CMSC 341

Splay Trees
Splay Trees

Concept

– adjust tree in response to accesses to make common operations efficient
– after access node is moved to root by *splaying*

Performance

– amortized such that m operations take $O(m \ lg \ n)$ where n is the number of insertions
Splay Operation

Traverse tree from node \( x \) to root, rotating along the way until \( x \) is the root.

Each rotation

- If \( x \) is root, do nothing.
- If \( x \) has no grandparent, rotate \( x \) about its parent.
- If \( x \) has a grandparent,
  - if \( x \) and its parent are both left children or both right children, rotate the parent about the grandparent, then rotate \( x \) about its parent.
  - if \( x \) and its parent are opposite type children (one left and the other right), rotate \( x \) about its parent, then rotate \( x \) about its new parent (former grandparent).
Node has no grandparent
Node and Parent are Same Side
Zig-Zig
Node and Parent are Different Sides
Zig-Zag
Operations in Splay Trees

insert
- first insert as in normal binary search tree
- then splay inserted node
- if there is a duplicate, the node holds the duplicate
  element is splayed

find
- search for node
- if found, splay to root; otherwise splay last node on
  path
Insertion in order into a Splay Tree

I

II

III

IV

Access Node 1
Operations on Splay Trees (cont)

remove

– splay selected element to root
– disconnect left and right subtrees from root
– do one of:
  • splay max item in $T_L$ (then $T_L$ has no right child)
  • splay min item in $T_R$ (then $T_R$ has no left child)
– connect other subtree to empty child
– if the item to be deleted is not in the tree, the node last visited in the search is splayed
After Splaying At Node "X"

Original Tree
Performance of Splay Trees

insert
  – regular bst insertion -- $O(\text{depth})$
  – splay: $O(1)$ for each rotation, $O(\text{depth})$ rotations