Description Logics

What Are Description Logics?
- A family of logic based KR formalisms
  - Descendants of semantic networks and KL-ONE
  - Describe domain in terms of concepts (classes), roles (relationships) and individuals
- Distinguished by:
  - Formal semantics (typically model theoretic)
    - Decidable fragments of FOL
    - Closely related to Propositional Modal & Dynamic Logics
  - Provision of inference services
    - Sound and complete decision procedures for key problems
    - Implemented systems (highly optimized)

Major focus of KR research in the 80’s
- Led by Ron Brachman – (AT&T Labs)
Major systems and languages –
- 80s: KL-ONE, NIKL, KANDOR, BACK, CLASSIC, LOOM
- 90s: FACT, RACER, ...
- 00s: DAML+OIL, OWL, Pellet, Jena, FACT++

Used as the basis for the Semantic web languages DAML+OIL and OWL
Some commercial systems

Thought to be well-suited for the representation of and reasoning about
- ontologies
- terminological knowledge
- Configurations and configuration problems
- database schemata
  - schema design, evolution, and query optimization
  - source integration in heterogeneous databases/data warehouses
  - conceptual modeling of multidimensional aggregation
Example of Network KR

- Person, Female, etc. are concepts
- hasChild is a property of Person
  - hasChild relates Parent to Person
  - Nil means infinity. A Parent is a Person with between 1 and infinity children
- Large arrows are “IS-A” links
  - A Mother is a (specialization of a) Parent
- Concepts either primitive or definitions
  - Primitive concepts have only necessary properties
  - Defined concepts have necessary and sufficient conditions

Necessary vs. Sufficient

- **Necessary** properties of an object are common to all objects of that type
  - Being a man is a necessary condition for being a father
- **Sufficient** properties allow one to identify an object as belonging to a type and need not be common to all members of the type
  - Speeding is a sufficient reason for being stopped by the police
- Definitions often specify both necessary and sufficient properties

DL Paradigm

- A Description Logic characterized by a set of constructors that allow one to build complex descriptions or terms out of concepts and roles from atomic ones
  - Concepts correspond to classes
    - and are interpreted as sets of objects,
  - Roles correspond to relations
    - and are interpreted as binary relations on objects
- Set of axioms for asserting facts about concepts, roles and individuals

Basic Concepts of a DL

- Individuals are treated exactly the same as constants in FOL
  - *john*
- Concepts are exactly the same as Unary Predicates in FOL
  - *Person(john)*
- Roles are exactly the same as Binary Predicates in FOL
  - *has_mother(john, mary)*
**Descriptions**

- Like FOL, we are dealing with (ultimately) sets of individuals and relations between them.
- Basic unit of semantic significance is a *Description*.
- "We are *describing* sets of individuals".
- Description logics differ in allowed operators.
- If a "happy father" is a man with both a son and daughter and all of whose children are either rich or happy, we describe it in DL as:
  \[
  \text{HappyFather} = \text{Man} \cap \exists\text{hasChild.Female} \cap \exists\text{hasChild.Male} \cap \forall\text{hasChild.}(\text{Rich} \cup \text{Happy})
  \]

**A family of languages**

- The expressiveness of a description logic is determined by the operators that it uses:
  - Add or eliminate operators (e.g., ~, ∨, ∩), and the statements that can be expressed are increased/reduced in number.
  - Higher expressiveness implies higher complexity.
- **AL** or **Attributive Language** is the base and includes just a few operators.
- Other DLs are described by the additional operators they include.

**Typical Architecture**

- The division into TBox and ABox doesn’t have a logical significance, but is made for conceptual and implementation convenience.

**AL: Attributive Language**

<table>
<thead>
<tr>
<th>Constructor</th>
<th>Syntax</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>atomic concept</td>
<td>C</td>
<td>Human</td>
</tr>
<tr>
<td>atomic negation</td>
<td>¬C</td>
<td>¬Human</td>
</tr>
<tr>
<td>atomic role</td>
<td>R</td>
<td>hasChild</td>
</tr>
<tr>
<td>conjunction</td>
<td>C ∨ D</td>
<td>Human ∧ Male</td>
</tr>
<tr>
<td>value restriction</td>
<td>R,C</td>
<td>Human ∃ hasChild.Blond</td>
</tr>
<tr>
<td>existential rest. (lim)</td>
<td>∃R</td>
<td>Human ∃ hasChild</td>
</tr>
<tr>
<td>Top (univ. conc.)</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>bottom (null conc)</td>
<td>⊥</td>
<td>⊥</td>
</tr>
</tbody>
</table>

*for concepts C and D and role R*
**ALC**

ALC is the smallest DL that is propositionally closed (i.e., includes full negation and disjunction) and includes boolean (and, or, not) and restrictions on role values.

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<th>Constructor</th>
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</tr>
</thead>
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<tr>
<td>atomic concept</td>
<td>C</td>
<td>Human</td>
</tr>
<tr>
<td>negation</td>
<td>~ C</td>
<td>~ (Human V Ape)</td>
</tr>
<tr>
<td>atomic role</td>
<td>R</td>
<td>hasChild</td>
</tr>
<tr>
<td>conjunction</td>
<td>C ^ D</td>
<td>Human ^ Male</td>
</tr>
<tr>
<td>disjunction</td>
<td>C V D</td>
<td>Nice V Rich</td>
</tr>
<tr>
<td>value restrict.</td>
<td>?R.C</td>
<td>Human ? hasChild.Blond</td>
</tr>
<tr>
<td>existential rest.</td>
<td>?R.C</td>
<td>Human ? hasChild.Male</td>
</tr>
<tr>
<td>Top (univ. conc.)</td>
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</tr>
<tr>
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<td>⊥</td>
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</tbody>
</table>

**Other Constructors**

<table>
<thead>
<tr>
<th>Constructor</th>
<th>Syntax</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number restriction</td>
<td>&gt;= n R</td>
<td>&gt;= 7 hasChild</td>
</tr>
<tr>
<td>Inverse role</td>
<td>R^-1</td>
<td>haschild^-1</td>
</tr>
<tr>
<td>Transitive role</td>
<td>R*</td>
<td>hasChild*</td>
</tr>
<tr>
<td>Role composition</td>
<td>R o R</td>
<td>hasParent o hasBrother</td>
</tr>
<tr>
<td>Qualified # restric.</td>
<td>&gt;= n R.C</td>
<td>&gt;= 2 hasChild.Female</td>
</tr>
<tr>
<td>Singleton concepts</td>
<td>&lt;name&gt;</td>
<td>{Italy}</td>
</tr>
</tbody>
</table>

**∀ and ∃ deserve special attention.**

- Note that they only can come before a Role:
  
  ∀HasChild.Girl       ∃isEmployedBy.Farmer

- Remember, they describe sets of individuals

- ∀HasChild.Girl would be interpreted as:
  The set \{ x | ∀(y)( HasChild(x,y) → Girl(y) ) \}
  Note the conditional: Are you in that set?

- ∃isEmployedBy.Farmer would be:
  The set \{ x | ∃(y)( isEmployedBy(x,y) ∧ Farmer(y) ) \}

**Special names and combinations**


- S = ALC + transitive properties
- H = role hierarchy, e.g., rdfs:subPropertyOf
- O = nominals, e.g., values constrained by enumerated classes, as in owl:oneOf and owl:hasValue
- I = inverse properties
- N = cardinality restrictions (owl:cardinality, maxCardonality)
- (D) = use of datatypes properties
- R = complex role axioms (e.g. (ir)reflexivity, disjointedness)
- Q = Qualified cardinality (e.g., at least two female children)
  - OWL-DL is SHOIN(D)
  - OWL 2 is SROIQ(D)
OWL as a DL

- OWL-DL is SHOIN(D)
- We can think of OWL as having three kinds of statements
  - Ways to specify classes
    - the intersection of humans and males
  - Ways to state axioms about those classes
    - Humans are a subclass of apes
  - Ways to talk about individuals
    - John is a human, john is a male, john has a child mary

### OWL Class Constructors

<table>
<thead>
<tr>
<th>Constructor</th>
<th>DL Syntax</th>
<th>Example</th>
<th>(Modal Syntax)</th>
</tr>
</thead>
<tbody>
<tr>
<td>intersectionOf</td>
<td>$C_1 \cap \ldots \cap C_n$</td>
<td>Human $\sqcap$ Male</td>
<td>$C_1 \land \ldots \land C_n$</td>
</tr>
<tr>
<td>unionOf</td>
<td>$C_1 \cup \ldots \cup C_n$</td>
<td>Doctor $\sqcup$ Lawyer</td>
<td>$C_1 \lor \ldots \lor C_n$</td>
</tr>
<tr>
<td>complementOf</td>
<td>$\neg C$</td>
<td>$\neg$Male</td>
<td>$\neg C$</td>
</tr>
<tr>
<td>oneOf</td>
<td>${ x_1 \ldots x_n }$</td>
<td>${john, mary}$</td>
<td>${x_1 \lor \ldots \lor x_n}$</td>
</tr>
<tr>
<td>allValuesFrom</td>
<td>$\forall x \in C$</td>
<td>hasChild.Doctor</td>
<td>$\forall (x \in C)[P(x)]$</td>
</tr>
<tr>
<td>someValuesFrom</td>
<td>$\exists x \in C$</td>
<td>hasChild.Lawyer</td>
<td>$\exists (x \in C)[P(x)]$</td>
</tr>
<tr>
<td>maxCardinality</td>
<td>$\leq nP$</td>
<td>$\leq$hasChild</td>
<td>$\leq \exists C_{n!-!1}$</td>
</tr>
<tr>
<td>minCardinality</td>
<td>$\geq nP$</td>
<td>$\geq$hasChild</td>
<td>$\geq \exists C_n$</td>
</tr>
</tbody>
</table>

- **XMLS datatypes** as well as classes in $\forall x \in C$ and $\exists x \in C$
- E.g., hasAge, nonNegativeInteger
- Arbitrarily complex nesting of constructors
  - E.g., Person $\sqcap$ hasChild.(Doctor $\sqcup$ hasChild.Doctor)

### OWL Axioms

<table>
<thead>
<tr>
<th>Axiom</th>
<th>DL Syntax</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>subClassOf</td>
<td>$C_1 \sqsubseteq C_2$</td>
<td>Human $\sqsubseteq$ Animal $\sqsubseteq$ Biped</td>
</tr>
<tr>
<td>equivalentClass</td>
<td>$C_1 \equiv C_2$</td>
<td>Man $\equiv$ Human $\equiv$ Male</td>
</tr>
<tr>
<td>disjointWith</td>
<td>$C_1 \sqcap C_2$</td>
<td>Male $\sqcap$ Female</td>
</tr>
<tr>
<td>sameIndividualAs</td>
<td>${x_1} \equiv {x_2}$</td>
<td>President_Bush $\equiv$ G_W_B</td>
</tr>
<tr>
<td>differentFrom</td>
<td>${x_1} \sqsubseteq \neg {x_2}$</td>
<td>(John) $\sqsubseteq$ (Peter)</td>
</tr>
<tr>
<td>subpropertyOf</td>
<td>$P_1 \sqsubseteq P_2$</td>
<td>hasDaughter $\sqsubseteq$ hasChild</td>
</tr>
<tr>
<td>equivalentProperty</td>
<td>$P_1 \equiv P_2$</td>
<td>cost $\equiv$ price</td>
</tr>
<tr>
<td>inverseOf</td>
<td>$P_1 \sqsubseteq P_2^\top$</td>
<td>hasChild $\equiv$ hasParent</td>
</tr>
<tr>
<td>transitiveProperty</td>
<td>$P^+ \subseteq P$</td>
<td>ancestor $^+ \subseteq$ ancestor</td>
</tr>
<tr>
<td>functionalProperty</td>
<td>$T \sqsubseteq \ell P$</td>
<td>$\exists hasMother</td>
</tr>
<tr>
<td>inverseFunctionalProperty</td>
<td>$T \sqsubseteq \ell P^\top$</td>
<td>$\exists hasSSN$</td>
</tr>
</tbody>
</table>

- $T$ satisfies $C_1 \sqsubseteq C_2$ iff $C_1 \subseteq C_2$; satisfies $P_1 \sqsubseteq P_2$ iff $P_1^\top \subseteq P_2^\top$
- $T$ satisfies ontology $\mathcal{O}$ (is a model of $\mathcal{O}$) iff satisfies every axiom in $\mathcal{O}$
Subsumption: $D \subseteq C$ ?

- Concept $C$ subsumes $D$ iff on every interpretation $I$
  - $I(D) \subseteq I(C)$
- This means the same as $\forall x (D(x) \rightarrow C(x))$ for complex statements $D \& C$
- Determining whether one concept logically contains another is called the *subsumption problem*.
- Subsumption is undecidable for reasonably expressive languages
  - e.g., for FOL: does one FOL sentence imply another
- and non-polynomial for fairly restricted ones

Other reasoning problems

These problems can be reduced to subsumption (for languages with negation) and to the satisfiability problem, as well
- **Concept satisfiability** is $C$ (necessarility) empty?
- **Instance Checking**  $\text{Father}$(john)?
- **Equivalence**  $\text{CreatureWithHeart} \equiv \text{CreatureWithKidney}$
- **Disjointness**  $C \cap D$
- **Retrieval**  $\text{Father}(X)$? $X = \{\text{john, robert}\}$
- **Realization**  $X(\text{john})$? $X = \{\text{Father}\}$

Definitions

- A **definition** is a description of a concept or a relationship
- It is used to assign a meaning to a term
- In description logics, definitions use a specialized logical language
- Description logics are able to do limited reasoning about concepts defined in their logic
- One important inference is classification (computation of subsumption)

Necessary vs. Sufficient

- **Necessary** properties of an object are common to all objects of that type
  - Being a man is a necessary condition for being a father
- **Sufficient** properties allow one to identify an object as belonging to a type and need not be common to all members of the type
  - Speeding is a sufficient reason for being stopped by the police
- Definitions often specify both *necessary and sufficient* properties
Subsumption

- Meaning of Subsumption
  
  A more general concept or description is said to **subsume** a more specific one. Members of a subsumed concept are necessarily members of a subsuming concept.

- Two ways to formalize the meaning of subsumption
  
  - Using logic
    
    - Satisfying a subsumed concept implies that the subsuming concept is satisfied also
      
      E.g., if John is a person, he is also an animal.
    
  - Using set theory
    
    - The instances of a subsumed concept are necessarily a subset of the subsuming concept’s instances
      
      E.g., the set of all persons is a subset of all animals.

How Does Classification Work?

- Using logic
  
  - Satisfying a subsumed concept implies that the subsuming concept is satisfied also
    
    E.g., if John is a person, he is also an animal.

- Using set theory
  
  - The instances of a subsumed concept are necessarily a subset of the subsuming concept’s instances
    
    E.g., the set of all persons is a subset of all animals.

Defining a “rabid dog”

- A rabid dog is defined as something that is both a dog and has at least one thing that is a kind of a rabies.

Classification as a “sick animal”

- We can easily prove that a rabid dog is a kind of a sick animal.
Defining “rabid animal”

A rabid animal is defined as something that is both an animal and has at least one thing that is a kind of a rabies

DL reasoners places concepts in hierarchy

Note: we can remove the subclass link from rabid animal to animal because it is redundant. We don’t need to. But humans like to see the simplest structure and it may be informative for agents as well.

We can easily prove that a rabid dog is a kind of rabid animal

Primitive versus Structured (Defined)

- Description logics reason with definitions
  - They prefer to have complete descriptions
  - A complete definition includes both necessary conditions and sufficient conditions
- This is often impractical or impossible, especially with natural kinds.
- A “primitive” definition is an incomplete definition
  - This limits the amount of classification that can be done automatically
- Example:
  - Primitive: a Person
  - Defined: Parent = Person with at least one child

Intentional versus Extensional Semantics

- Extensional Semantics are a model-theoretic idea.
  They define the meaning of a description by enumerating the set of objects that satisfy the description.
- Intensional Semantics defines the meaning of a description based on the intent or use of the description.
- Example:
  - Morning-Star  Evening-Star
    - Extensional: Same object, namely Venus
    - Intensional: Different objects, one meaning Venus seen in the morning and one in the evening.
**Definition vs. Assertion**

- A **definition** describes *intrinsic* properties of an object. The parts of a description have meaning as a part of a composite description of an object.
- An **assertion** is used to describe an *incidental* property of an object. Asserted facts have meaning on their own.
- Example: “a black telephone”
  Could be either a description or an assertion, depending on the meaning and import of “blackness” on the concept telephone.

**Classification is very useful**

- Classification is a powerful kind of reasoning that is very useful
- Many expert systems can be usefully thought of as doing “heuristic classification”
- Logical classification over structured descriptions and individuals is also quite useful
- But... can classification ever deduce something about an individual other than what classes it belongs to?
- And what does *that* tell us?

**Definition versus Assertion**

- In English, “a black telephone” is ambiguous
  (1) A black telephone is a common sight in an office
  (2) A black telephone is on the corner of my desk
- KR languages should not be ambiguous so typically distinguish between descriptions of classes and descriptions of individuals
- KR languages often allow additional assertions to be made that are not part of the definition (e.g., annotation properties in OWL)

**Example: Blood Pressure**

A Non-Critical Blood Pressure is “a Systolic B.P. between 85 and 160.”
Example: Blood Pressure

Normal Systolic B.P. is “a Systolic B.P. between 90 and 140.

If Joe’s BP is Normal is it also Non-Critical?

With Classified Concepts the Answer is Easy to Compute

Non-Critical Systolic BP

Normal Systolic BP

Systolic Blood Pressure
Incidental properties

- If we allow incidental properties (e.g., ones that don’t participate in the description mechanism) then these can be deduced via classification

Some DL reasoners

  - CEL, free (for non-commercial use), LISP
  - Cerebra Engine, commercial, C++
  - FaCT++, free, open-source, C++
  - KAON2 free (for non-commercial usage), Java
  - MSPASS free, open-source, C
  - Pellet free, open-source, Java
  - RacerPro commercial, LISP
- DIG is a standard interface to a DL reasoner that predates RDF and today uses XML
- Protégé uses DIG and can thus use any of several DL reasoners that have a DIG interface