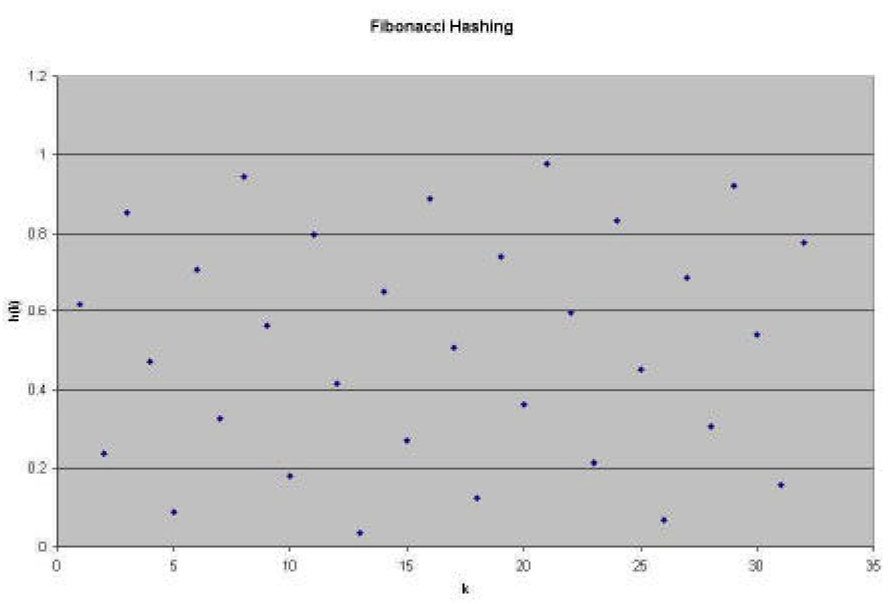
CMSC 341
Lecture 12

Announcements

Postponing Priority Queues until later



HashTable Class

```
template <class HashedObj>
class HashTable {
public:
   explicit HashTable(const HashedObj) &notFound, size=101);
   HashTable(const HashTable &rhs) :
   ITEM_NOT_FOUND(rhs.ITEM_NOT_FOUND),theLists(rhs.theLists){
   }
   const HashedObj &find(const HashedObj &x) const;
   void makeEmpty();
   void insert (const HashedObj &x);
   void remove (const HashedObj &x);
   const HashTable &operator=(const HashTable &rhs);
private:
   vector<List<HashedObj>> theLists;
   const HashedObj ITEM_NOT_FOUND;
};
```

Handling Collisions Revisited

Open addressing (aka closed hashing)

- all elements stored in the table itself (so table should be large. Rule of thumb: M>= 2N)
- upon collision, item is hashed to a new (open) slot.

Hash function

$h: U \times \{0,1,2,\ldots.\} \to \{0,1,\ldots,M-1\}$

$h(k,I) = (h'(k) + f(I)) \bmod m$

for some $h': U \to \{0,1,\ldots,M-1\}$

and $f(0) = 0$

Each try is called a *probe*

Linear Probing

Function:

$f(i) = ci$

Example:

$h'(k) = k \bmod 10$ in a table of size 10 , $f(i) = i$

U={89,18,49,58,69}

Linear Probing (cont)

Problem: Clustering

- when table starts to fill up, performance → O(N)

Asymptotic Performance

- insertion and unsuccessful find, average
  - # probes $\cong 1/2(1+1/(1-\lambda)^2)$
  - if $\lambda \cong 1$, the denominator goes to zero and the number of probes goes to infinity

Linear Probing (cont)

Remove

- Can't just use the hash function(s) to find the object,and remove it, because objects that were inserted after x were hashed based on x's presence.
- Can just mark the cell as deleted so it won't be found anymore.
  - Other elements still in right cells
  - Table can fill with lots of deleted junk

Quadratic Probing

Function:

$f(i) = c_2 i^2 + c_1 i + c_0$

Example:

$f(i) = i^2$, $m=10$

$U=\{89,18,49,58,69\}$

Quadratic Probing (cont.)

Advantage:

- reduced clustering problem

Disadvantages:

- reduced number of sequences
- no guarantee that empty slot will be found if tablesize is not prime

Double Hashing

Use two hash functions: $h'_1(k)$, $h'_2(k)$

$$h(k,I) = (h'_1(k) + ih'_2(k)) \bmod M$$

Choosing $h'_2(k)$

- don't allow $h'_2(k) = 0$ for any k.
- a good choice:
  - $h'_2(k) = R - (k \bmod R)$ with R a prime smaller than M

Characteristics

- No clustering problem
- Requires a second hash function

Balanced Trees

Problem

- specific tree configuration dependent on order in which nodes are inserted
- may become noticeably unbalanced, leading to performance approaching worst case -- O(n)

Solution

- ensure that trees remain balanced (or pretty balanced), no matter what

AVL Trees

Method

- impose *balance condition*: The height of the left and right subtrees of a node can differ by no more than one.
- adjust structure after each insertion or deletion to maintain balance condition; uses *rotation*

Pros and Cons

- guarantee O(lg n) performance
- lots of adjustments can make insertion expensive

Rotation Operation

To rotate about a node t and its left child l:

```
t->left = l->right;
l->right = t;
```

To rotate about a node t and its right child r:

```
t->right = r->left;
r->left = t;
```

Splay Trees

Concept

- adjust tree in response to accesses to make common operations efficient
- after access node is moved to root by *splaying*

Performance

- amortized such that m operations take O(m lg n) where n is the number of insertions

Splay Operation

Traverse tree from node x to root, rotating along the way until x is the root

Each rotation

- If x is root, do nothing.
- If x has no grandparent, rotate x about its parent.
- If x has a grandparent,
  - if x and its parent are both left children or both right children, rotate the parent about the grandparent, then rotate x about its parent
  - if x and its parent are opposite type children (one left and the other right), rotate x about its parent, then rotate x about its new parent (former grandparent)

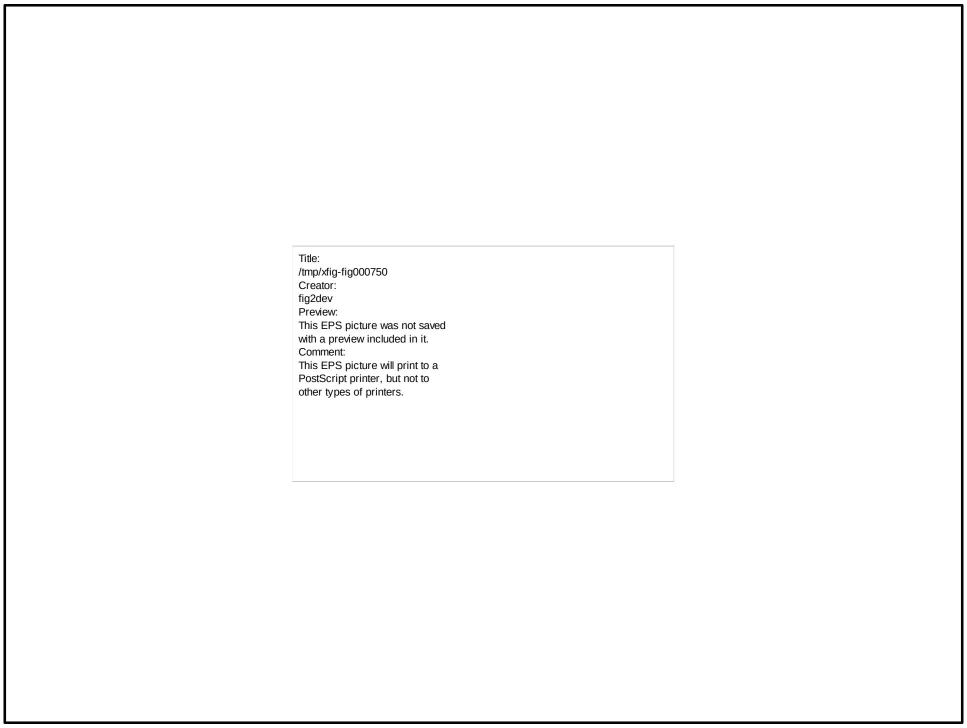

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fig2dev
Preview:
This EPS picture was not saved
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Comment:
This EPS picture will print to a
PostScript printer, but not to
other types of printers.