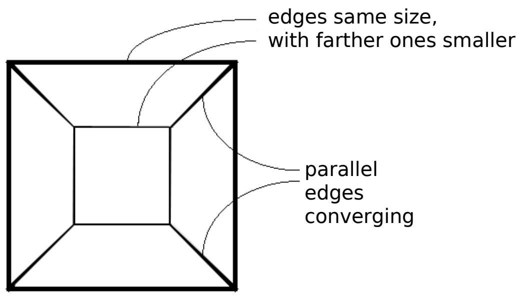
3D Viewing

From 3D to 2D: Orthographic and Perspective Projection—Part 1

- Geometrical Constructions
- Types of Projection
- Projection in Computer Graphics

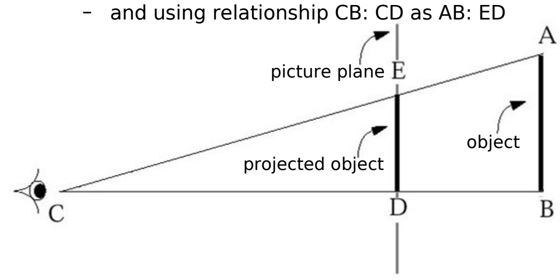
Most Striking Features of Linear Perspective

- || lines converge (in 1, 2, or 3 axes) to *vanishing point*
- Objects farther away are more *foreshortened* (i.e., smaller) than closer ones
- Example: perspective cube



The Visual Pyramid and Similar Triangles

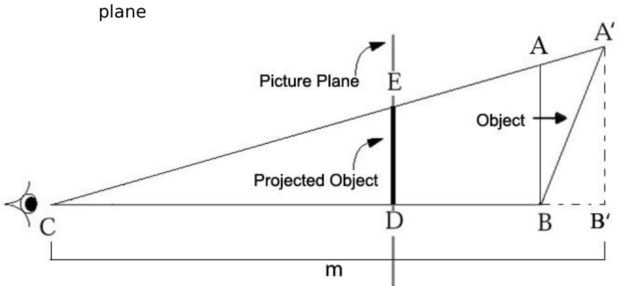
- Projected image is easy to calculate based on
  - height of object (AB)
  - distance from eye to object (CB)
  - distance from eye to picture (projection) plane (CD)
  - and using relationship CB: CD as AB: ED



CB : CD as AB : ED

The Visual Pyramid and Similar Triangles Cont.

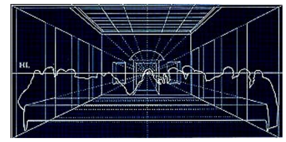
- The general case: the object we're considering is not parallel to the picture plane
  - AB is component of A'B in a plane parallel to the picture plane

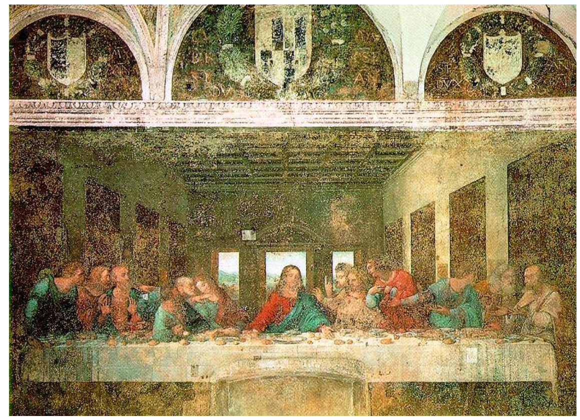


- Find the projection (B') of A' on the line CB.
  - Normalize CB
  - dot(CA', normalize(CB)) gives magnitude, m, of projection of CA' in the direction of CB
  - Travel from C in the direction of B for distance m to get B'
- A'B':ED as CB':CD
  - We can use this relationship to calculate the projection of A'B on ED

Leonardo da Vinci The Last Supper (1495)

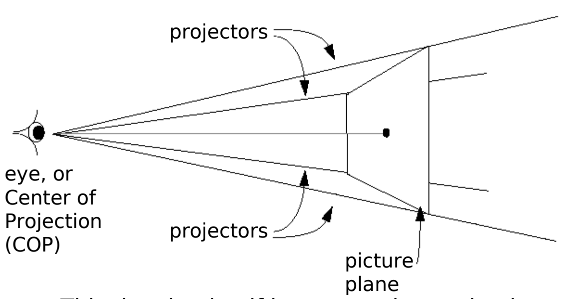
- Perspective plays very large role in this painting





Planar Geometric Projection

- Projectors are straight lines.
- Projection surface is plane (picture plane, projection plane)

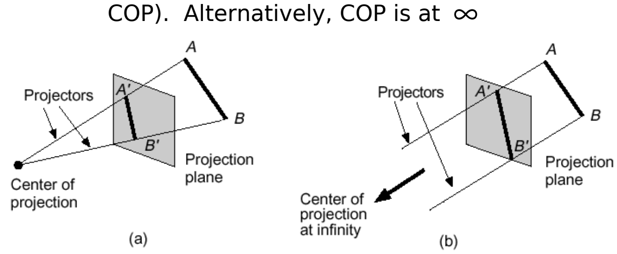


- This drawing itself is perspective projection
- What other types of projections do you know?
  - Hint:maps

Main Classes of Planar Geometrical Projections

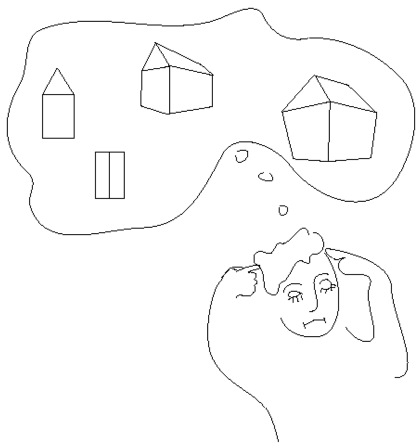
a) Perspective: determined by Center of Projection (COP) (in our diagrams, the "eye")

b) Parallel: determined by Direction of Projection (DOP) (projectors are parallel—do not converge to "eye" or COP). Alternatively, COP is at ∞

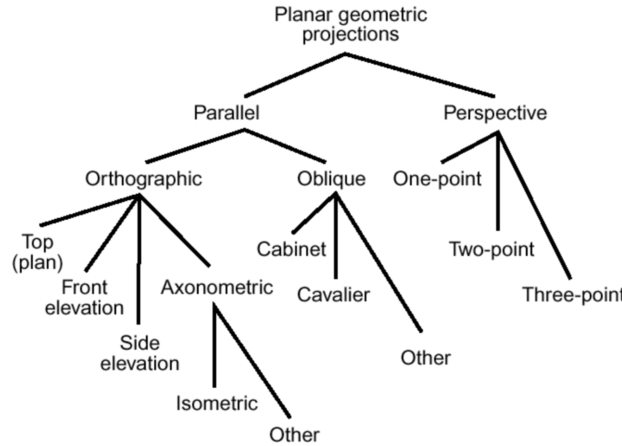


- In general, a projection is determined by where you place the projection plane relative to principal axes of object (relative angle and position), and what angle the projectors make with the projection plane

Types of Projection



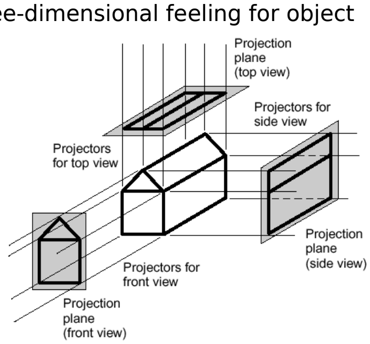
Logical Relationship Between Types of Projections



- Parallel projections used for engineering and architecture because they can be used for measurements
- Perspective imitates eyes or camera and looks more natural

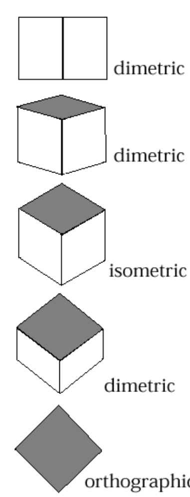
Multiview Orthographic

- Used for:
  - engineering drawings of machines, machine parts
  - working architectural drawings
- Pros:
  - accurate measurement possible
  - all views are at same scale
- Cons:
  - does not provide “realistic” view or sense of 3D form
- Usually need multiple views to get a three-dimensional feeling for object



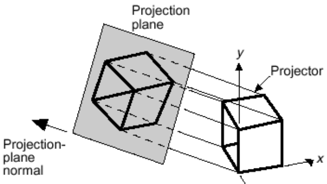
Axonometric Projections

- Same method as multiview orthographic projections, except projection plane not parallel to any of **coordinate planes**; parallel lines equally foreshortened
- *Isometric*: Angles between all three principal axes equal (120º). Same scale ratio applies along each axis
- *Dimetric:* Angles between two of the principal axes equal; need two scale ratios
- *Trimetric:* Angles different between three principal axes; need three scale ratios
- Note: different names for different views, but all part of a continuum of parallel projections of cube; these differ in where projection plane is relative to its cube

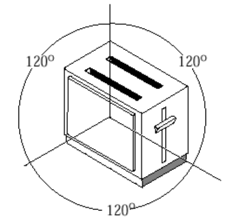


Carlbom Fig.3-8

Isometric Projection (1/2)



Construction of an isometric projection: projection plane cuts each principal axis by 45°



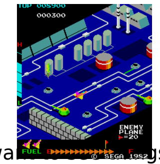
Example

Carlbom Fig.2.2

- Used for:
  - catalogue illustrations
  - patent office records
  - furniture design
  - structural design
  - 3d Modeling in real time (Maya, AutoCad, etc.)
- Pros:
  - don't need multiple views
  - illustrates 3D nature of object
  - measurements can be made to scale along principal axes
- Cons:
  - lack of foreshortening creates distorted appearance
  - more useful for rectangular than curved shapes

Isometric Projection (2/2)

Video games have been using isometric projection for ages. It all started in 1982 with *Q*Bert* and *Zaxxon* which were made possible by advances in raster graphics hardware



- Still in use today when you wa… gs in distance as well as things close up (e.g. strategy, simulation games)

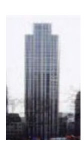
SimCity IV (Trimetric)

StarCraft II

Oblique Projections

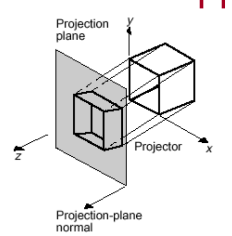
- Projectors at oblique angle to projection plane; view cameras have accordion housing, used for skyscrapers
- Pros:
  - can present exact shape of one face of an object (can take accurate measurements): better for elliptical shapes than axonometric projections, better for "mechanical" viewing
  - lack of perspective foreshortening makes comparison of sizes easier
  - displays some of object's 3D appearance
- Cons:
  - objects can look distorted if careful choice not made about position of projection plane (e.g., circles become ellipses)
  - lack of foreshortening (not realistic looking)

perspective

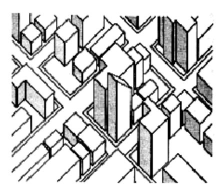


oblique

Examples of Oblique Projections

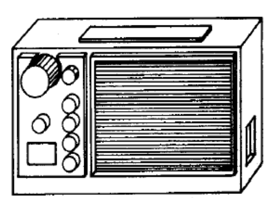


Construction of oblique parallel projection



(Carlbom Fig. 2-6)

Plan oblique projection of city

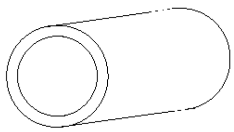


(Carlbom Fig. 2-4)

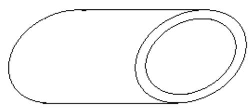
Front oblique projection of radio

Example: Oblique View

- Rules for placing projection plane for oblique views: projection plane should be chosen according to one or several of following:
  - parallel to most irregular of principal faces, or to one which contains circular or curved surfaces
  - parallel to longest principal face of object
  - parallel to face of interest



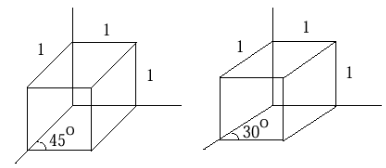
Projection plane parallel to circular face



Projection plane not parallel to circular face

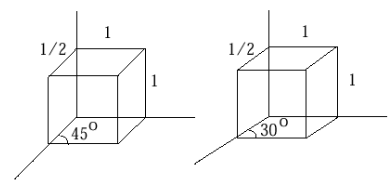
Main Types of Oblique Projections

- *Cavalier:* Angle between projectors and projection plane is 45º. Perpendicular faces projected at full scale



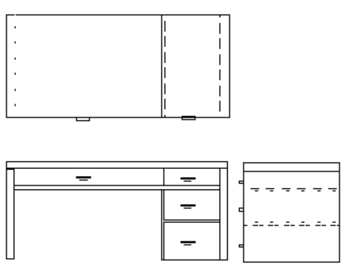
cavalier projection of unit cube

- *Cabinet:* Angle between projectors & projection plane: arctan(2) = 63.4º. Perpendicular faces projected at 50% scale

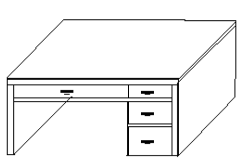


cabinet projection of unit cube

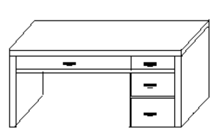
Examples of Orthographic and Oblique Projections



multiview orthographic



cavalier

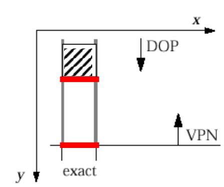


cabine
t

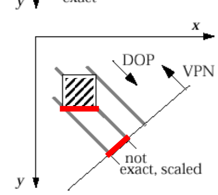
Carlbom Fig. 3-2

Summary of Parallel Projections

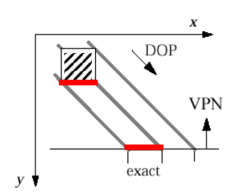
- Assume object face of interest lies in principal plane, i.e., parallel to *xy*, *yz*, or *zx* planes. (DOP = Direction of Projection, VPN = View Plane Normal)



1) Multiview Orthographic
  - VPN || a principal coordinate axis
  - DOP || VPN
  - shows single face, exact measurements

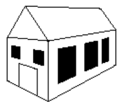


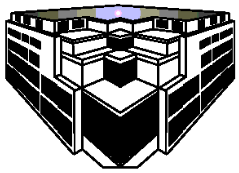
2) Axonometric
  - VPN ∦ a principal coordinate axis
  - DOP || VPN
  - adjacent faces, none exact, uniformly foreshortened (function of angle between face normal and DOP)



3) Oblique
  - VPN || a principal coordinate axis
  - DOP ∦ VPN
  - adjacent faces, one exact, others uniformly foreshortened

Perspective Projections

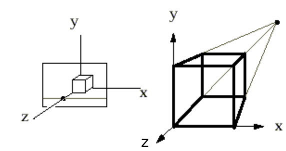




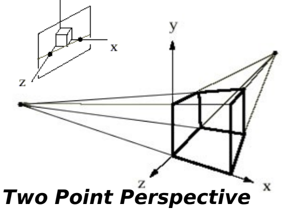
- Used for:
  - advertising
  - presentation drawings for architecture, industrial design, engineering
  - fine art
- Pros:
  - gives a realistic view and feeling for 3D form of object
- Cons:
  - does not preserve shape of object or scale (except where object intersects projection plane)
- Different from a parallel projection because
  - parallel lines not parallel to the projection plane converge
  - size of object is diminished with distance
  - foreshortening is not uniform

Vanishing Points (1/2)

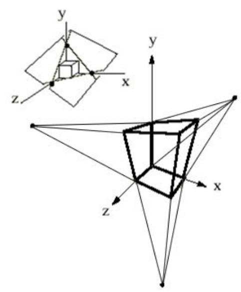
- For right-angled forms whose face normals are perpendicular to the *x, y, z* coordinate axes, number of vanishing points = number of principal coordinate axes intersected by projection plane



One Point Perspective
(*z*-axis vanishing point)



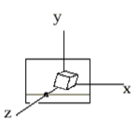
Two Point Perspective
(*z*, and *x*-axis vanishing points)



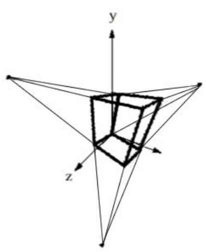
Three Point Perspective
(*z*, *x*, and *y*-axis vanishing points)

Vanishing Points (2/2)

- What happens if same form is turned so its face normals are *not* perpendicular to *x*, *y*, *z* coordinate axes?



Unprojected cube depicted here with parallel projection

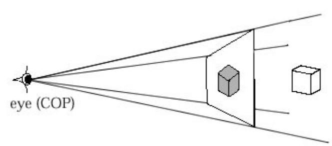


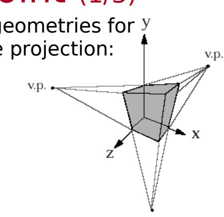
Perspective drawing of the rotated cube

- New viewing situation: cube is rotated, face normals no longer perpendicular to any principal axes
- Although projection plane only intersects one axis (*z*), three vanishing points created
- But… can achieve final results identical to previous situation in which projection plane intersected all three axes
  - Note: the projection plane still intersects all three of the cube's edges, so if you pretend the cube is unrotated, and it's edges the axes, then your projection plane is intersecting the three axes

Vanishing Points and the View Point (1/3)

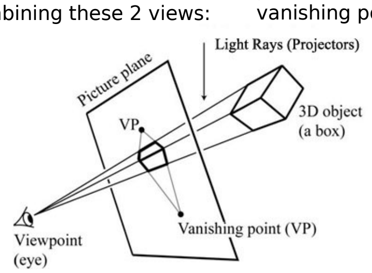
- We've seen two pyramid geometries for understanding perspective projection:



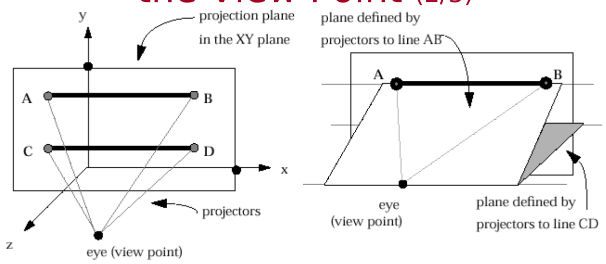


1. perspective image is intersection of a plane with light rays from object to eye (COP)
2. perspective image is result of foreshortening due to convergence of some parallel lines toward vanishing points

- Combining these 2 views:

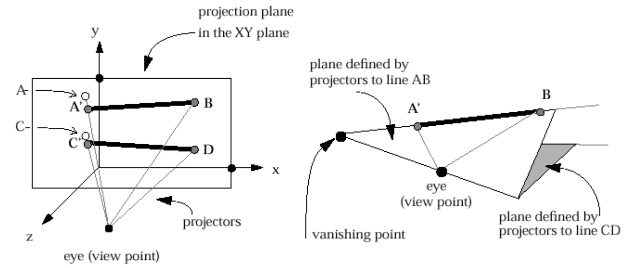


Vanishing Points and the View Point (2/3)



- Project parallel lines *AB*, *CD* on *xy* plane
- Projectors from eye to *AB* and *CD* define two planes, which meet in a line which contains the view point, or eye
- This line **does not** intersect projection plane (*XY*), because parallel to it. Therefore there is no vanishing point

Vanishing Points and the View Point (3/3)



- Lines *AB* and *CD* (this time with *A* and *C* behind the projection plane) projected on *xy* plane: *A'B* and *C'D*
- Note: *A'B* not parallel to *C'D*
- Projectors from eye to *A'B* and *C'D* define two planes which meet in a line which contains the view point
- This line **does** intersect projection plane
- Point of intersection is vanishing point

Next Time: Projection in Computer Graphics