These are some review questions to test your understanding of the material. Some of these questions may appear on an exam.

1 Binary Trees

1.1 Define binary tree, full binary tree, complete binary tree, and perfect binary tree.

1.2 Define internal node and external node of a rooted tree.

1.3 Define internal path length and external path length of a rooted tree.

1.4 Define augmented binary tree.

1.5 Prove: A perfect binary tree of height $h$ has $2^h$ leaf nodes.

1.6 Prove: A perfect binary tree of height $h$ has $2^{h+1} - 1$ vertices.

1.7 Prove: A full binary tree with $n_i$ internal nodes has $n_i + 1$ leaf nodes.

1.8 Prove: In any binary tree of $n$ nodes, there are $n + 1$ “null pointers.”

1.9 Consider a binary tree that has $n_i$ internal nodes. Let $E(n_i)$ and $I(n_i)$ denote the external and internal path lengths of the tree, respectively. Show that for a full binary tree, $E(n_i) = I(n_i) + 2n_i$.

1.10 Prove: the internal path length of an augmented binary tree $T$ with $n$ internal nodes is

$$I(n) = I(n_L) + I(n - n_L - 1) + n - 1$$

where $n_L$ is the number of internal nodes in the left subtree.

1.11 Suppose you have the following two traversal sequences from the same binary tree:

- pre-order: A D F G H K L P Q R W 2
- in-order: G F H K D L A W R Q P 2

Draw the tree.

1.12 Suppose you have the following two traversal sequences from the same binary tree:

- post-order: F G H D A L P Q R Z W X
- in-order: G F H K D L A W R Q P 2

Draw the tree.