The Place where
1 + 1 = 1....

~~or~~

Boolean Functions and Truth Tables

Review of Objectives

After this lecture, you should be able to…….

- Simplify Boolean Algebra expressions
- Develop combinational logic solutions:
  - Sum of Product
  - Product of Sum
- Configure positive and negative logic circuits
- Identify critical parameters for logic gates from their datasheets
- Carry out "bubble matching"

The Basic Properties of Boolean Algebra

Principle of duality: The dual of a Boolean function is gotten by replacing AND with OR and OR with AND, constant 1s by 0s, and 0s by 1s

	Relationship	Dual	Property
Postulates	$A\,B = B\,A$	$A + B = B + A$	Commutative
	$A\,(B + C) = A\,B + A\,C$	$A + B\,C = (A + B)\,(A + C)$	Distributive
	$1\,A = A$	$0 + A = A$	Identity
	$A\,\overline{A} = 0$	$A + \overline{A} = 1$	Complement
Theorems	$0\,A = 0$	$1 + A = 1$	Zero and one theorems
	$A\,A = A$	$A + A = A$	Idempotence
	$A\,(B\,C) = (A\,B)\,C$	$A + (B + C) = (A + B) + C$	Associative
	$\overline{\overline{A}} = A$		Involution
	$\overline{A\,B} = \overline{A} + \overline{B}$	$\overline{A + B} = \overline{A}\,\overline{B}$	DeMorgan's Theorem
	$AB + \overline{A}C + BC = AB + \overline{A}C$	$(A+B)(\overline{A}+C)(B+C) = (A+B)(\overline{A}+C)$	Consensus Theorem
	$A\,(A + B) = A$	$A + A\,B = A$	Absorption Theorem

Postulates

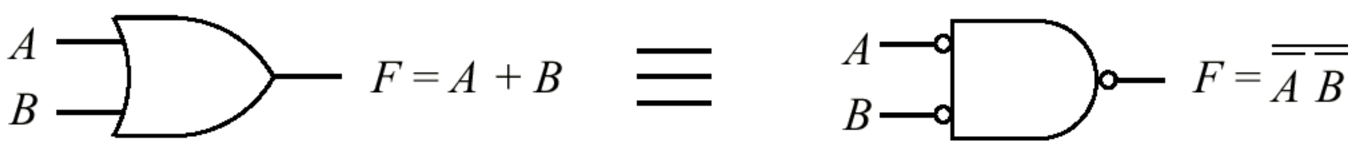
Theorems

A, B, etc. are Literals; 0 and 1 are constants.

DeMorgan's Theorem

$A \quad B$	$\overline{AB} = \overline{A} + \overline{B}$		$\overline{A+B} = \overline{A}\,\overline{B}$	
0 0	1	1	1	1
0 1	1	1	0	0
1 0	1	1	0	0
1 1	0	0	0	0

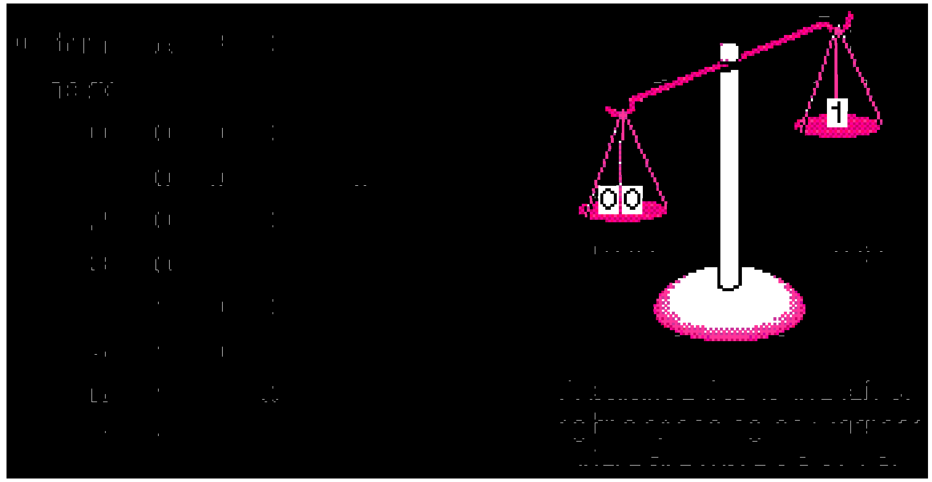
DeMorgan's theorem: $A + B = \overline{\overline{A + B}} = \overline{\overline{A}\;\overline{B}}$



Discuss: Applying DeMorgan's theorem by "pushing the bubbles," and "bubble tricks."

The Sum-of-Products (SOP) Form

Fig. A.15—Truth Table for The Majority Function

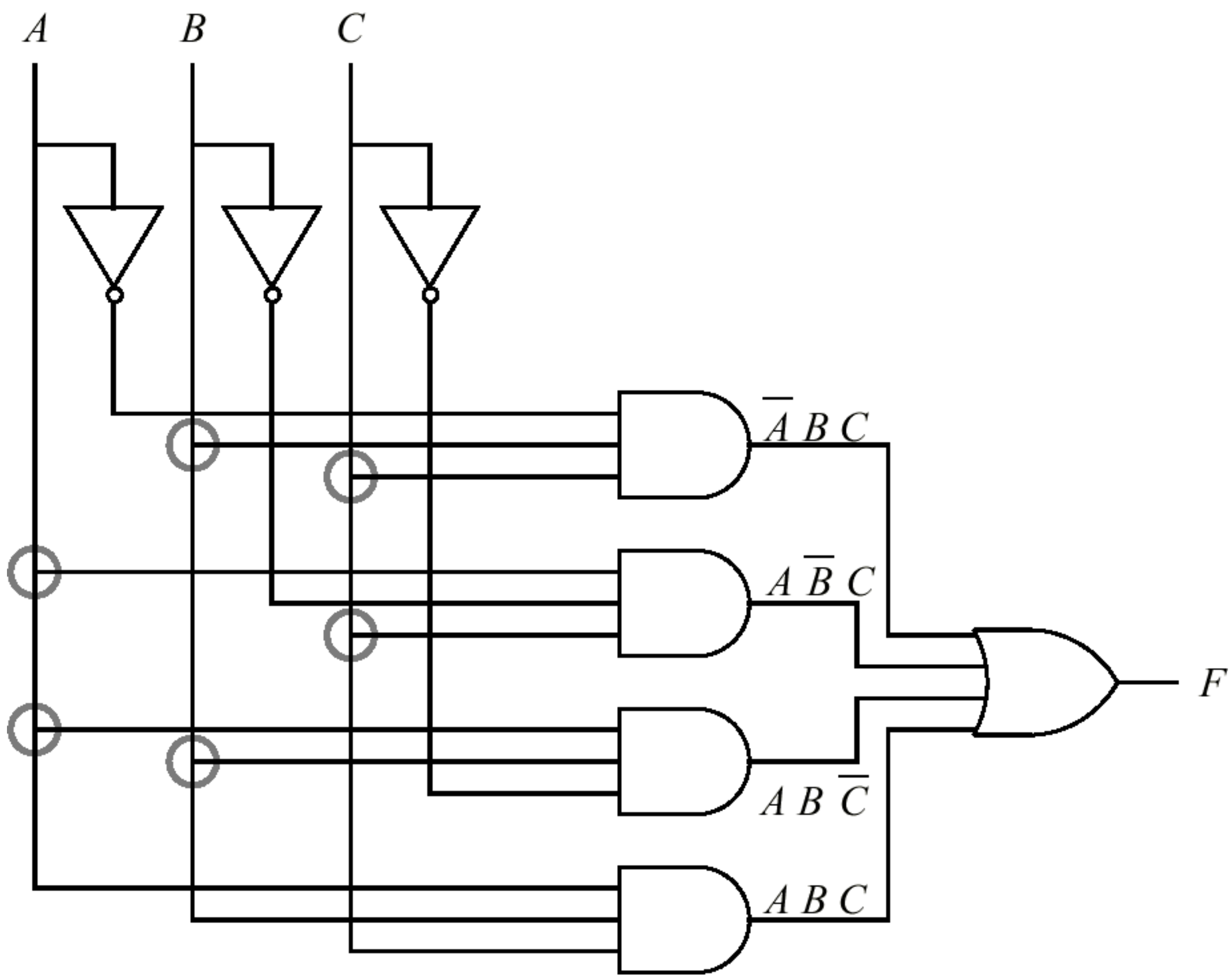


- **Transform the function into a two-level AND-OR equation**
- **Implement the function with an arrangement of logic gates from the set {AND, OR, NOT}**
- **F is true when A=0, B=1, and C=1, or when A=1, B=0, and C=1, and so on for the remaining cases.**
- **Represent logic equations by using the sum-of-products (SOP) form**

The SOP Form of the Majority Gate

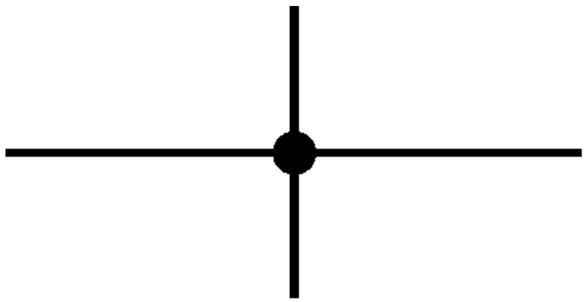
- The SOP form for the 3-input majority gate is:
- $M = \overline{A}BC + A\overline{B}C + AB\overline{C} + ABC = m3 + m5 + m6 + m7 = \Sigma(3, 5, 6, 7)$
- Each of the 2^n terms are called minterms, running from 0 to $2^n - 1$
- Note the relationship between minterm number and boolean value.
- Discuss: common-sense interpretation of equation.

A 2-Level AND-OR Circuit that Implements the Majority Function

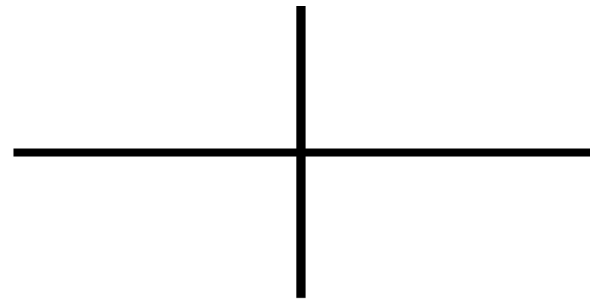


Discuss: What is the Gate Count?

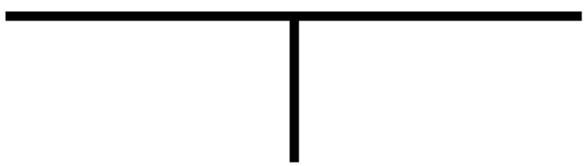
Notation Used at Circuit Intersections



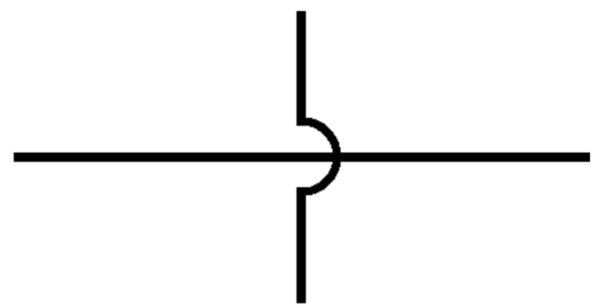
Connection



No connection

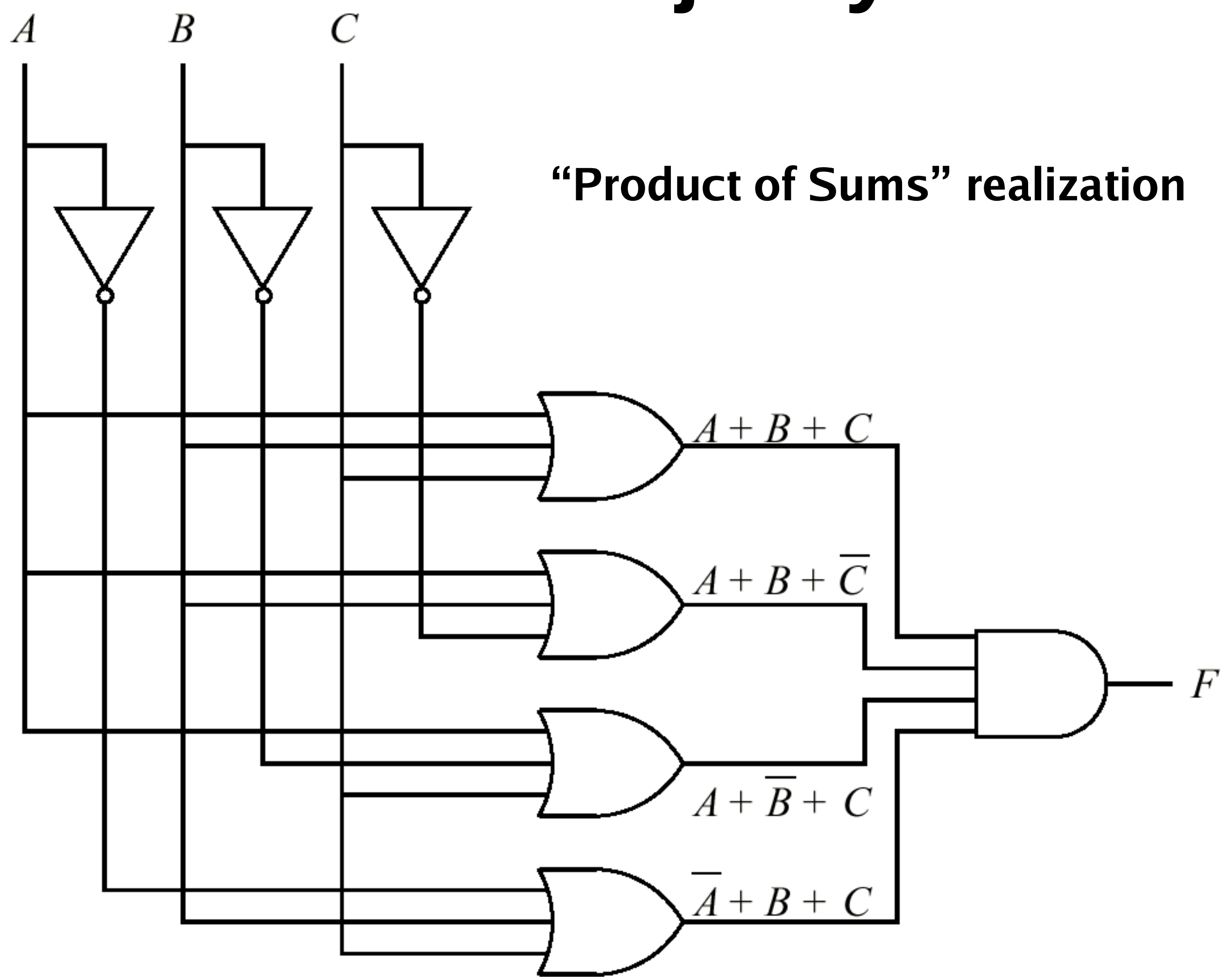


Connection



No connection

A 2-Level OR-AND Circuit that Implements the Majority Function



Positive vs. Negative Logic

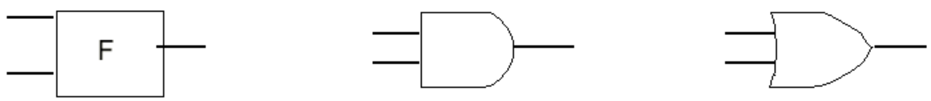
•Positive logic: truth, or assertion is represented by logic 1, higher voltage; falsity, de- or unassertion, logic 0, is represented by lower voltage.

•Negative logic: truth, or assertion is represented by logic 0 , lower voltage; falsity, de- or unassertion, logic 1, is represented by lower voltage

Gate Logic: Positive vs. Negative Logic

Normal Convention: Postive Logic/Active High
Low Voltage = 0; High Voltage = 1

Alternative Convention sometimes used: Negative Logic/Active Low



Voltage Truth Table

A	B	F
low	low	low
low	high	low
high	low	low
high	high	high

Positive Logic

A	B	F
0	0	0
0	1	0
1	0	0
1	1	1

Negative Logic

A	B	F
1	1	1
1	0	1
0	1	1
0	0	0

Behavior in terms of Electrical Levels

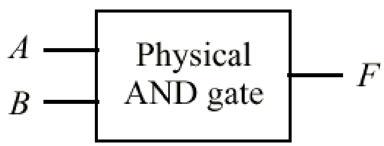
Two Alternative Interpretations
Positive Logic AND
Negative Logic OR

Dual Operations

Positive and Negative Logic (Cont’d.)

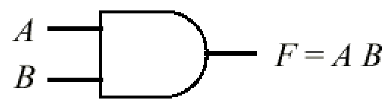
Voltage Levels

A	*B*	*F*
low	low	low
low	high	low
high	low	low
high	high	high



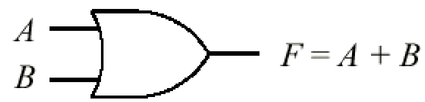
Positive Logic Levels

A	*B*	*F*
0	0	0
0	1	0
1	0	0
1	1	1



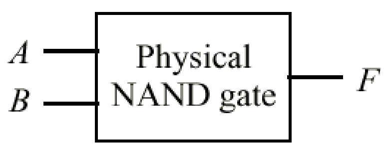
Negative Logic Levels

A	*B*	*F*
1	1	1
1	0	1
0	1	1
0	0	0



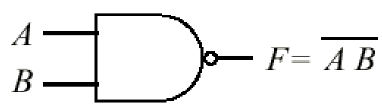
Voltage Levels

A	*B*	*F*
low	low	high
low	high	high
high	low	high
high	high	low



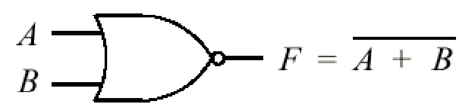
Positive Logic Levels

A	*B*	*F*
0	0	1
0	1	1
1	0	1
1	1	0



Negative Logic Levels

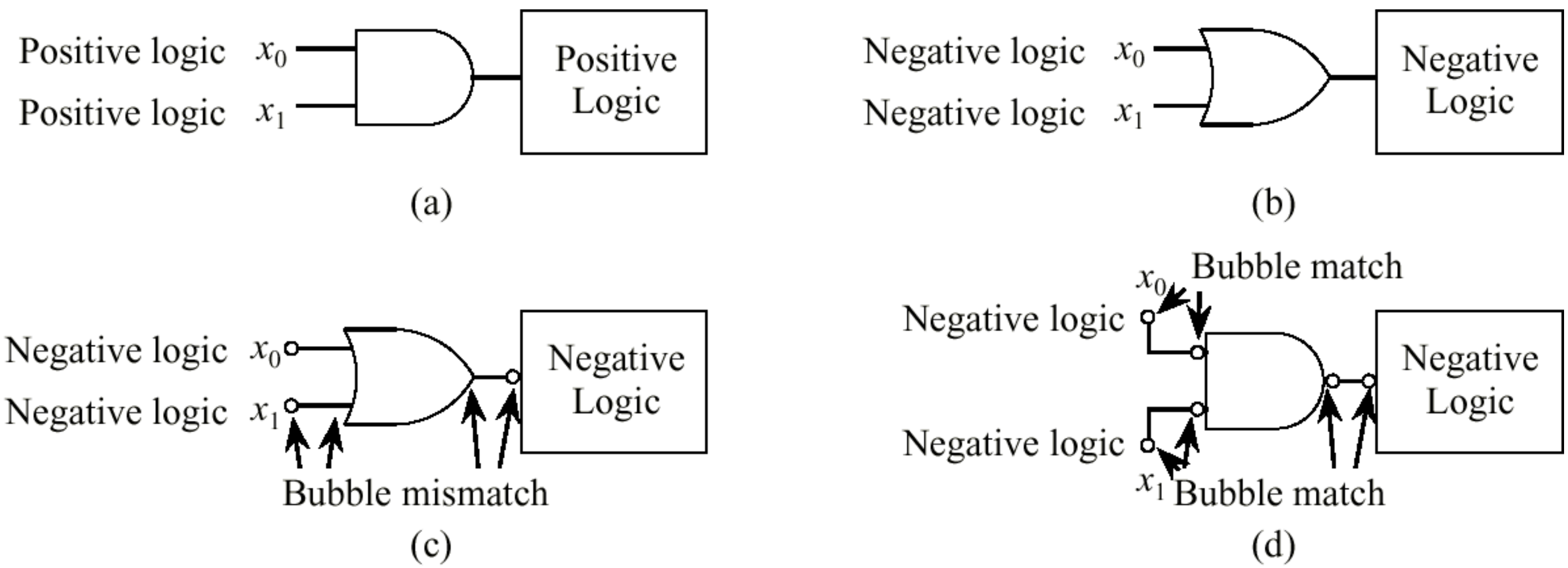
A	*B*	*F*
1	1	0
1	0	0
0	1	0
0	0	1



Bubble Matching

- Active low signals are signified by a prime or overbar or /.
- Active high: enable ______
- Active low: enable’, enable, enable/
- Discuss microwave oven control:
- Active high: Heat = DoorClosed • Start
- Active low: ? (hint: begin with AND gate as before.)

Bubble Matching (Cont'd.)



Digital Components

- High level digital circuit designs are normally made using collections of logic gates referred to as components, rather than using individual logic gates. The majority function can be viewed as a component.
- Levels of integration (numbers of gates) in an integrated circuit (IC):
- Small scale integration (SSI): 10-100 gates.
- Medium scale integration (MSI): 100 to 1000 gates.
- Large scale integration (LSI): 1000-10,000 logic gates.
- Very large scale integration (VLSI): 10,000-upward.
- These levels are approximate, but the distinctions are useful in comparing the relative complexity of circuits.
- Let us consider several useful MSI components:

Objectives Completed

- Reviewed rules of Boolean algebra
- Investigated two combinational logic forms:
  - Sum of Product : SOP
  - Product of Sum : POS
- Distinguished between positive and negative logic
- Identified critical parameters for logic gates from datasheets
- Carried out “bubble matching”

Next time we will....

- Examine logic components
- Describe and apply typical component functions
- Develop a ripple carry adder using logic components