CMSC 435 / 634 **Introduction to Computer Graphics** **Spring 2016**
March 30. 2016 2:30-3:45pm

- Print your name on the cover of the exam booklet. If you need extra space, write "on the second booklet" on the last page of the first booklet and staple them together when you hand it in.
- Clearly label the problem number on the booklet(s)
- Carefully read each question before answering it
- **There are 3 pages 4 questions 100 points total in this exam.**
- **There are 20 point extra credit question. There is no partial credit for this question.**

Strategy: Read through the entire question before you begin it. If you get bogged down on a question, go on and come back later. Even if you don't think you know the entire answer to a question, do what you can in order to get partial credit. If something isn't clear to you, ask.

Part 1: Odds, Ends, and Concepts (15 points)

1. Please answer the following questions.

1. (2 point) What is the most amusing bug you've written in this class?

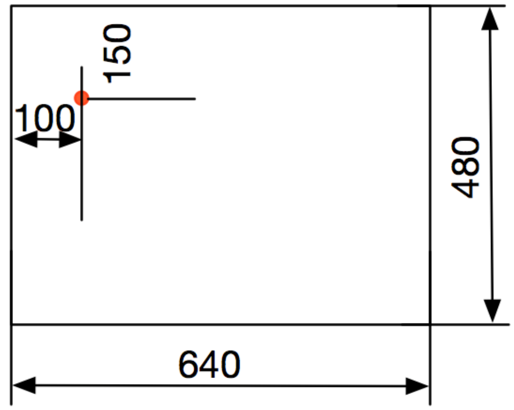
For me Dr. Chen, it took me a day to figure out that glFloat was different from float. When I did not cast the type from float to glFloat, nothing would be shown on the screen. I always pay attention to all warnings in OpenGL code since then.

(You will get full credits if you write down something here.)

2. (2 point) What is a pixel?

A pixel is a physical point in a raster image or the smallest addressable element in a display device.

3. (3 points) Where is the point (100, 150) going to appear on a screen with a resolution of (640, 480) assuming the default OpenGL (0, 0) coordinate system? You can answer these questions by drawing the screen coordinates.



4. (5 points) Two vectors can define a plane. Given two vectors, v_1 (x_1, y_1, z_1) and v_2 (x_2, y_2, z_2), what is the normal direction of the plane defined by these two vectors?

$$V1 \times V2 = \begin{bmatrix} i & j & k \\ x1 & y1 & z1 \\ x2 & y2 & z2 \end{bmatrix} = (y1z2 - y2z1)\, i + (x2z1 - x1\, z2)\, j + (x1y2 - x2y1)\, k$$

therefore, the normal direction of the plane is

$$\left(\frac{y1z2 - y2z1}{L}, \quad \frac{x2z1 - x1\, z2}{L}, \quad \frac{x1y2 - x2y1}{L} \right),$$

where

$$L = \mathrm{sqrt}\,(\,(y1z2-y2z1)\text{^}2 + (x2z1-x1z2)\text{^}2 + (x1y2-x2y1)\text{^}2))$$

5. (3 points) Write a parametric equation using radius, and two angles, for a sphere at the origin (x, y, z) with radius of 2.5.

x = 2.5 cos(alpha) sin (beta)
y = 2.5 sin(alpha) sin (beta)
z = 2.5 cos(beta)
(Should draw a picture to show the beta and alpha as well.)

Part 2: Problem Solving (85 points)

2. (30 points)

(1) Express the homogeneous 3D transformation defined by the matrix

$$\begin{bmatrix} 0 & -1 & 0 & 2 \\ 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

as a sequence of transformations in the following ways:

- (10 points) A rotation followed by a translation.

  TR =
```
[1 0 0 2] [0 -1 0 0]
[0 1 0 3] [1  0 0 0]
[0 0 1 4] [0  0 1 0]
[0 0 0 1] [0  0 0 1]
```

- (10 points) A translation followed by a rotation.

  RT =

```
[0 -1 0 0] [1 0 0  3]
[1  0 0 0] [0 1 0 -2]
[0  0 1 0] [0 0 1  4]
[0  0 0 1] [0 0 0  1]
```

(2) (10 points) Write the OpenGL code for performing the transformation in (1) using glRotatef and glTranslatef

A rotation followed by a translation:

```
glPushMatrix();
glTranslatef(2, 3, 4);
glRotatef(90, 0, 0, 1);
DrawSomethingHere();
glPop();
```

A translation followed by a rotation

```
glPushMatrix();
glRotatef(90, 0, 0, 1);
glTranslatef(3, -2, 4);
DrawSomethingHere();
glPop();
```

3. (30 points) Here is an unedited photograph of two normal-sized people:

The image above is 450 pixels high, and the two heads measure 90 and 15 pixels high. Assume all heads are 30 cm high. The image on the camera's film plane is 24 mm high.

(1) If I know that the person in the foreground is 2 meters from the camera, what is the camera's image plane distance (the focal length) and how far away is the other person?

This problem tests whether or not you understand the coordinate systems in computer graphics. An object can be described in three ways: (1) # of pixels appeared on your computer screen (here 90 pixels for the front person's head), (2) the size of the object on the camera plane (here 24mm), and (3) the physical size of an object (here 30cm). These numbers represent exactly the same object (the front person) but are in different coordinate systems.

Now back to the question,

*At the distance of the close person, 90 pixels correspond to 0.3 m. At the same distance, 450 pixels correspond to 1.5m. So the image is 1.5m high at 2m – a ratio of 3:4. When the image is 24 mm high it is at a distance of (4:3) * 24 = 32 mm. This is the image plane distance. The second person is 6 times the distance of the first person, so 12 meters.*

(2) If I know that the two people are standing 20 meters apart, what is the image plane distance and how far from the camera is the closer person?
(Photo courtesy of Seth Teller, who says, "no computers were used to make this picture.")

Let the distance to the near person be d. The far person is at distance 6d. The distance between the people is then 5d=20 meters. So d = 4 meters. This distance is twice that in the part 1, so the image plane distance is also doubled and it is 64 mm.

4. **(25 points)** In class, we learned about transform curves from the parametric form to the matrix form. One benefit of doing so is that curves in the physical world can be represented

as a set of curve segments subsequently. We studied cubic curve in class and it is possible to use the continuity constraints to drive curve matrix.

Answer the two questions:

(10 points) Please explain the following continuity concepts given the parametric equation $f(t) = a_0 + a_1 t + a_2 * t^2 + a_3 t^3 + \ldots + a_n t^n$. Please describe the concept in plain English and in math equation.

G^0 continuity
C^1 continuity
C^2 continuity

Continuity is the team used in alias to describe how surface patches meet. G^0 continuity means that end point in the first curve is at the same location as the first point in the second curve. Or

$f_1(t=1) = f_2(t=0)$ or $a_{10} + a_{11} + a_{12} + a_{13} + \ldots + a_{1n} = a_{20}$

where in a_{ij}, $_I$ is the curve index and j is the coefficient index in the original equation.

C^1 continuity means that (1) the first derivative of the two curves / patches must be the same and (1) C^0 must be true or

$f_1'(t=1) = f_2'(t=0)$

Here, $f'(t) = a_1 + 2a_2 t + 3a_3 t^2 + \ldots + na_n t^{n-1}$

$f_1'(t=1) = f_2'(t=0)$ or $a_{11} + 2a_{12} + 3a_{13} + \ldots + na_{1n} = a_{21}$

and

$a_{10} + a_{11} + a_{12} + a_{13} + \ldots + a_{1n} = a_{20}$

Do the same for C^2 continuity which means that (1) the second derivative of the two patches / curves must be the same and (2) must satisfy C^1 continuity.

(15 points) Write the matrix representation of the cubic curve under the following constraints. Here q_i (i=[1,4]) are the four control points on the polygon. P_0-P_n are the points on the curve defined by the four control points. V_0 is the velocity or the first order derivative at q_0 and V_1 is the derivative at the end point q_3.

Hint, the third order parametric equation is $f(t) = a_0 + a_1\ t + a_2\ t^2 + a_3\ t^3$. Thus V_t is the derivative of f(t), which is $f'(t) = a_1 + 2\ a_2\ t + 3a_3\ t^2$. When q_0 and p_0 is at t=0 and q_n and p_i is at t=1.

$$\mathbf{p}_0 = \mathbf{q}_0$$
$$\mathbf{p}_1 = \mathbf{q}_3$$
$$\mathbf{v}_0 = 3(\mathbf{q}_1 - \mathbf{q}_0)$$
$$\mathbf{v}_1 = 3(\mathbf{q}_3 - \mathbf{q}_2)$$

$$\mathbf{p}(t) = [t^3 \quad t^2 \quad t \quad 1] \begin{bmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} q0 \\ q1 \\ q2 \\ q3 \end{bmatrix}$$

$f'(t) = a_1\ + 2\ a_2\ t + 3\ a_3\ t^2$

$Q_{(0\text{-}3)}$ are the control points. Ps are the points on the curve.

$q_0 = f(t=0) = a_0$
$q_3 = f(t=1) = a_0 + a_1\ + a_2\ + a_3$
$3(q_1 - q_0) = v_0 = f'(t=0) = a_1$
$3(q_3 - q_2) = v_1 = f'(t=1) = a_1 + 2\ a_2\ + 3\ a_3$

Thus,
$q_0 = f(t=0) = a_0$
$q_1 = a_0 + 1/3\ a_1$
$q_2 = a_0 + 2/3\ a_1\ + 1/3\ a_2$
$q_3 = a_0 + a_1\ + a_2\ + a_3$

Thus
c = [1 0 0 0]
[1 1/3 0 0]
[1 2/3 1/3 0]
[1 1 1 1]

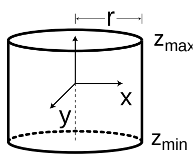
det (c) = 1/9.

$$B = c^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

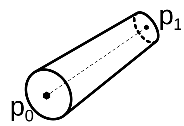
This is exactly how we get the Bezier curves. Please also refer to textbook p367 (Section 15.6.1)

Part 5: Extra credits (20 points). There is no partial credits for this question. You either get 20 or nothing.

How would you build transform functions to create a cylinder of radius r2 between points p0=(x0,y0,z0) and p1=(x1,y1,z1)?



RiCylinder (r, zmin, zmax)



newCylinder(r, x_0,y_0,z_0, x_1,y_1,z_1)

As long as the order is T_2SRT_1 (or T_2RST_1) and the T_1, T_2, and S are written correctly, you will get full points.