Propositional Logic

Chapter 7.4–7.7

Some material adopted from notes by Andreas Geyer-Schulz and Chuck Dyer

Propositional logic syntax

• Users specify

- Set of propositional symbols (e.g., P, Q) whose values can be True or False
- -What each means, e.g.: P: "It's hot", Q: "It's humid "
- A sentence (well formed formula) is defined as:
 - -Any symbol is a sentence
 - If S is a sentence, then -**S** is a sentence
 - -If S is a sentence, then (S) is a sentence
 - -If S and T are sentences, then so are $(S \lor T)$, $(S \land T)$, $(S \rightarrow T)$, and $(S \leftrightarrow T)$
 - -A finite number of applications of the rules

Examples of PL sentences

•Q

"It's humid"

 $\bullet \mathbf{Q} \to \mathbf{P}$

"If it's humid, then it's hot"

• (P \land Q) \rightarrow R

"If it's hot and it's humid, then it's raining"

 We're free to choose better symbols, e.g.: Hot for "It's hot" Humid for "It's humid" Raining for "It's raining"

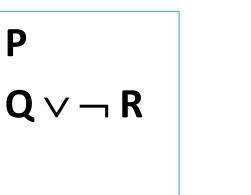
Some terms

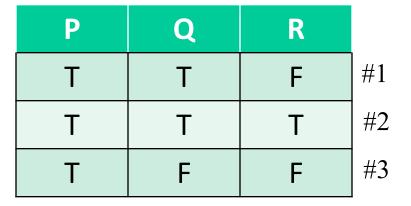
- Given the truth values of all symbols in a sentence, it can be *evaluated* to determine its truth value (True or False)
- We consider a Knowledge Base (KB) to be a set of sentences that are all True
- A model for a KB is a possible world an assignment of truth values to propositional symbols that makes each KB sentence true

A simple example

The KB

Models for the KB





The KB has 2 sentences.

The KB has 3 variables.

The KB has 3 models. Each model has a value for every variable in the KB such every sentence evaluates to true.

Another simple example

The KB

P ∧ Q R ∧ ¬ P

Models for the KB



The KB has 2 sentences.

The KB has 3 variables.

The KB has no models. There is no assignment of True or False to every variable that makes every sentence in the KB true

More terms

- A valid sentence or tautology: one that's True under all interpretations, no matter what the world is actually like or what the semantics is.
 Example: "It's raining or it's not raining" (P V ¬P)
- An inconsistent sentence or contradiction: a sentence that's False under all interpretations. The world is never like what it describes, as in "It's raining and it's not raining." (P ∧ ¬P)

Truth tables

Used to define meaning of logical connectives

Truth tables for the five logical connectives

Р	Q	$\neg P$	$P \land \underline{O}$	$P \lor \underline{O}$	$P \Rightarrow \underline{Q}$	$P \Leftrightarrow \underline{Q}$
False False True True	False True False True	True True False False	False False False True	False True True True	True True False True	True False False True

Given a value for P and for Q, the truth table defines the value of $P \lor Q$

Truth tables

Used to define meaning of logical connectives and to determine when a complex sentence is true given values of its symbols

Truth tables for the five logical connectives. The value of complex sentences							
P	Q	$\neg P$	$P \wedge U$	can be determined from the			
False	False	Тпие	False		eir elements	True	
False	True	Тпие	<i>ruise</i>	values of th	en ciements	Fallse	
True	False	False	False	True	False	Eulse	
Тrue	True	False	True	Тпие	True	True	

Example of a truth table used for a complex sentence

Р	Н	$P \lor H$	$(P \lor H) \land \neg H$	$((P \lor H) \land \neg H) \implies P$
False	False	False	False	True
False	Тпие	Тгие	False	True
True	False	True	True	True
Тгие	Тпие	True	False	True

The implies connective: $P \rightarrow Q$

\rightarrow is a logical connective

- $P \rightarrow Q$ is a **logical sentence** and has a truth value, i.e., is either **True** or **False**
- If the sentence is in a KB, it can be used by a rule (<u>Modes Ponens</u>) to infer that Q is True if P is True in the KB
- Given a KB where P=True and Q=True, we can derive/infer/prove that $P \rightarrow Q$ is True
- Note: $P \rightarrow Q$ is equivalent to $\sim P \lor Q$

$P \rightarrow Q$

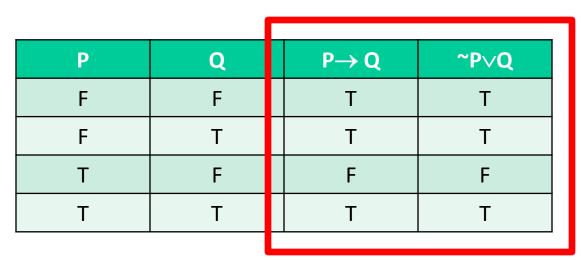
- When is *P*→*Q* true? Check all that apply
 - □ P=Q=true
 - P=Q=false
 - P=true, Q=false
 - □ P=false, Q=true

$P \rightarrow Q$

- When is $P \rightarrow Q$ true? Check all that apply
 - ☑ P=Q=true
 - ☑ P=Q=false
 - P=true, Q=false
 - ☑ P=false, Q=true
- $\bullet\, {\rm We}\ {\rm can}\ {\rm get}\ {\rm this}\ {\rm from}\ {\rm the}\ {\rm truth}\ {\rm table}\ {\rm for} \rightarrow$
- Note: in FOL it's much harder to prove that a conditional true, e.g., prime(x) → odd(x) you must prove it's true for every possible value of x

$P \rightarrow Q \equiv P \lor Q$

- P \rightarrow Q is equivalent to \sim P \lor Q
- We can show this by looking at a truth table



These two columns are the same

Models for a KB

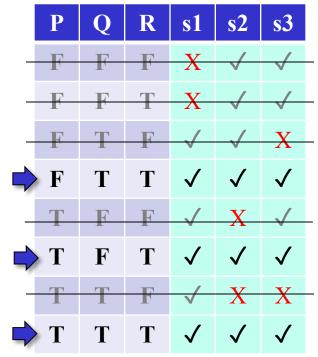
- KB: $[P \lor Q, P \rightarrow R, Q \rightarrow R]$
- What are the sentences?
 - s1: PVQs2: $P \rightarrow R$ s3: $Q \rightarrow R$
- What are the propositional variables? P, Q, R
- What are the candidate models?
- 1) Consider all **eight** possible assignments of T|F to P, Q, R
- 2) Check if each sentence is consistent with the model

P	Q	R	s1	s2	s3
F	F	F	Χ	\checkmark	\checkmark
F	F	Τ	Χ	\checkmark	>
F	Τ	F	\checkmark	\checkmark	Χ
F	Τ	Τ	\checkmark	\checkmark	\checkmark
Τ	F	F	\checkmark	Χ	>
Τ	F	Τ	\checkmark	\checkmark	\checkmark
Τ	Τ	F	\checkmark	Χ	Χ
Τ	Τ	Τ	\checkmark	\checkmark	\checkmark

Here X means the model makes the sentence False and ✓ means it doesn't make it False

Models for a KB

- KB: $[P \lor Q, P \rightarrow R, Q \rightarrow R]$
- What are the sentences?
 - s1: PVQs2: $P \rightarrow R$ s3: $Q \rightarrow R$
- What are the propositional variables?
 P, Q, R
- What are the candidate models?
- 1) Consider all possible assignments of T|F to P, Q, R
- 2) Check truth tables for consistency, eliminate a row that does not make every KB sentence true



- Only 3 models are consistent with KB
- R true in **all** of them
- Therefore, R is true and can be added to KB

