CMSC 203 - Homework Assignment 4 - Due November 14, 2002

1. Show algebraically that $\binom{25}{12} = \binom{24}{12} + \binom{24}{11}$
2. Show that the sequence \( \{a_n\} \) generated by \( a_n = 5n + 3 \) satisfies the recurrence relation:
\[
a_n = 2a_{n-1} - a_{n-2}.
\]
3. Find the general solution for the linear, homogeneous recurrence relation:

\[ a_n = 12a_{n-1} - 36a_{n-2} \text{ for } n > 1. \]
4. Find the Particular Solution for the linear, homogeneous recurrence relation whose general solution is:

\[ a_n = 3(2^n) - 2(5^n) \]

subject to the initial conditions \( a_0 = 1 \) and \( a_1 = -4 \).
5. Draw the directed graph of the relation \( R \) on \( A = \{1,2,3,4,5,6,7,8\} \) defined as
\[ R = \{(a,b) \mid a,b \in A \text{ and } a \equiv b \mod 3\}. \]
6. Let $F$ be a function on the integers.
(a) Show that the relation $R = \{(x,y) \mid x,y \text{ are integers and } F(x) = F(y)\}$ is a Reflexive, Symmetric, and Transitive relation.

(b) Describe the partition of the integers induced by $R$. 