1. For each of the following program fragments, give an analysis of the running time (Big-Oh will suffice). (2 points each)

1. \( \text{sum} = 0 \)
   \[ \text{for}(i = 0; i < n; i++) \]
   \[ \text{sum}++; \]
   \[ \text{Answer: } O(n) \]

2. \( \text{sum} = 0 \)
   \[ \text{for}(i = 0; i < n; i++) \]
   \[ \text{for}(j = 0; j < n; j++) \]
   \[ \text{sum}++; \]
   \[ \text{Answer: } O(n^2) \]

3. \( \text{sum} = 0 \)
   \[ \text{for}(i = 0; i < n; i++) \]
   \[ \text{for}(j = 0; j < n * n; j++) \]
   \[ \text{sum}++; \]
   \[ \text{Answer: } O(n^3) \]

4. \( \text{sum} = 0 \)
   \[ \text{for}(i = 0; i < n; i++) \]
   \[ \text{for}(j = 0; j < i; j++) \]
   \[ \text{sum}++; \]
   \[ \text{Answer: } O(n^2) \]

5. \( \text{sum} = 0 \)
   \[ \text{for}(i = 0; i < n; i++) \]
   \[ \text{for}(j = 0; j < i * i; j++) \]
   \[ \text{for}(k = 0; k < j; k++) \]
   \[ \text{sum}++; \]
   \[ \text{Answer: } O(n^5) \]

6. \( \text{sum} = 0 \)
   \[ \text{for}(i = 0; i < n; i++) \]
   \[ \text{for}(j = 1; j < i * i; j++) \]
   \[ \text{if}(j \% i == 0) \]
   \[ \text{for}(k = 0; k < j; k++) \]
   \[ \text{sum}++; \]
   \[ \text{Answer: } O(n^4) \text{ if you expand the problem, A will run every time } j \text{ is a multiple of } i \text{ which is } N \text{ times. Other occasions the loop will terminate in constant time.} \]
2. Order the following functions by growth rate from slowest to fastest (indicate any that grow at the same rate):

\[ N, \sqrt{N}, N^{1.5}, N^2, N \log N, N \log^2 N, N \log N^2, 2/N, 2^N, 2^{N/2}, 37, N^2 \log N, N^3 \]

(14 total points, 1 point each being in the correct order)

Answer:

\[ 2/ n, 37, \sqrt{n}, n, n \log n, n \log n \land n \log n^2, n \log^2 n, n^{1.5}, n^2, n^2 \log n, n^3, 2^{n/2} 2^N \]

2/N vs 37 - if you look, 37 is a constant rate of growth where 2/N is a line that approaches zero that will never achieve a constant slope of 0.

3. Find two functions \( f(N) \) and \( g(N) \) such that neither \( f(N) = O(g(N)) \) nor \( g(N) = O(f(N)) \). Prove your answer.

(6 points, 3 points for functions and 3 points for justification)

Answer: \( f(n) = \cos(n), g(n) = \sin(n) \)

Since neither function bounds the other from the top for all \( n > n_0 \) neither function be considered Big-O based on the definition. Since these two functions are oscillatory neither will prove \( f(n) = O(g(n)) \) if \( f(n) \leq cg(n), c > 0 \land n > n_0 \).