## Homework I

1. Five jobs are waiting to be run. Their expected run times are $9,6,3,5$ and X . In what order should they be run in order to minimize average waiting time? (Hint: Though the answer is simple, you need to consider all cases).

In class we've discussed that shortest job first leads to minimum average waiting time. See class text, p 159. You don't need to prove it for this question, just remember that fact. In that case the answer is
a. $\mathbf{0}<\mathbf{X} \beta 3$ 3: $\mathbf{X}, 3,5,6,9$
b. $\mathbf{3}<\mathbf{X} \beta$ 5: 3, $X, 5,6,9$
c. $\mathbf{5}<\mathbf{X} \boldsymbol{\beta} \mathbf{6}: 3,5, \mathrm{X}, 6,9$
d. $\mathbf{6}<\mathbf{X} \boldsymbol{\beta}$ 9: 3, 5, 6, X, 9
e. $\mathbf{X}>9: 3,5,6,9, X$
2. This problem deals with real time systems with N CPU's. Events that real time systems have to respond to are classified as periodic or aperiodic. Consider the first case where jobs occur at regular intervals: Let there be $m$ periodic events and event $i$ occurs with period $P_{i}$ and $C_{i}$ seconds of CPU time are required to handle each event, derive an inequality that can be used to show the conditions under which the periodic load can be handled. (Hint: Define CPU utilization, ignore context switch overhead.)
Utilization is defined as $\sum_{i=1}^{m} \frac{C_{i}}{P_{i}}$ and this must be less than or equal to N, i.e., CPU utilization cannot exceed the number of CPU's available, and we have assumed N CPU's.
3. For the bakery algorithm we define a processor is in the bakery from when it sets choosing $[\mathrm{i}]=$ false till it fails or leaves the critical section. The two lines before it is in the CS it called as being in the doorway. Prove the following assertions:
a. Assertion 1: If processors $\mathrm{i}, \mathrm{k}$ are in the bakery and i entered the bakery before $k$ entered the doorway, then number[i] < number[k].
b. Assertion 2: If processor i is in its critical section, processor k is in the bakery, and k .not equal. i , then (number[i] , i) < (number[k], k ).

Assertion 1: If processors $\mathrm{i}, \mathrm{k}$ are in the bakery and i entered the bakery before k entered the doorway, then number $[\mathbf{i}]$ < number $[\mathbf{k}]$.
Proof: By hypothesis, number[i] had its current value while k was choosing the current value of number $[k]$.Hence, $k$ must have chosen number $[k]>=1+$ number $[i]$.

[^0]for $\mathrm{j}=\mathrm{k}$, so TL2 $<$ TL3. When processor k was choosing its current value of number[k], let Te be the time at which it entered the doorway, Tw the time at which it finished writing the value of number [k], and Tc the time at which it left the doorway. $\mathrm{Te}<\mathrm{Tw}<$ Tc. Since choosing[k] was equal to zero at time TL2, we have either a) TL2 $<\mathrm{Te}$ or b) Tc < TL2. In case a), Assertion 1 implies number[i] < number[k] so Assertion 2 holds.
4. A computer has six tape drives, with n processes competing for them. Each process may need two drives. For which values of n is this system safe?

With $\mathrm{n}=6$, i.e., six processes, each one holding one tape drive, and wanting another one, we have a deadlock. For $\mathrm{n}<6$, the system is deadlock-free (check it out for yourself).
5. Consider a system consisting of $m$ resources of the same type, being shared by $n$ processes. Resources can be requested and released by processes only one at a time. Show that the system is deadlock free if the following two conditions hold:
a. The max need of each process is between 1 and $m$ resources
b. The sum of all max needs is less than $m+n$

This is problem 8.9 in the textbook. You should have a solutions manual available and should refer to it. A) $\sum_{i=1}^{n} \operatorname{Max}_{i}<m+n$. B) $\operatorname{Max}_{i} \geq 1$ for all i. $\left(\operatorname{Need}_{i}=\operatorname{Max}_{i}-\right.$ Allocation $_{i}$ ). If there exists a deadlock then C) $\sum_{i=1}^{n}$ Allocation $_{i}=m$. Use A) to get $\sum_{i=1}^{n}$ Allocation $_{i}+\sum_{i=1}^{n}$ Need $_{i}=\sum_{i=1}^{n}$ Max $_{i}<m+n$. Use C) to get $m+\sum_{i=1}^{n}$ Need $_{i}=\sum_{i=1}^{n} \operatorname{Max}_{i}<m+n$ or $\sum_{i=1}^{n} \operatorname{Need}_{i}=\sum_{i=1}^{n} \operatorname{Max}_{i}<n$. This implies that there exists a process $P_{i}$ such that $\sum_{i=1}^{n}$ Need $_{i}=0$. Because of B) it follows that $P_{i}$ has at least one resource that it can release. Hence the system cannot be in a deadlock state.
6. If an instruction takes 1 microsec and a page fault takes an extra n microsec, derive a formula for the effective instruction time if page faults occur every k instructions (on average).

Every k instructions we add n microseconds therefore for 1 instruction we add $\mathrm{n} / \mathrm{k}$ microseconds. The effective instruction time on average is $1+\mathrm{n} / \mathrm{k}$.
7. The figure below shows a virtual address space from 0 to 64 K and 32 K of physical memory. There are 16 pages and 8 frames and transfers between memory and disk are in pages. Give the physical address corresponding to the following virtual addresses: a) 20 b) 4100 c) 8300

a) 8212 b) 4100 c) 24684
8. Solve problem 10.10 in our textbook.
a) 5000 page faults. b) 50 page faults.
9. Solve problem 10.11 in our textbook.

| \# of frames | LRU | FIFO | Optimal |
| :--- | :--- | :--- | :--- |
| 1 | 20 | 20 | 20 |
| 2 | 18 | 18 | 15 |
| 3 | 15 | 16 | 11 |
| 4 | 10 | 14 | 8 |
| 5 | 8 | 10 | 7 |
| 6 | 7 | 10 | 7 |
| 7 | 7 | 7 | 7 |


[^0]:    Assertion 2: If processor i is in its critical section, processor k is in the bakery, and k .not equal. i, then (number[i] , i) < (number [k] , k).
    Proof: Let TL2 be the time at which processor i read choosing $[\mathbf{k}]$ during its last execution of L2 for $\mathrm{j}=\mathrm{k}$, and let TL3 be the time at which i began its last execution of L3

