CMSC 478 Machine Learning - Spring 2024
Homework Assignment 5
Due at 11:59PM on April 22^{nd}

Consider the dataset with instances in $\mathbb{R}^2$ below:

x_1	x_2	class
–2	–2	1
-2	-1	1
1	2	1
2	1	1
-2	2	-1
0	2	-1
0	-1	-1
2	-1	-1

Consider the non-linear mapping from the input space to a two-dimensional feature space given by the following:

$$(x_1, x_2) \rightarrow (x_1^2, x_1 x_2)$$

(a) Plot on graph paper the training instances in the input space and label them according to the class to which they belong. State whether the patterns from the two classes are linearly separable in this space.

(b) Create a new plot of the training instances in the feature space and label them according to the class to which they belong.

(c) Find the maximum margin classifier in the feature space. More specifically, find the equations of the classification boundary and of the two margin boundaries. Plot these three boundaries on the same graph that was used in step (b). Also indicate which are the support vectors in the feature space. Note that the boundaries and the support vectors are easy to find by inspection.

(d) Find which vectors in the input space correspond to the support vectors found in step (c).

(e) Plot the classification boundary in input space on the same graph that was used in step (a). Note that the classification boundary is a line in $(x_1^2, x_1 x_2)$ space with some slope m and some intercept b. That is, assuming that x_1^2 is the vertical axis in the feature space, it will look like this: $x_1^2 = m x_1 x_2 + b$. If you solve that equation for x_2, assuming it is the vertical axis in the input space, you'll have the equation for the separator in the input space.

(f) Plot on the same graph the boundaries of the classification margin zone in the input space. Shade the area between these two boundaries to better visualize the classification margin zone in the input space.