Red Black Trees

Top-Down Insertion
Review of Bottom-Up Insertion

• In B-Up insertion, “ordinary” BST insertion was used, followed by correction of the tree on the way back up to the root

• This is most easily done recursively
  – Insert winds up the recursion on the way down the tree to the insertion point
  – Fixing the tree occurs as the recursion unwinds
Top-Down Insertion Strategy

• In T-Down insertion, the corrections are done while traversing down the tree to the insertion point.
• When the actual insertion is done, no further corrections are needed, so no need to traverse back up the tree.
• So, T-Down insertion can be done iteratively which is generally faster
Goal of T-D Insertion

- Insertion is always done as a leaf (as in ordinary BST insertion)
- Recall from the B-Up flow chart that if the uncle of a newly inserted node is Black, we restore the RB tree properties by one or two local rotations and recoloring – we do not need to make changes further up the tree
Goal (2)

• Therefore, the goal of T-D insertion is to traverse from the root to the insertion point in such a way that RB properties are maintained, and at the insertion point, the uncle is Black.

• That way we may have to rotate and recolor, but not propagate back up the tree
Possible insertion configurations

If a new node is inserted as a child of Y or Z, there is no problem since the new node’s parent is Black
Possible insertion configurations

If new node is child of Z, no problem since Z is Black.

If new node is child of Y, fixable problem since the new node’s uncle (Z) is Black – do a few rotations and recolor…. done
Possible insertion configurations

If new node is inserted as child of Y or Z, its uncle will be Red and we will have to go back up the tree. This is the only case we need to avoid.
Top-Down Traversal

As we traverse down the tree and encounter this case, we recolor and possibly do some rotations.

There are 3 cases.

Remember the goal – to create an insertion point at which the parent of the new node is Black, or the uncle of the new node is Black.
Case 1 – X’s Parent is Black

Just recolor and continue down the tree
Case 2

• X’s Parent is Red (so Grandparent is Black) and X and P are both left/right children
  – Rotate P around G
  – Color P Black
  – Color G Red

• Note that X’s uncle, U, must be Black because it (a) was initially Black, or (b) would have been made Black when we encountered G (which would have had two Red children -- X’s Parent and X’s uncle)
Case 2 diagrams

Rotate P around G. Recolor X, Y, Z, P and G
Case 3

- X’s Parent is Red (so Grandparent is Black) and X and P are opposite children
  - Rotate P around G
  - Color P Black
  - Color G Red

- Again note that X’s uncle, U, must be Black because it (a) was initially Black, or (b) would have been made Black when we encountered G (which would have had two Red children -- X’s Parent and X’s uncle)
Case 3 Diagrams (1 of 2)

Step 1 – recolor X, Y and Z. Rotate X around P.
Case 3 Diagrams (2 of 2)

Step 2 – Rotate X around G. Recolor X and G
An exercise — insert F
Top-Down Insert Summary

Case 1
P is Black
Just Recolor

Case 2
P is Red
X & P both left/right

Case 3
P is Red
X and P are opposite children

Recolor X, Y, Z
Rotate X around P
Recolor X, G
Rotate X around G
Recolor P, G
Recolor X, Y, Z