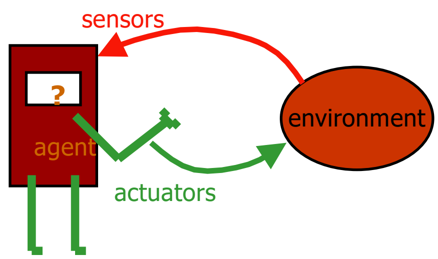
Decision Making Under Uncertainty

AI Class 10 (Ch. 15.1-15.2.1, 16.1-16.3)



X_t = unobserved
E_t = observed

Material from Marie desJardin, Lise Getoor, Jean-Claude Latombe, Daphne Koller, and Paula Matuszek

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Bookkeeping; reminders

- **Only the professor can change grades**
  - The TA cannot, although they can help you understand your grade
  - Grade change requests must be submitted:
    - **In writing**
    - To the professor, with the TA Cc'd
    - With a **clear justification** for the request
- Everyone in this class is expected to behave professionally at all times
  - Toward one another and toward the instructional staff
- Start homework well in advance
  - Bring questions, extension requests, etc. with time to spare

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Today's Class

- Making Decisions Under Uncertainty
  - Tracking Uncertainty over Time
  - Decision Making under Uncertainty
  - Decision Theory
  - Utility

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Introduction

- The world is not a well-defined place.
- Sources of uncertainty
  - Uncertain **inputs**: What's the temperature?
  - Uncertain (imprecise) **definitions**: Is Trump a good president?
  - Uncertain (unobserved) **states**: What's the top card?
- There is uncertainty in **inferences**
  - If I have a blistery, itchy rash and was gardening all weekend I **probably** have poison ivy

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Sources of Uncertainty

- Uncertain **inputs**
  - Missing data
  - Noisy data
- Uncertain **knowledge**
  - >1 cause → >1 effect
  - Incomplete knowledge of causality
  - Probabilistic effects
- Uncertain **outputs**
  - All uncertain:
    - Reasoning-by-default
    - Abduction & induction
    - Incomplete deductive inference
- Result is derived correctly but wrong in real world

Probabilistic reasoning only gives **probabilistic results**
(summarizes uncertainty from various sources)

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Reasoning Under Uncertainty

- People constantly make decisions anyhow.
  - Very successfully!
  - How?
  - More formally: how do we reason under uncertainty with inexact knowledge?
- Step one: **understanding what we know**

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Part I: Modeling Uncertainty Over Time

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States and Observations

- Agents don't have a continuous view of world
  - People don't either!
- We see things as a series of snapshots:
- **Observations**, associated with **time slices**
  - $t_1, t_2, t_3, \ldots$
- Each snapshot contains all variables, observed or not
  - $\mathbf{X}_t$ = (unobserved) state variables at time t; observation at t is $\mathbf{E}_t$
- This is **world state at time t**

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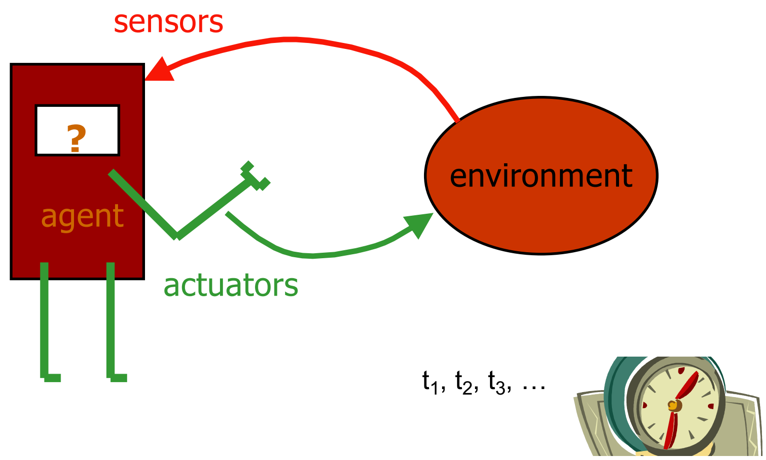
Temporal Probabilistic Agent



Uncertainty and Time

- The world changes; we need to track and predict it
  - Examples: diabetes management, traffic monitoring
  - How does blood sugar change over time?
- Tasks: **track** changes; **predict** changes
- Basic idea:
  - For each time step, copy state and evidence variables
  - Model uncertainty in **change over time** (the Δ)
  - Incorporate new observations as they arrive

Uncertainty and Time

- Basic idea:
  - Copy state and evidence variables for each time step
  - Model uncertainty in change over time
  - Incorporate new observations as they arrive
- $\mathbf{X}_t$ = unobserved/unobservable state variables at time t:
  $BloodSugar_t$, $StomachContents_t$
- $\mathbf{E}_t$ = evidence variables at time t:
  $MeasuredBloodSugar_t$, $PulseRate_t$, $FoodEaten_t$
- Assuming discrete time steps

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States (more formally)

- Change is viewed as series of snapshots
  - Time slices/timesteps
  - Each describing the **state** of the world at a particular time
    - So we also refer to these as *states*
- Each time slice/timestep/state is represented as a set of random variables indexed by *t*:
  1. the set of unobservable **state variables** $\mathbf{X}_t$
  2. the set of observable **evidence variables** $\mathbf{E}_t$

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Observations (more formally)

- Time slice (a set of random variables indexed by t):
  1. the set of unobservable **state variables** $\mathbf{X}_t$
  2. the set of observable **evidence variables** $\mathbf{E}_t$
- An **observation** is a set of observed variable instantiations at some timestep
- Observation at time t: $\mathbf{E}_t = e_t$
  - (for some values e_t)
- $\mathbf{X}_{a:b}$ denotes the set of variables from $\mathbf{X}_a$ to $\mathbf{X}_b$

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Transition and Sensor Models

- **So how do we model change over time?**
- Transition model
  - Models how the world changes over time
  - Specifies a probability distribution…
    - Over state variables at time t
    - Given values at previous times

$$P(\mathbf{X}_t \mid \mathbf{X}_{0:t-1})$$

This can get exponentially large…

- Sensor model
  - Models how evidence (sensor data) gets its values
  - E.g.: $\text{BloodSugar}_t \rightarrow \text{MeasuredBloodSugar}_t$

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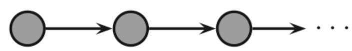
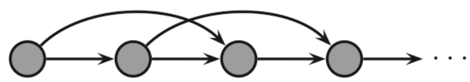
Markov Assumption(s)

- **Markov Assumption**:
  - $\mathbf{X}_t$ depends on some finite (usually fixed) number of previous $\mathbf{X}_i$'s
- **First-order Markov process**: $P(\mathbf{X}_t | \mathbf{X}_{0:t-1}) = P(\mathbf{X}_t | \mathbf{X}_{t-1})$



  - k^{th} order: depends on previous k time steps



- **Sensor Markov assumption**: $P(\mathbf{E}_t | \mathbf{X}_{0:t}, \mathbf{E}_{0:t-1}) = P(\mathbf{E}_t | \mathbf{X}_t)$
  - Agent's *observations* depend *only* on actual current state of the world

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Stationary Process

- Infinitely many possible values of t
  - Does each timestep need a distribution?
    - That is, do we need a distribution of what the world looks like at t_3, given t_2 AND a distribution for t_{16} given t_{15} AND …
- Assume **stationary process**:
  - Changes in the world state are governed by laws that do not themselves change over time
  - Transition model $P(\mathbf{X}_t | \mathbf{X}_{t-1})$ and sensor model $P(\mathbf{E}_t | \mathbf{X}_t)$ are time-invariant, i.e., they are the same for all t

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Complete Joint Distribution

- Given:
  - Transition model: $P(\mathbf{X}_t | \mathbf{X}_{t-1})$
  - Sensor model: $P(\mathbf{E}_t | \mathbf{X}_t)$
  - Prior probability: $P(\mathbf{X}_0)$
- Then we can specify a **complete joint distribution** of a sequence of states:

$$P(X_0, X_1, \ldots, X_t, E_1, \ldots, E_t) = P(X_0) \prod_{i=1}^{t} P(X_i \mid X_{i-1}) P(E_i \mid X_i)$$

- What's the joint probability of specific instantiations?

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Inference Tasks

- **Filtering** or monitoring: $P(\mathbf{X}_t | e_1,\ldots,e_t)$:
  - Compute the current belief state, given all evidence to date
- **Prediction**: $P(\mathbf{X}_{t+k} | e_1,\ldots,e_t)$:
  - Compute the probability of a future state
- **Smoothing**: $P(\mathbf{X}_k | e_1,\ldots,e_t)$:
  - Compute the probability of a past state (hindsight)
- **Most likely explanation**: $\arg\max_{x_1,\ldots x_t} P(x_1,\ldots,x_t | e_1,\ldots,e_t)$
  - Given a sequence of observations, find the sequence of states that is most likely to have generated those observations

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Inference Tasks

- **Filtering**: What is the probability that it is raining today, given all of the umbrella observations up through today?
- **Prediction**: What is the probability that it will rain the day after tomorrow, given all of the umbrella observations up through today?
- **Smoothing**: What is the probability that it rained yesterday, given all of the umbrella observations through today?
- **Most likely explanation**: If the umbrella appeared the first three days but not on the fourth, what is the most likely weather sequence to produce these umbrella sightings?

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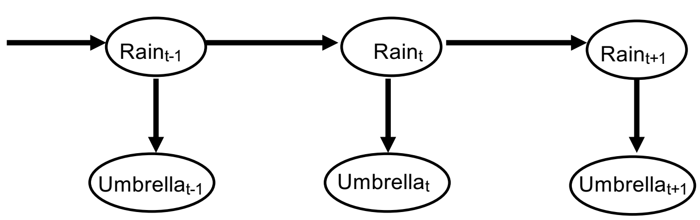
Example: Is it raining, given umbrellas?

R_{t-1}	$P(R_t \mid R_{t-1})$
t	0.7
f	0.3

Weather has a 30% chance of changing and a 70% chance of staying the same.



R_t	$P(U_t \mid R_t)$
t	0.9
f	0.2

If it's raining, the probability of someone carrying an umbrella is .9; if it's raining, the probability of NOT carrying an umbrella is .2

Fully worked out HMM for rain: http://www.sci.brooklyn.cuny.edu/~parsons/courses/740-fall-2011/notes/lect07.pdf

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Filtering

- For each day t, $\mathbf{E}_t$ contains variable U_t (whether the umbrella appears) and $\mathbf{X}_t$ contains state variable R_t (whether it's raining)
- Compute the current belief state, given all evidence to date
- Maintain a **current** state estimate and update it
  - Instead of looking at all observed values in history
  - Also called **state estimation**
- Given result of filtering up to time t, agent must compute result at t+1 from new evidence $\mathbf{e}_{t+1}$:

$$P(\mathbf{X}_{t+1} \mid \mathbf{e}_{1:t+1}) = f(\mathbf{e}_{t+1},\ P(\mathbf{X}_t \mid \mathbf{e}_{1:t}))$$

… for some function f.

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Filtering

- A good algorithm for filtering will maintain a current state estimate and update it at each point.
- $P(X_{t+1}|e_{1:t+1}) = f\,(P(X_t|e_{1:t}), e_{t+1})$
- Where X is the random variable and e is evidence
- Saves recomputation.
- It turns out that this is easy enough to come up with.

http://www.sci.brooklyn.cuny.edu/~parsons/courses/740-fall-2011/notes/lect07.pdf

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Filtering

- We rearrange the formula for:
  - $P(X_{t+1}|e_{1:t+1})$
- First, we divide up the evidence:
  - $P(X_{t+1}|e_{1:t+1}) = P(X_{t+1}|e_{1:t}, e_{t+1})$
- Then we apply Bayes rule, remembering the use of the normalization factor α:
  - $P(X_{t+1}|e_{1:t+1}) = \alpha P(e_{t+1}|X_{t+1}, e_{1:t})\, P(X_{t+1}|e_{1:t})$
- And after that we use the Markov assumption on the sensor model:
  - $P(X_{t+1}|e_{1:t+1}) = \alpha P(e_{t+1}|X_{t+1})P(X_{t+1}|e_{1:t})$
- The result of this assumption is to make that first term on the right hand side ignore all the evidence — the probability of the observation at $t + 1$ only depends on the value of X_{t+1}.

http://www.sci.brooklyn.cuny.edu/~parsons/courses/740-fall-2011/notes/lect07.pdf

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Filtering

- Let's look at that expression some more:
  - $P(X_{t+1}|e_{1:t+1}) = \alpha P(e_{t+1}|X_{t+1})P(X_{t+1}|e_{1:t})$
- The first term on the right updates with the new evidence and the second term on the right is a one step prediction from the evidence up to t to the state at $t + 1$.
- Next we condition on the current state P(X):
  - $P(X_{t+1}|e_{1:t+1}) = \alpha P(e_{t+1}|X_{t+1})\ \Sigma x_t\ P(X_{t+1}|x_t, e_{1:t})P(x_t|e_{1:t})$
- Finally, we apply the Markov assumption again:
  - $P(X_{t+1}|e_{1:t+1}) = \alpha P(e_{t+1}|X_{t+1})\ \Sigma x_t\ P(X_{t+1}|x_t)P(x_t|e_{1:t})$
- We'll call the bit on the right $f_{1:t}$

http://www.sci.brooklyn.cuny.edu/~parsons/courses/740-fall-2011/notes/lect07.pdf

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Filtering

- $f_{1:t}$ gives us the required recursive update.
  - The probability distribution over the state variables at $t + 1$ is a function of the transition model, the sensor model, and what we know about the state at time t.
- Space and time constant, independent of t.
- This allows a limited agent to compute the current distribution for any length of sequence.

http://www.sci.brooklyn.cuny.edu/~parsons/courses/740-fall-2011/notes/lect07.pdf

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Recursive Estimation

- We use ***recursive estimation*** to compute $P(X_{t+1} \mid e_{1:t+1})$ as a function of e_{t+1} and $P(X_t \mid e_{1:t})$

1. Project current state forward (t → t+1)
2. Update state using new evidence $\mathbf{e}_{t+1}$

$\mathrm{P}(\mathbf{X}_{t+1} \mid \mathbf{e}_{1:t+1})$ as function of $\mathbf{e}_{t+1}$ and $\mathrm{P}(\mathbf{X}_{t} \mid \mathbf{e}_{1:t})$:

$$\mathrm{P}(\mathbf{X}_{t+1} \mid \mathbf{e}_{1:t+1}) = \mathrm{P}(\mathbf{X}_{t+1} \mid \mathbf{e}_{1:t}, \mathbf{e}_{t+1})$$

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Recursive Estimation

- $P(\mathbf{X}_{t+1} \mid \mathbf{e}_{1:t+1})$ as a function of $\mathbf{e}_{t+1}$ and $P(\mathbf{X}_t \mid \mathbf{e}_{1:t})$:

$$P(X_{t+1} \mid e_{1:t+1}) = P(X_{t+1} \mid e_{1:t}, e_{t+1}) \quad \text{dividing up evidence}$$
$$= \alpha P(e_{t+1} \mid X_{t+1}, e_{1:t}) P(X_{t+1} \mid e_{1:t}) \quad \text{Bayes rule}$$
$$= \alpha P(e_{t+1} \mid X_{t+1}) P(X_{t+1} \mid e_{1:t}) \quad \text{sensor Markov assumption}$$

- $P(\mathbf{e}_{t+1} \mid \mathbf{X}_{1:t+1})$ updates with new evidence (from sensor)
- One-step prediction by conditioning on current state X:

$$= \alpha P(e_{t+1} \mid X_{t+1}) \sum_{x_t} P(X_{t+1} \mid x_t) P(x_t \mid e_{1:t})$$

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Recursive Estimation

- One-step prediction by conditioning on current state X:

$$P(\mathbf{X}_{t+1} \mid \mathbf{e}_{1:t+1}) = \alpha P(e_{t+1} \mid X_{t+1}) \sum_{x_t} \underbrace{P(X_{t+1} \mid x_t)}_{\text{transition model}} \underbrace{P(x_t \mid e_{1:t})}_{\text{current state}}$$

- …which is what we wanted!
- So, think of $P(\mathbf{X}_t \mid \mathbf{e}_{1:t})$ as a "message" $f_{1:t+1}$
  - Carried forward along the time steps
  - Modified at every transition, updated at every new observation
- This leads to a recursive definition:

$$f_{1:t+1} = \alpha \text{ FORWARD}(f_{1:t}, e_{t+1})$$

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Filtering: Umbrellas example

- The prior is ⟨0.5, 0.5⟩. (R=*t,* R=*f*)
- We can first predict whether it will rain on day 1 given what we already know:
- $\mathbf{P}(\mathbf{R}_1) = \Sigma_{r_0} \mathbf{P}(R_1|r_0)\, P(r_0)$
  $= \langle 0.7,0.3\rangle \times 0.5 + \langle 0.3,0.7\rangle \times 0.5$
  $= \langle 0.35,0.15\rangle + \langle 0.15,0.35\rangle$
  $= \langle 0.5,0.5\rangle$
- As we should expect, this just gives us the prior — that is the probability of rain when we don't have any evidence.

http://www.sci.brooklyn.cuny.edu/~parsons/courses/740-fall-2011/notes/lect07.pdf

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Filtering: Umbrellas example

- However, we have observed the umbrella, so that U_1 = *true*, and we can update using the sensor model:
- $\mathbf{P}(\mathbf{R}_1|U_1) = \alpha \mathbf{P}(u_1|R_1)\mathbf{P}(R_1)$
  $= \alpha\langle 0.9,0.2\rangle\langle 0.5,0.5\rangle$
  $= \alpha\langle 0.45,0.1\rangle$
  $\approx \langle 0.818,0.182\rangle$
- So, since umbrella is strong evidence for rain, the probability of rain is much higher once we take the observation into account.

http://www.sci.brooklyn.cuny.edu/~parsons/courses/740-fall-2011/notes/lect07.pdf

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Filtering: Umbrellas example

- We can then carry out the same computation for Day 2, first predicting whether it will rain on day 2 given what we already saw:
- $\mathbf{P}(\mathbf{R}_2 | u_1) = \Sigma_{r_1} \mathbf{P}(R_2 | r_1) P(r_1 | u_1)$
  $= \langle 0.7, 0.3 \rangle \times 0.818 + \langle 0.3, 0.7 \rangle \times 0.182$
  $\approx \langle 0.627, 0.373 \rangle$
- So even without evidence of rain on the second day there is a higher probability of rain than the prior because rain tends to follow rain.
  - (In this model rain tends to persist.)

http://www.sci.brooklyn.cuny.edu/~parsons/courses/740-fall-2011/notes/lect07.pdf

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Filtering: Umbrellas example

- Then we can repeat the evidence update, u_2 ($U_2 = true$), so:
- $\mathbf{P}(\mathbf{R}_2 | u_1, u_2) = \alpha \mathbf{P}(u_2 | R_2) \mathbf{P}(R_2 | u_1)$
  $= \alpha \langle 0.9, 0.2 \rangle \langle 0.627, 0.373 \rangle$
  $= \alpha \langle 0.565, 0.075 \rangle$
  $\approx \langle 0.883, 0.117 \rangle$
- So, the probability of rain increases again, and is higher than on day 1.

http://www.sci.brooklyn.cuny.edu/~parsons/courses/740-fall-2011/notes/lect07.pdf

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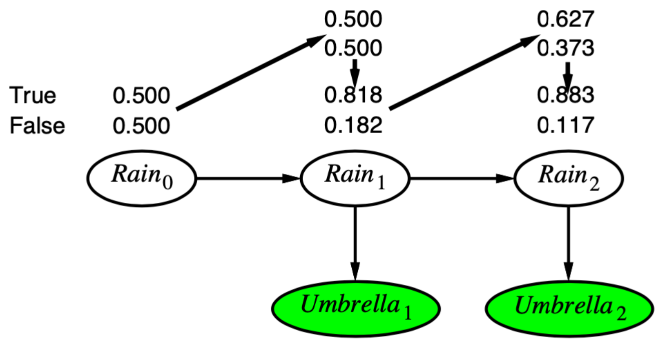
Filtering: Umbrellas example

- Put more succinctly:



- We can think of the calculation as messages passed along the chain

http://www.sci.brooklyn.cuny.edu/~parsons/courses/740-fall-2011/notes/lect07.pdf

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Umbrellas, summarized

- $P(Rain_1 = t)$
  $= \Sigma_{Rain_0} P(Rain_1 = t \mid Rain_0)\, P(Rain_0)$
  $= 0.70 * 0.50 + 0.30 * 0.50 =$ **0.50**
- $P(Rain_1 = t \mid Umbrella_1 = t)$
  $= \alpha\, P(Umbrella_1 = t \mid Rain_1 = t)\, P(Rain_1 = t)$
  $= \alpha * 0.90 * 0.50 = \alpha * 0.45 \approx$ **0.818**
- $P(Rain_2 = t \mid Umbrella_1 = t)$
  $= \Sigma_{Rain_1} P(Rain_2 = t \mid Rain_1)\, P(Rain_1 \mid Umbrella_1 = t)$
  $= 0.70 * 0.818 + 0.30 * 0.182 \approx$ **0.627**
- $P(Rain_2 = t \mid Umbrella_1 = t, Umbrella_2 = t)$
  $= \alpha\, P(Umbrella_2 = t \mid Rain_2 = t)\, P(Rain_2 = t \mid Umbrella_1 = t)$
  $= \alpha * 0.90 * 0.627 \approx \alpha * 0.564 \approx$ **0.883**

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Group Exercise: Filtering

$$P(X_{t+1} \mid e_{1:t+1}) = \alpha P(e_{t+1} \mid X_{t+1}) \sum_{X_t} P(X_{t+1} \mid X_t) P(X_t \mid e_{1:t})$$

$P(R_2 \mid U_1, U_2) = \alpha P(U_2 \mid R_2) \Sigma_{R1} P(R_2 \mid R_1) P(R_1 \mid U_1)$
$= 0.883$

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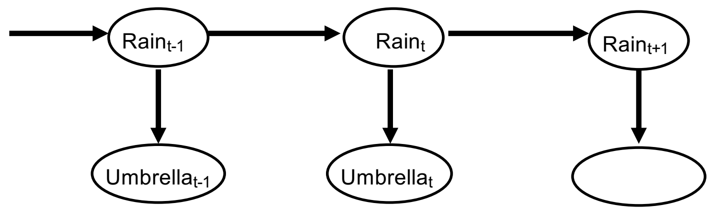
T	0.7
F	0.3

Weather has a 30% chance of changing and a 70% chance of staying the same.



What is the probability of rain on Day 2, given a uniform prior of rain on Day 0, U_1 = true, and U_2 = true?

R_t	$P(U_t \mid R_t)$
T	0.9
F	0.2

If it's raining, the probability of someone carrying an umbrella is .9; if it's raining, the probability of NOT carrying an umbrella is .2

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PART II: DECISION MAKING UNDER UNCERTAINTY

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Decision Making Under Uncertainty

- Many environments have multiple possible outcomes
- Some outcomes may be good; others may be bad
- Some may be very likely; others unlikely
- **What's a poor agent to do?**

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Reasoning Under Uncertainty

- How do we **reason** under uncertainty and with inexact knowledge?
- Heuristics
  - Mimic heuristic knowledge processing methods used by experts
- Empirical associations
  - Experiential reasoning based on limited observations
- **Probabilities**
  - Objective (frequency counting)
  - Subjective (human experience)

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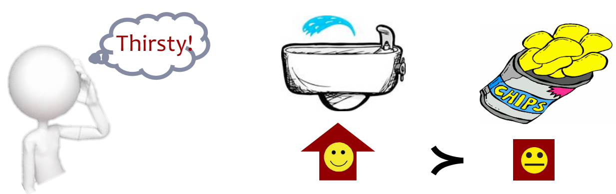
Decision-Making Tools

- Decision Theory
  - Normative: how *should* agents make decisions?
  - Descriptive: how *do* agents make decisions?
- **Utility** and utility functions
  - Something's **perceived ability to satisfy needs or wants**
  - A mathematical function that ranks alternatives by utility



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What is Decision Theory?

- Mathematical study of strategies for optimal decision-making
  - Options involve different risks
  - **Expectations** of gain or loss
- The study of identifying:
  - The values, uncertainties and other issues relevant to a decision
  - The resulting optimal decision for a rational agent

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Decision Theory

- Combines **probability** and **utility** → Agent that makes **rational** decisions (takes rational actions)
  - On average, lead to desired outcome
- First-pass simplifications:
  - Want most desirable *immediate* outcome (episodic)
  - Nondeterministic, partially observable world
- Definition of **action**:
- An action *a* in state *s* leads to outcome *s'*, RESULT:
  - RESULT(*a*) is a random variable; domain is possible outcomes
  - $P(\text{RESULT}(a) = s' \mid a, e))$

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Expected Value

- Expected Value
  - The **predicted future value** of a variable, calculated as:
  - The sum of all possible values
  - Each multiplied by the probability of its occurrence

A $1000 bet for a 20% chance to win $10,000?
EV = [20%($10,000) + 80%($0)] = $2000

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Satisficing

- Satisficing: achieving a goal sufficiently
  - Achieving the goal "more" does not increase utility of resulting state
  - Portmanteau of "satisfy" and "suffice"

Win a baseball game by 1 point now, or 2 points in another inning?

Full credit for a search is <= 3K nodes visited. You're at 2K. Spend an hour making it 1K?

Do you stop the coin flipping game at 1-0, or continue playing, hoping for 2-0?

At the end of semester, you can stop with a B. Do you take the exam?

You're thirsty. Water is good. Is more water better?

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Value Function

- Provides a **ranking** of alternatives, but not a meaningful metric scale
- Also known as an "ordinal utility function"
- Sometimes, only relative judgments (value functions) are necessary
- At other times, absolute judgments (utility functions) are required

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Rational Agents

- Rationality (an overloaded word).
- A rational agent…
  - Behaves according to a **ranking over possible outcomes**
  - Which is:
    - Complete (covers all situations)
    - Consistent
    - Optimizes over strategies to best serve a desired interest
- Humans are none of these.

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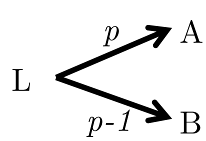
Preferences

- An agent chooses among:
  - *Prizes* (A, B, etc.)
  - *Lotteries* (situations with uncertain prizes and probabilities)



- Notation:
  - $A \succ B$ A preferred to B
  - $A \sim B$ Indifference between A and B
  - $A \succsim B$ B **not preferred** to A

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Expected Utility

- Goal: find best of expected outcomes
- Random variable X with:
  - n values $x_1,\dots,x_n$
  - Distribution $(p_1,\dots,p_n)$
- X is the state reached after doing an action A under uncertainty
  - state = some state of the world at some timestep
- Utility function U(s) is the utility of a state, i.e., **desirability**

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Expected Utility

- X is state reached after doing an action A under uncertainty
- U(s) is the utility of a state ← **desirability**
- EU(*a*|**e**): The **expected utility** of action A, given evidence, is the *average utility of outcomes* (states in S), weighted by probability an action occurs:

$$\mathrm{EU}[\mathrm{A}] = \mathrm{S}_{\mathrm{i}=1,\dots,\mathrm{n}}\, \mathrm{P}(\mathrm{x_i}|\mathrm{A})\mathrm{U}(\mathrm{x_i})$$

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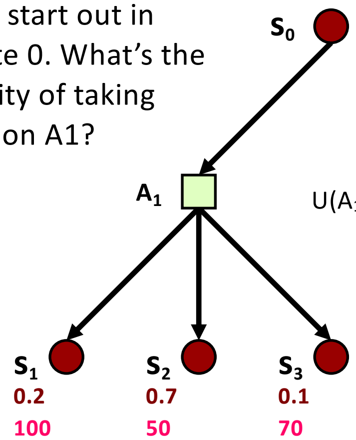
One State/One Action Example

- We start out in state 0. What's the utility of taking action A1?



$U(A_1, S_0) = \underline{100 \times 0.2} + \underline{50 \times 0.7} + \underline{70 \times 0.1}$
$= 20 + 35 + 7$
$= 62$

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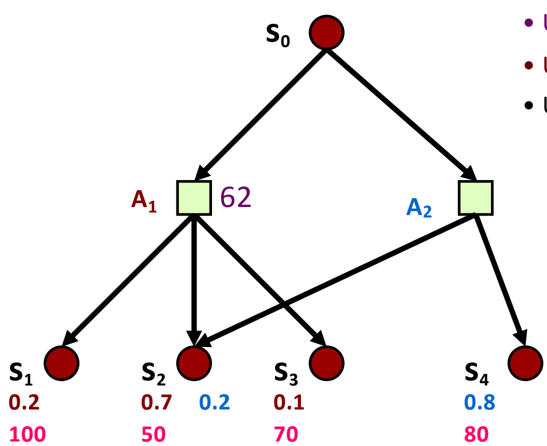
One State/Two Actions Example



- $U(A_1, S_0) = 62$
- $U(A_2, S_0) = 74$
- $U(S_0) = \max_a\{U(a,S_0)\}$
  $= 74$

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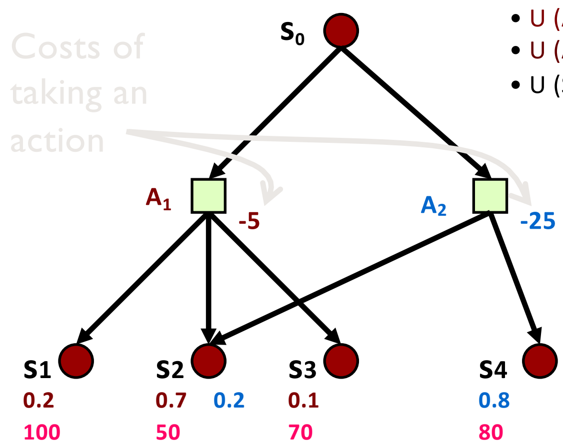
Introducing Action Costs



- $U(A_1, S_0) = 62 - 5 = 57$
- $U(A_2, S_0) = 74 - 25 = 49$
- $U(S_0) = \max_a\{U(a, S_0)\}$
  $= 57$

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MEU Principle

- A **rational agent** should choose the action that maximizes agent's expected utility
- This is the basis of the field of **decision theory**
- The MEU principle provides a normative criterion for rational choice of action
- Decision-making is solved!
  - Not quite…

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Rational Preferences

- Preferences of a rational agent must obey constraints
  - Transitivity $(A \succ B) \land (B \succ C) \Rightarrow (A \succ C)$
  - Monotonicity $(A \succ B) \Rightarrow [p > q \Leftrightarrow [p, A; 1 - p, B] \succ [q, A; 1 - q, B])$
  - Orderability $(A \succ B) \lor (B \succ A) \lor (A \sim B)$
  - Substitutability $(A \sim B) \Rightarrow [p,A; 1 - p, C] \sim [p,B; 1 - p,C]$)
  - Continuity $(A \succ B \succ C \Rightarrow \exists p\ [p,A; 1-p,C] \sim B$)
- Rational preferences give behavior that **maximizes expected utility**
- Violating these constraints leads to irrationality
  - For example: an agent with intransitive preferences can be induced to give away all its money.

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Not Quite…

- Must have a **complete** model of:
  - Actions
  - Utilities
  - States
- Even if you have a complete model, decision making is computationally **intractable**
- In fact, a truly rational agent takes into account the utility of reasoning as well (**bounded rationality**)
- Nevertheless, great progress has been made in this area
  - We are able to solve much more complex decision-theoretic problems than ever before

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Money

- Money does not behave as a utility function
  - That is, people don't maximize expected value of dollars.
- People are risk-averse:
  - Given a lottery L with expected monetary value EMV(L), usually U(L) < U(EMV(L))

Want to bet $1000 for a 20% chance to win $10,000?
[20%($10,000)+80%($0)] = $2000 > [100%($1000)]

- Expected Utility Hypothesis
  - rational behavior maximizes the expectation of some function u… which need not be monetary

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Money Versus Utility

- Money Utility
  - More money is better, but not always in a linear relationship to the amount of money
- Expected Monetary Value
- Risk-averse: $U(L) < U(S_{EMV(L)})$
- Risk-seeking: $U(L) > U(S_{EMV(L)})$
- Risk-neutral: $U(L) = U(S_{EMV(L)})$

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Maximizing Expected Utility

- Utilities map **states** to **real numbers**.
  - Which numbers?
- People are **terrible** at mapping their preferences
  - Give each of these things a utility between 1 and 10:
    - Winning the lottery
    - Getting an A on an exam
    - Failing a class (you won't though)
    - Getting hit by a truck

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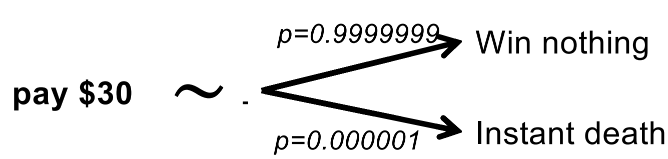
Maximizing Expected Utility

- Standard approach to assessment of human utilities:
  - Compare a state A to a standard lottery L_p that has
    - "best possible prize" $u\top$ with probability p
    - "worst possible catastrophe" $u^\perp$ with probability $(1-p)$
  - adjust lottery probability p until $A \sim L_p$



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On a Less Grim Note

- You are designing a cool new robot-themed attraction for Disneyworld!
- You could add a part that takes the project from $500M to $750M
- What piece of information do you need to decide whether this is the best action to take?

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