \mathbb{R} \to \mathbb{R} \) given by \( f(x) = 7x + 4 \) is a bijection.
\[
\text{Show } f \text{ is } 1-1 \text{ and onto.}
\]
\[
\text{1-1: Let } a, b \in \mathbb{R} \Rightarrow f(a) = f(b), \text{ show } a = b.
\]
Now, \( f(a) = f(b) \Rightarrow 7a + 4 = 7b + 4 \Rightarrow 7a = 7b \Rightarrow a = b. \)
Thus, \( f \) is 1-1.
\[
\text{Onto: Let } y \in \mathbb{R}, \text{ show } \exists x \in \mathbb{R} \text{ s.t. } f(x) = y.
\]
Now, \( y \in \mathbb{R} \Rightarrow (y-4) \in \mathbb{R} \Rightarrow (y-4)/7 \in \mathbb{R}. \) Denoting \( x = \frac{y-4}{7} \),
we see that \( f(x) = f \left( \frac{y-4}{7} \right) = \frac{7(y-4)}{7} + 4 = y-4 + 4 = y. \)
Thus, \( f \) is onto.

8. (10 pts.) Use Valid Reasoning to obtain the given conclusion:
\(
\begin{align*}
1. & \quad r \land S \\
2. & \quad S \rightarrow p \\
3. & \quad q \rightarrow \neg P \\
4. & \quad q \land \neg v \land t \\
5. & \quad t \land v \land t
\end{align*}
\)
\[
\begin{align*}
& q \rightarrow \neg v \\
& r \rightarrow s \\
& p \land s \\
& q \rightarrow \neg p \\
& \therefore \quad r
\end{align*}
\]
\[
\begin{align*}
& q \rightarrow \neg v \\
& \therefore \quad q \rightarrow \neg v \\
& \therefore \quad t, \neg v, \text{ QEO}
\end{align*}
\]
9. (20 pts.) Prove TWO of the THREE theorems below:

**Theorem 1:** (by Contradiction) There is no greatest integer.

**Theorem 2:** The product of 3 consecutive integers is an even integer.

**Theorem 3:** (by Contrapositive) For all integers, n, if \( n^2 \) is odd, then \( n \) is odd.

**Theorem 1:** Proof (Contradiction) Assume there is a greatest integer, call it \( M \).

Now, since \( M \in \mathbb{Z} \), we see that \( (M+1) \in \mathbb{Z} \). Also, since \( M \) is the greatest integer, we conclude that

\[
M + 1 \leq M.
\]

Subtracting \( M \) from both sides, we conclude that \( 1 \leq 0 \times \).

Therefore, there is no greatest integer. \( \Box \)

**Theorem 2:** Proof: Let \( a \) be an integer. Show \( a(a+1)(a+2) \) is even.

Now, since \( a \in \mathbb{Z} \), \( a \) is even or \( a \) is odd.

**Case 1:** \( a \) is even. Thus, \( \exists k \in \mathbb{Z} \), \( a = 2k \).

Now, \( a(a+1)(a+2) = 2k(2k+1)(2k+2) = 4k(k+1)(k+1) \).

Since \( k \in \mathbb{Z} \), we see that \( [k(k+1)(k+1)] \in \mathbb{Z} \). Therefore, \( a(a+1)(a+2) \) is even.

**Case 2:** \( a \) is odd. Thus, \( \exists k \in \mathbb{Z} \), \( a = 2k+1 \).

Now, \( a(a+1)(a+2) = (2k+1)(2k+2)(2k+3) = 2[(2k+1)(k+1)(2k+3)] \).

Since \( k \in \mathbb{Z} \), we conclude that \( [2k+1)(k+1)(2k+3)] \in \mathbb{Z} \). Therefore, \( a(a+1)(a+2) \) is even. \( \Box \)
Theorem 3

Proof (Contraposition) Show $\forall n \in \mathbb{Z},$ if $n$ even implies $n^2$ even.

Let $n \in \mathbb{Z}$ be even. Thus $\exists k \in \mathbb{Z}$ such that $n = 2k$.

Now, $n^2 = (2k)^2 = 4k^2$. Since $k \in \mathbb{Z}$, we see

that $k^2 \in \mathbb{Z}$ and hence, $(2k^2) \in \mathbb{Z}$. Therefore, for

$n$ even, we can conclude that $n^2$ is even. \[QED\]