

# Cryptographic Hashes

CMSC 426/626 - Computer Security  
Fall 2014

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## Outline

- Authentication vs. Confidentiality
- Simple Hash Functions
- Secure Hash Functions
- HMAC

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## Authentication

- If Alice and Bob share a secret key and Alice sends Bob an encrypted message, can Bob assume the message is "authentic?"
- What do we mean by *authentic*?

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# Consider

- A **block cipher in ECB** mode...

An attacker could re-order blocks in the message without affecting Bob's ability to decrypt it.

- A **block cipher in CFB or CTR** mode, or a **stream cipher**...

If the plaintext is highly structured, an attacker can modify the plaintext without decrypting the message.

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In fact, it is possible to *authenticate* a message without *encrypting* the message.

Authentication and Confidentiality are distinct.

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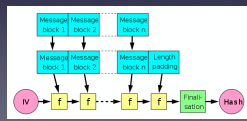
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# Hash Functions

- Given a message  $M$  of arbitrary length, a hash function  $H$  produces a fixed size *digest*  $H(M)$ .
- It should be "easy" to compute  $H(M)$  for any  $M$ .
- Hashes are an alternative to MACs (we'll cover those later)



(Merkle-Damgard Construction; from Wikipedia, public domain)

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## Uses for Hashes

- For authentication and integrity.
- **With encryption:** append hash to  $M$  before encrypting.
- **Keyed hash:** Alice and Bob share a secret authentication key  $K$ ; Alice authenticates message  $M$  by appending

$$h_K = H(M || K)$$

- **Digital signature:** Alice public-key encrypts the hash of  $M$  with her private key  $A_{priv}$

$$s = E[A_{priv}, H(M)]$$

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## Questions

Suppose Alice and Bob are using a keyed hash scheme with shared key  $K_{AB}$ . Alice sends Bob the message  $M$  along with  $H(M || K_{AB})$ .

1. How does Bob verify the message is really from Alice?
2. How does Bob verify that the message has not been altered?

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## Pre-Image Resistance

- For **any** given hash code  $h$ , it should be infeasible to construct an  $M$  such that  $H(M) = h$ .
- In the keyed hash case, pre-image resistance prevents an attacker from recovering  $M || K$ , and thus  $K$ .

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## Weak Collision Resistance

- For **any given** message  $M$ , it should be infeasible to construct a different message  $N$  such that  $H(M) = H(N)$ .
- In digital signature applications, lack of weak collision resistance allows an attacker to find a different message with the same signature.



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## Strong Collision Resistance

- It should be infeasible to construct a pair of different messages  $(M, N)$  such that  $H(M) = H(N)$ .
- Subtly different from weak collision resistance.
- Prevents the following sort of attack:
  1. Eve constructs two messages with the same hash value. One is an I.O.U. for \$10, the other is an I.O.U. for \$10,000.
    - Eve gets Alice to sign the \$10 I.O.U.
    - Eve insists on being paid her \$10,000.

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Suppose  $H()$  is a strongly collision resistant hash function that maps messages of arbitrary length to an  $n$ -bit hash value.

1. Is it true that for all distinct messages  $x$  and  $y$ ,  $H(x) \neq H(y)$  ?

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## Simple Hash Functions

- Break the  $M$  into  $b$ -bit blocks  $M_1, M_2, \dots, M_n$ .

$$h = M_1 \oplus M_2 \oplus \dots \oplus M_n$$

- A variation: let  $r(x, n)$  denote the left circular shift of  $x$  by  $n$  bits

$$h = M_1 \oplus r(M_2, 1) \oplus \dots \oplus r(M_n, n-1)$$

- There are  $2^b$  possible hash codes, so if the message is modified or corrupted, there is probability  $2^{-b}$  that the hash code  $h$  will be unchanged.

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- Unfortunately, neither of these schemes is **collision resistant** (weak or strong).

- Suppose I construct the following messages:

$$M = M_1, M_2$$

$$N = N_1, N_2, M_1 \oplus M_2 \oplus N_1 \oplus N_2$$

$$N' = N_1, N_2, r(M_1 \oplus r(M_2, 1) \oplus N_1 \oplus r(N_2, 1), -2)$$

- If  $H$  is the first simple hash, then  $H(M) = H(N)$ .
- If  $H$  is the variation, then  $H(M) = H(N')$

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## One More Example

- Another simple hash: let a message be represented by a list of integers

$$M = (a_1, a_2, \dots, a_i)$$

- Let  $N$  be a positive integer and define  $H(M)$  by

$$h = (a_1 + a_2 + \dots + a_i) \bmod N$$

- Is  $H$  **pre-image resistant**?

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# Brute Force Costs

For a hash with digest of size  $n$ :

- Constructing a **pre-image**:  $2^n$  hash computations
- Finding a **weak collision**:  $2^n$  hash computations
- Finding a **strong collision**:  $2^{n/2}$  hash computations (this is due to the birthday problem)

For example, the MD5 message digest is 128 bits, so it should take  $2^{64}$  hash computations to find a strong collision pair.

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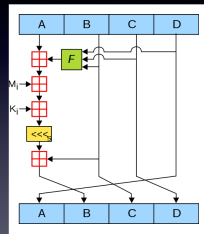
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# MD5

- Ron Rivest, 1992
- Operates on 32-bit words with addition mod  $2^{32}$
- Message processed in 512-bit "chunks" broken into 16 32-bit words.
- Basic function applied 64 times per chunk.



(from Wikipedia by Surachit; CC A-SA 3.0)

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# MD5 Attacks

- 2004 - Wang, Fang, Lai, and Yu demonstrate first practical collision
- 2005 - Lenstra, Wang, de Weger produce colliding X.509 certificates
- 2008 - "normal" certificate converted to intermediate CA certificate
- 2012 - Flame malware uses fraudulent MS code signing certificate; constructed using collision

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# The SHA Family

Algorithm	Comments	Reference
SHA-0	Had problems	FIPS PUB 180 (1993)
SHA-1	Corrected problems in SHA-0; similar to MD5	FIPS PUB 180-1 (1995)
SHA-2	Family of algorithms (SHA-256, SHA-512, etc.)	FIPS PUB 180-2 (2002)
SHA-3	Very different algorithm; selected in 2012	FIPS PUB 202 (DRAFT)

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## Current Status

- SHA-0 and SHA-1 produce a 160 bit digest, so 80 bits of security for strong collision resistance. Too small?!
- SHA-2 provides 256-, 384-, and 512-bit options. No known attacks against SHA-2, but mathematics is similar to MD5, so NIST wanted an alternative...just in case.
- SHA-3 selected in 2012 after an open competition. It is quite different from SHA-2.

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## SHA-512

- Processes message in 1024-bit blocks.
- Maintains 512-bit internal state.
- Uses an 80-round function to update state for each block.
- Digest is state after processing the last message block.

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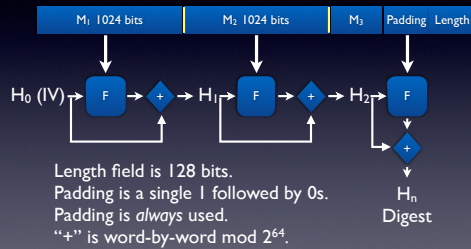
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## SHA-512

Two full blocks and one partial block



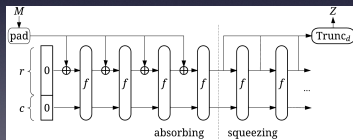
## The F-function

- The F-function consists of 80 rounds.
- Each round involves basic boolean operations (AND, OR, XOR, NOT).
- Each round incorporates a portion of the message block ( $W$ ) and a constant ( $K_i$ ).

**The F-function provides good mixing.**  
Each digest bit is a function of every input bit.

## SHA-3

- "Sponge" construction
- $f$ -function operates on 1600-bit state
- Message blocks xor-ed with state





## HMAC

- HMAC - *Hash-based MAC* - published in RFC 2104.
- Improves on security of basic keyed hash.
- Security of HMAC depends only on security of the hash function.
- Later we will see MACs based on block ciphers.

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$$\text{HMAC}(K, M) = H[ (K \oplus op) \parallel H[K \oplus ip] \parallel M ]$$

- $H[ ]$  is the hash function.
- $K$  is the secret key, padded with zeros on the left to match the hash block size.
- $op$  is a constant (0x5c repeated).
- $ip$  is a constant (0x36 repeated).

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## Using an HMAC

Use an HMAC just as we would a keyed hash:

- Alice and Bob have secret key  $K$
- Alice computes HMAC of message  $M$  using key  $K$  and sends  $M$  and HMAC to Bob.
- Bob computes HMAC of received message using key  $K$  and checks it against the value Alice sent; if they match, all is good!

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Finished. See the website for exercises.

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