9

Bottom Up Parsing

Motivation

- In the last lecture we looked at a table driven, top-down parser
  - A parser for LL(1) grammars
- In this lecture, we'll look a a table driven, bottom up parser
  - A parser for LR(1) grammars
- In practice, bottom-up parsing algorithms are used more widely for a number of reasons

Right Sentential Forms

```
1 E -> E+T
2 E -> T
3 T -> T*F
4 T -> F
5 F -> (E)
6 F -> id
```

- Recall the definition of a derivation and a rightmost derivation
- Each of the lines is a (right) sentential form
- A form of the parsing problem is finding the correct RHS in a right-sentential form to reduce to get the previous right-sentential form in the derivation

generation ↓ / parsing ↑

E
E+T
E+T*F
E+T*id
E+F*id
E+id*id
T+id*id
F+id*id
id+id*id

Right Sentential Forms

Consider this example

```
1 E -> E+T
2 E -> T
3 T -> T*F
4 T -> F
5 F -> (E)
6 F -> id
```

- We start with id+id*id
- What rules can apply to some portion of this sequence?
  - Only rule 6: `F -> id`
- Are there more than one way to apply the rule?
  - Yes, three
- Apply it so the result is part of a "right most derivation"
  - If there is a derivation, there is a right most one
  - If we always choose that, we can't get into trouble

generation ↓ / parsing ↑

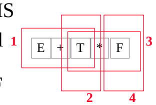
E

F+id*id
id+id*id

Bottom up parsing

```
1 E -> E+T
2 E -> T
3 T -> T*F
4 T -> F
5 F -> (E)
6 F -> id
```

- A bottom up parser looks at a sentential form and selects a contiguous sequence of symbols that matches the RHS of a grammar rule, and replaces it with the LHS
- There might be several choices, as in the sentential form E+T*F
- Which one should we choose?



parsing ↑

E
E+T
E+T*F
E+T*id
E+F*id
E+id*id
T+id*id
F+id*id
id+id*id

Bottom up parsing

```
1 E -> E+T
2 E -> T
3 T -> T*F
4 T -> F
5 F -> (E)
6 F -> id
```

- If the wrong one is chosen, it leads to failure
- E.g.: replacing E+T with E in E+T*F yields E+F, which can't be further reduced using the given grammar
- The **handle** of a sentential form is the RHS that should be rewritten to yield the next sentential form in the right most derivation

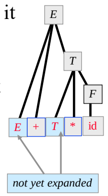
parsing ↑

error
E*F
E+T*F
E+T*id
E+F*id
E+id*id
T+id*id
F+id*id
id+id*id

Sentential forms

- Think of a sentential form as one of the entries in a derivation that begins with the start symbol and ends with a legal sentence
- It's like a sentence but it may have *unexpanded* non-terminals
- We can also think of it as a parse tree where some leaves are as yet unexpanded non-terminals

```
1 E -> E+T
2 E -> T
3 T -> T*F
4 T -> F
5 F -> (E)
6 F -> id
```



generation ↓ / parsing ↑

E
E+T
*E+T*F*
*E+T*id*
*E+F*id*
*E+id*id*
*T+id*id*
*F+id*id*
id+id*id

Handles

- A handle of a sentential form is a substring α such that :
  - α matches the RHS of some production A -> α ; and
  - replacing α by the LHS **A** represents a step in the reverse of a rightmost derivation of s.
- For this grammar, the rightmost derivation for the input **abbcde** is
  S => aABe => aAde => aAbcde => abbcde

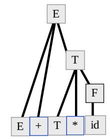
```
1: S -> aABe
2: A -> Abc
3: A -> b
4: B -> d
```

- The string **aAbcde** can be reduced in two ways:
  (1) aAbcde => aAde *(using rule 2)*
  (2) aAbcde => aAbcBe *(using rule 4)*
- But (2) isn't a rightmost derivation, so Abc is the only handle.
- Note: the string to the right of a handle will only contain terminals (why?)

a A b c d e

Phrases

- A **phrase** is a subsequence of a sentential form that is eventually "reduced" to a single non-terminal.
- A **simple phrase** is a phrase that is reduced in a single step.
- The **handle** is the left-most simple phrase.



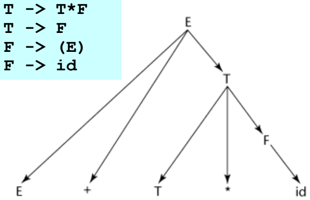
For sentential form E+T*id what are the
•phrases: E+T*id, T*id, id
•simple phrases: id
•handle: id

Phrases, simple phrases and handles

- **Def:** β is the *handle* of the right sentential form $\gamma = \alpha\beta w$ if and only if $S \Rightarrow^*_{rm} \alpha A w \Rightarrow_{rm} \alpha\beta w$
- **Def:** β is a *phrase* of the right sentential form γ if and only if $S \Rightarrow^* \gamma = \alpha_1 A \alpha_2 \Rightarrow^+ \alpha_1\beta\alpha_2$
- **Def:** β is a *simple phrase* of the right sentential form γ if and only if $S \Rightarrow^* \gamma = \alpha_1 A \alpha_2 \Rightarrow \alpha_1\beta\alpha_2$
- The handle of a right sentential form is its leftmost simple phrase
- Given a parse tree, it is now easy to find the handle
- Parsing can be thought of as handle pruning

Phrases, simple phrases and handles

```
E -> E+T
E -> T
T -> T*F
T -> F
F -> (E)
F -> id
```



E
E+T
E+T*F
E+T*id
E+F*id
E+id*id
T+id*id
F+id*id
id+id*id

On to shift-reduce parsing

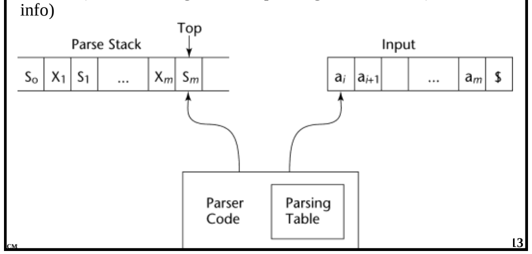
- How to do it w/o having a parse tree in front of us?
- Look at a shift-reduce parser - the kind that *yacc* uses
- A shift-reduce parser has a queue of input tokens & an initially empty stack. It takes one of 4 possible actions:
  - **Accept:** if the input queue is empty and the start symbol is the only thing on the stack
  - **Reduce:** if there is a handle on the top of the stack, pop it off and replace it with the rule's LHS
  - **Shift:** push the next input token onto the stack
  - **Fail:** if the input is empty and we can't accept
- In general, we might have a choice of (1) shift, (2) reduce, or (3) maybe reducing using one of several rules
- The algorithm we next describe is deterministic

Shift-Reduce Algorithms

A shift-reduce parser scans input, at each step decides to:

•**Shift** next token to top of parse stack (along with state info) or

•**Reduce** the stack by POPing several symbols off the stack (& their state info) and PUSHing the corresponding non-terminal (& state info)



Shift-Reduce Algorithms

The stack is always of the form

bottom ... top

S_0 X_1 S_1 X_2 S_2 … X_n S_n

state ... terminal or non-terminal

- A reduction step is triggered when we see the symbols corresponding to a rule's RHS on the top of the stack

bottom ... top

S_0 X_1 S_1 …T S_6 * S_7 F S_8

T -> T*F

S_0 X_1 S_1 …T S_6'

LR parser table

LR shift-reduce parsers can be efficiently implemented by precomputing a table to guide the processing

	Action						Goto		
State	id	+	*	(	)	$	E	T	F
0	S5		S4				1	2	3
1		S6				accept			
2		R2	S7		R2	R2			
3		R4	R4		R4	R4			
4	S5			S4			8	2	3
5		R6	R6		R6	R6			
6	S5			S4				9	3
7	S5			S4					10
8		S6			S11				
9		R1	S7		R1	R1			
10		R3	R3		R3	R3			
11		R5	R5		R5	R5			

More on this Later . . .

When to shift, when to reduce

- Key problem in building a shift-reduce parser is deciding whether to shift or to reduce
  - repeat: *reduce* if a handle is on top of stack, *shift* otherwise
  - *Succeed* if there is only S on the stack and no input
- A grammar may not be appropriate for a LR parser because there are <u>conflicts</u> which can not be resolved
- Conflict occurs when the parser can't decide whether to:
  - shift or reduce the top of stack (a shift/reduce conflict), or
  - reduce the top of stack using one of two possible productions (a reduce/reduce conflict)
- There are several varieties of LR parsers (LR(0), LR(1), SLR and LALR), with differences depending on amount of lookahead and on construction of the parse table

Conflicts

Shift-reduce conflict: can't decide whether to shift or to reduce

- Example : "dangling else"

  Stmt -> if Expr then Stmt

  | if Expr then Stmt else Stmt

  | ...
- What to do when else is at the front of the input?

Reduce-reduce conflict: can't decide which of several possible reductions to make

- Example :

  Stmt -> id (params)

  | Expr := Expr

  | ...

  Expr -> id (params)
- Given the input a(i, j) the parser does not know whether it is a procedure call or an array reference.

LR Table

- An LR configuration stores the state of an LR parser

  $(S_0X_1S_1X_2S_2…X_mS_m, a_ia_{i+1}…a_n\$)$
- LR parsers are table driven, where the table has two components, an ACTION table and a GOTO table
- The ACTION table specifies the action of the parser (shift or reduce) given the parser state and next token
  - Rows are state names; columns are terminals
- The GOTO table specifies which state to put on top of the parse stack after a reduce
  - Rows are state names; columns are non-terminals

	Action						Goto		
State	id	+	*	(	)	$	E	T	F
0	S5		S4				1	2	3
1		S6				accept			
2		R2	S7		R2	R2			
3		R4	R4		R4	R4			
4	S5			S4			8	2	3
5		R6	R6		R6	R6			
6	S5			S4				9	3
7	S5			S4					10
8		S6			S11				
9		R1	S7		R1	R1			
10		R3	R3		R3	R3			
11		R5	R5		R5	R5			

If in state 0 and the next input is id, then SHIFT and go to state 5

If in state 1 and no more input, we are done

If in state 5 and the next input is *, then REDUCE using rule 6. Use goto table and exposed state to select next state

```
1: E -> E+T
2: E -> T
3: T -> T*F
4: T -> F
5: F -> (E)
6: F -> id
```


Parser actions

Initial configuration: (S0, a1…an$)

Parser actions:

1 If ACTION[S_m, a_i] = Shift S, the next configuration is: $(S_0X_1S_1X_2S_2 \ldots X_mS_ma_iS, a_{i+1} \ldots a_n\$)$

2 If ACTION[S_m, a_i] = Reduce $A \rightarrow \beta$ and S = GOTO[S_{m-r}, A], where r = the length of β, the next configuration is

$$(S_00X_1S_1X_2S_2 \ldots X_{m-r}S_{m-r}AS, a_ia_{i+1} \ldots a_n\$)$$

3 If ACTION[S_m, a_i] = Accept, the parse is complete and no errors were found

4 If ACTION[S_m, a_i] = Error, the parser calls an error-handling routine

Example

```
1: E -> E+T
2: E -> T
3: T -> T*F
4: T -> F
5: F -> (E)
6: F -> id
```

Stack	Input	action
0	Id + id * id $	Shift 5
0 id 5	+ id * id $	Reduce 6 *goto(0,F)*
0 F 3	+ id * id $	Reduce 4 *goto(0,T)*
0 T 2	+ id * id $	Reduce 2 *goto(0,E)*
0 E 1	+ id * id $	Shift 6
0 E 1 + 6	id * id $	Shift 5
0 E 1 + 6 id 5	* id $	Reduce 6 *goto(6,F)*
0 E 1 + 6 F 3	* id $	Reduce 4 *goto(6,T)*
0 E 1 + 6 T 9	* id $	Shift 7
0 E 1 + 6 T 9 * 7	id $	Shift 5
0 E 1 + 6 T 9 * 7 id 5	$	Reduce 6 *goto(7,E)*
0 E 1 + 6 T 9 * 7 F 10	$	Reduce 3 *goto(6,T)*
0 E 1 + 6 T 9	$	Reduce 1 *goto(0,E)*
0 E 1	$	Accept

	Action						Goto		
State	id	+	*	(	)	$	E	T	F
0	S5		S4				1	2	3
1		S6				accept			
2		R2	S7		R2	R2			
3		R4	R4		R4	R4			
4	S5			S4			8	2	3
5		R6	R6		R6	R6			
6	S5			S4				9	3
7	S5			S4					10
8		S6			S11				
9		R1	S7		R1	R1			
10		R3	R3		R3	R3			
11		R5	R5		R5	R5			

Yacc as a LR parser

- The Unix yacc utility is just such a parser.
- It does the heavy lifting of computing the table
- To see the table information, use the –v flag when calling yacc, as in

  yacc –v test.y

```
 0  $accept : E $end
   1  E : E '+' T
   2    | T
   3  T : T '*' F
   4    | F
   5  F : '(' E ')'
   6    | "id"

state 0
        $accept : . E $end  (0)
        '('  shift 1
        "id"  shift 2
        .  error
        E  goto 3
        T  goto 4
        F  goto 5
state 1
        F : '(' . E ')'  (5)
        '('  shift 1
        "id"  shift 2
        .  error
        E  goto 6
        T  goto 4
        F  goto 5
. . .
```