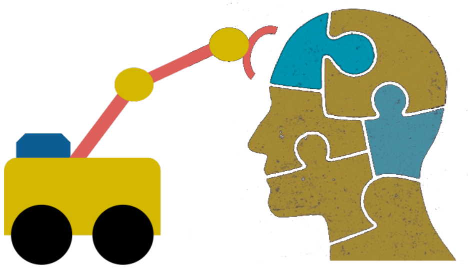
9.3.2



First-Order Logic (FOL) part 2

Overview

- We'll first give some examples of how to translate between FOL and English
- Then look at modelling family relations in FOL
- And finally touch on a few other topics

Translating English to FOL

Every gardener likes the sun

$\forall x\ gardener(x) \rightarrow likes(x,Sun)$

All purple mushrooms are poisonous

$\forall x\ (mushroom(x) \wedge purple(x)) \rightarrow poisonous(x)$

No purple mushroom is poisonous (two ways)

$\neg\exists x\ purple(x) \wedge mushroom(x) \wedge poisonous(x)$

$\forall x\ (mushroom(x) \wedge purple(x)) \rightarrow \neg poisonous(x)$

English to FOL: Counting

Use = predicate to identify different individuals

There are <u>at least</u> two purple mushrooms

$\exists x\ \exists y\ mushroom(x) \wedge purple(x) \wedge mushroom(y) \wedge purple(y) \wedge \neg\mathbf{(x{=}y)}$

This says that there exisit an x and a y such that

– "x is a purple mushroom" and
– "y is a purple mushroom" and
– "x and y are not the same objects"

English to FOL: Counting

There are <u>exactly</u> two purple mushrooms

$\exists x\ \exists y\ \text{mushroom}(x) \wedge \text{purple}(x) \wedge \text{mushroom}(y) \wedge \text{purple}(y) \wedge \neg\boldsymbol{(x=y)} \wedge$
$\forall z\ (\text{mushroom}(z) \wedge \text{purple}(z)) \rightarrow \boldsymbol{((x=z) \vee (y=z))}$

This says that

- "x is a purple mushroom" and
- "y is a purple mushroom" and
- "x and y are not the same objects"
- If there's a purple mushroom z, then either z=x or z=y

Saying there are 802 different <u>Pokemon</u> is hard!

Direct use of FOL is not for everything!

Translating English to FOL

What do these mean?

- **You can fool *some of* the people *all of* the time**
- **You can fool *all of* the people *some of* the time**

Translating English to FOL

What do these mean?

Both English statements are ambiguous

- **You can fool *some of* the people *all of* the time**

  #1 There is a nonempty subset of people so easily fooled that you can fool that subset every time*

  #2 For any given time, there is a non-empty subset at that time that you can fool

- **You can fool *all of* the people *some of* the time**

  #1 There are one or more times when it's possible to fool everyone*

  #2 Each individual can be fooled at some point in time

* Most common interpretation, I think

Some terms we will need

- **person(x):** True iff x is a person
- **time(t):** True iff t is a point in time
- **canFool(x, t):** True iff x can be fooled at time t

Note: *iff* = *if and only if* = ↔

Translating English to FOL

You can fool *some of* the people *all of* the time

#1 There is a nonempty group of people so easily fooled that you can fool that group every time*

≡ There's (at least) one person you can fool every time

$\exists \mathbf{x}\ \forall \mathbf{t}\ \ \text{person}(x) \land \text{time}(t) \rightarrow \text{canFool}(x, t)$

#2 For any given time, there is a non-empty group at that time that you can fool

≡ For every time, there's a person at that time that you can fool

$\forall \mathbf{t}\ \exists \mathbf{x}\ \ \text{person}(x) \land \text{time}(t) \rightarrow \text{canFool}(x, t)$

* Most common interpretation, I think

Translating English to FOL

You can fool *all of* the people *some of* the time

#1 There's at least one time when you can fool everyone*

$\exists \mathbf{t}\ \forall \mathbf{x}\ \text{time}(t) \land \text{person}(x) \rightarrow \text{canFool}(x, t)$

#2 Everybody can be fooled at some point in time

$\forall \mathbf{x}\ \exists \mathbf{t}\ \text{person}(x) \land \text{time}(t) \rightarrow \text{canFool}(x, t)$

* Most common interpretation, I think

Representation Design

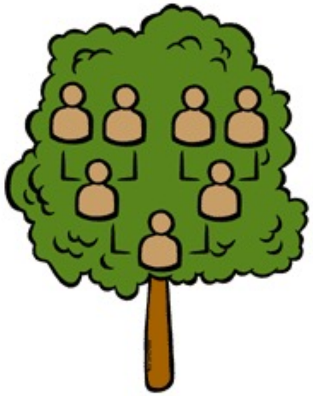
- Many options for representing even a simple fact, e.g., something's color as red, green or blue, e.g.:
  - green(kermit)
  - color(kermit, green)
  - hasProperty(kermit, color, green)
- Choice can influence how easy it is to use
- Last option of representing properties & relations as triples used by modern knowledge graphs
  - Easy to ask: What color is Kermit? What are Kermit's properties?, What green things are there? What properties are there? What thinks Tell me everything you know, …

Simple genealogy KB in FOL

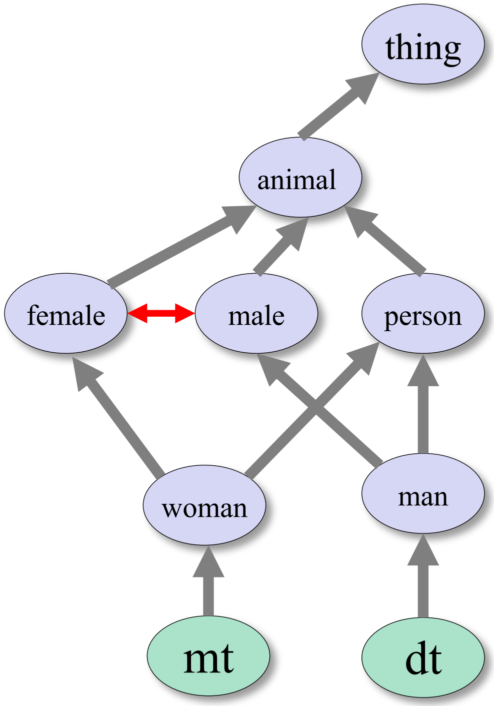
Design a knowledge base using FOL that

- Has facts of immediate family relations, e.g., spouses, parents, etc.
- Defines more complex relations (ancestors, relatives)
- Detect inconsistencies, e.g., a person is her own parent
- Infers relations, e.g., grandparent from parent
- Answers queries about relationships between people

How do we approach this?



- Design an initial ontology of types, e.g.
  - person, animal, man, woman, …
- Types form a taxonomy or lattice*, e.g.
  - person(X) <=> man(X) ∨ woman(Y)
  - man(X) <=> person(X) ∧ male(X)
  - woman(X) <=> person(X) ∧ female(X)
  - female(X) <=> ~ male(X)
- Make assertions about individuals, e.g.
  - man(donaldTrump)
  - woman(melaniaTrump)



* In a lattice, objects can have multiple immediate types

Extend with relations and constraints

- Simple two argument relations, e.g.
  - spouse, has_child, has_parent
- Add general constraints to relations, e.g.
  - spouse(X,Y) => ~ (X = Y)
  - spouse(X,Y) => person(X) ∧ person(Y)
  - spouse(X,Y) => (man(X) ∧ woman(Y)) ∨
    (woman(X) ∧ man(Y))*
- Add FOL sentences for inference, e.g.
  - spouse(X,Y) ⇔ spouse(Y,X)
- Add instance data
  - e.g., spouse(djt, mt)

* *Note this constraint is a traditional one than no longer holds*

Example: A simple genealogy KB in FOL

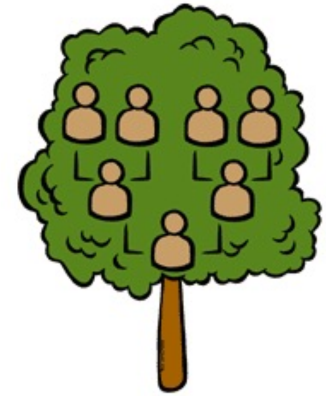
Predicates:

- parent(X, Y), child(X, Y), father(X, Y), daughter(X, Y), etc.
- spouse(X, Y), husband(X, Y), wife(X,Y)
- ancestor(X, Y), descendant(X, Y)
- male(X), female(Y)
- relative(X, Y)

Facts:

- husband(joe, mary), son(fred, joe)
- spouse(john, nancy), male(john), son(mark, nancy)
- father(jack, nancy), daughter(linda, jack)
- daughter(liz, linda)
- etc.

Example Axioms



$(\forall X,y)$ parent(X, Y) $\leftrightarrow$ child (Y, X)

$(\forall X,Y)$ father(X, Y) $\leftrightarrow$ parent(X, Y) $\wedge$ male(X)

$(\forall X,Y)$ mother(X, Y) $\leftrightarrow$ parent(X, Y) $\wedge$ female(X)

$(\forall X,Y)$ daughter(X, Y) $\leftrightarrow$ child(X, Y) $\wedge$ female(X)

$(\forall X,Y)$ son(X, Y) $\leftrightarrow$ child(X, Y) $\wedge$ male(X)

$(\forall X,Y)$ husband(X, Y) $\leftrightarrow$ spouse(X, Y) $\wedge$ male(X)

$(\forall X,Y)$ spouse(X, Y) $\leftrightarrow$ spouse(Y, X)

…

Axioms, definitions and theorems

- **Axioms**: facts and rules that capture (important) facts & concepts in a domain; used to prove **theorems**
  - Mathematicians dislike unnecessary (dependent) axioms, i.e., ones that can be derived from others
  - Dependent axioms can make reasoning faster, however
  - Choosing a good set of axioms is a design problem
- A **definition** of a predicate is of the form "p(X) $\leftrightarrow$ …" and can be decomposed into two parts
  - **Necessary** description: "p(X) $\rightarrow$ …"
  - **Sufficient** description "p(X) $\leftarrow$ …"
  - Some concepts have definitions (e.g., triangle) and some don't (e.g., person)

More on definitions

Example: define father(x, Y) by parent(X, Y) & male(X)

- **parent(X, Y)** is a **necessary** (but not sufficient) description of father(X, Y)

  $\text{father}(X, Y) \rightarrow \text{parent}(X, Y)$

- **parent(X, Y) ^ male(X) ^ age(X, 35)** is a **sufficient** (but not necessary) description of father(X, Y):

  $\text{father}(X, Y) \leftarrow \text{parent}(X, Y) \wedge \text{male}(X) \wedge \text{age}(X, 35)$

- **parent(X, Y) ^ male(X)** is a **necessary and sufficient** description of father(X, Y)

  $\text{parent}(X, Y) \wedge \text{male}(X) \leftrightarrow \text{father}(X, Y)$

Necessary and sufficient descriptions are definitions

Higher-order logic

- FOL only lets us quantify over variables, and **variables can only range over objects**
- HOL allows us to quantify over relations, e.g.

  "two functions are equal iff they produce the same value for all arguments"

  $\forall f \, \forall g \, (f = g) \leftrightarrow (\forall x \, f(x) = g(x))$

- E.g.: (quantify over predicates)

  $\forall r \text{ transitive}(r) \rightarrow (\forall xyz) \, r(x,y) \wedge r(y,z) \rightarrow r(x,z))$

- More expressive, but reasoning is undecide-able, in general

Examples of FOL in use

- Semantics of W3C's Semantic Web stack (RDF, RDFS, OWL) is defined in FOL
- OWL Full is equivalent to FOL
- Other OWL profiles support a subset of FOL and are more efficient
- FOL oriented knowledge representation systems have many user friendly tools
- E.g.: Protégé for creating, editing and exploring OWL ontologies

Examples of FOL in use

Many practical approaches embrace the approach that "some data is better than none"

- The semantics of schema.org is only defined in natural language text
- Wikidata's knowledge graph has a rich schema
  - Many constraint/logical violations are flagged with warnings
  - However, not all, see this Wikidata query that finds people who are their own grandfather

Wikidata knowledge graph

- Community knowledge graph with ~1B statements about ~100M items
- Fine-grained **ontology** has ~2M types & ~9K properties
- Multilingual: all text values tagged with language id
- Has both a human and query interface
- Many community tools for editing, search, visualization, update

Wikidata web interface for the UMBC entity, Q735049

Huge Ontology

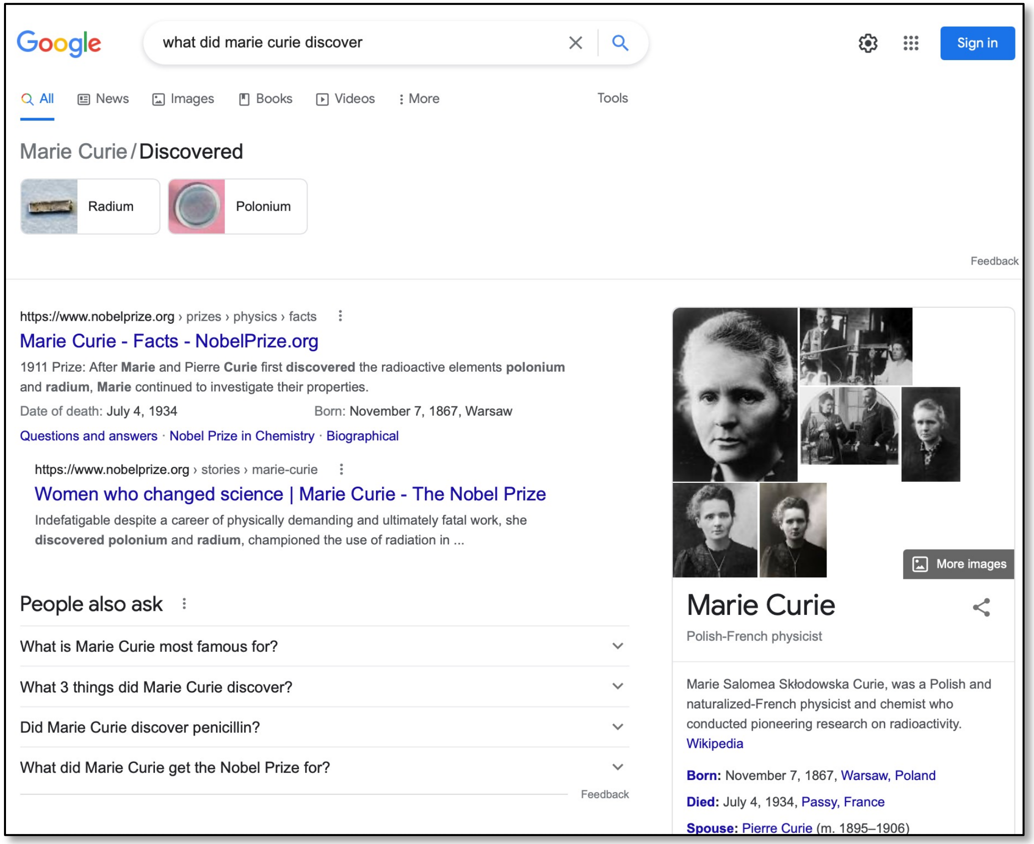
How can we understand an ontology with more than two million types?

- wdtaxonomy is a useful tool for exploring the ontology
- Given a type (e.g, Q3918, university) you can quickly see
  - Subtypes or supertypes (immediate or inferred),
  - Number of instances (immediate or inferred),
  - Direct instances
  - Number Wikimedia sites it’s in
- Implemented in javascript with a command line script

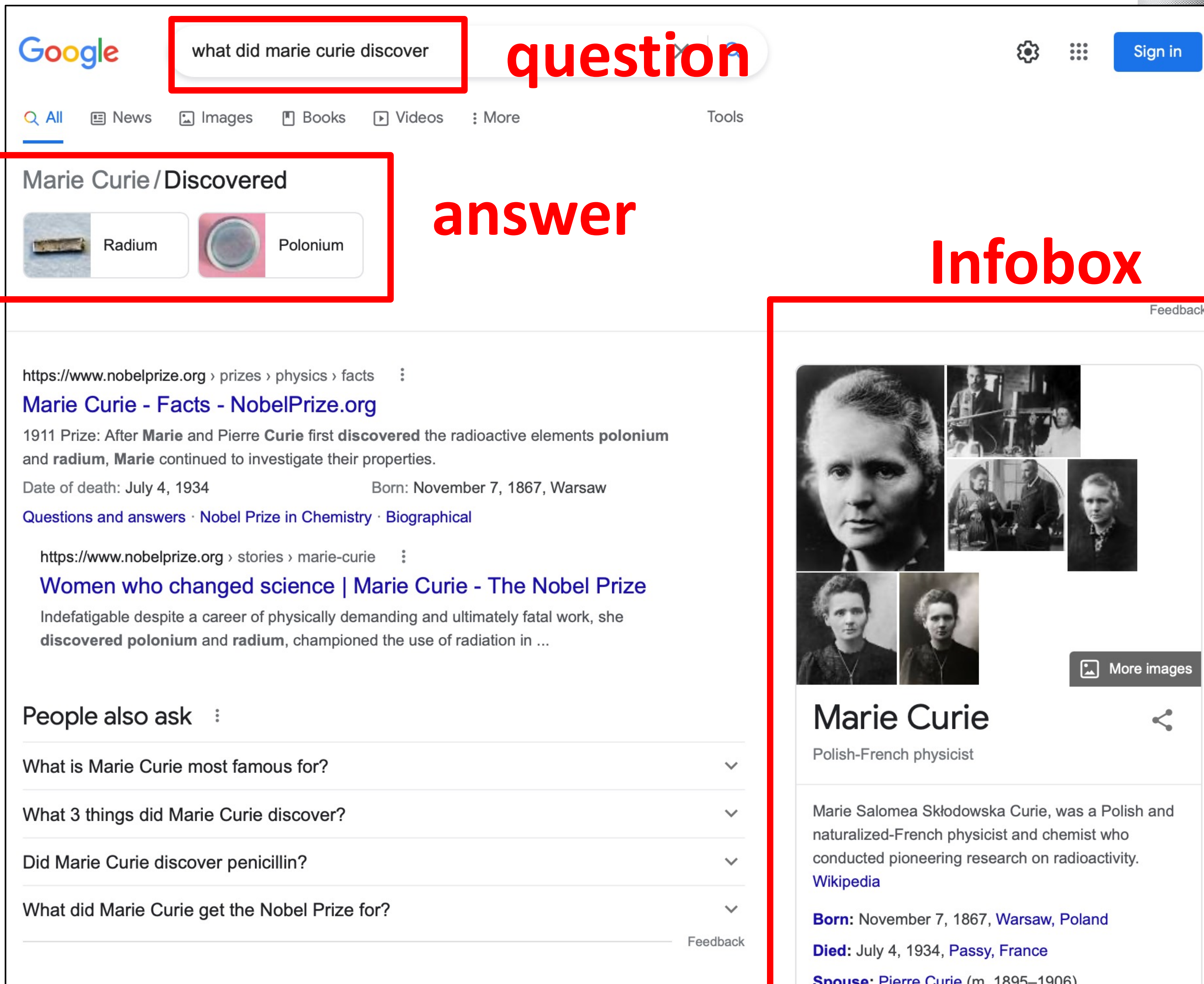
```
$$ wdtaxonomy Q3918 -c -t
university (Q3918) •163 ×15380 ↑↑
├──Universities in Germany (Q212462) •2
├──national university (Q265662) •11 ×73
├──National University (Q366354) •5
├──Imperial universities of Japan (Q562092) •12
├──Byzantine university (Q622870) •4
├──college and university rankings (Q847843) •23 ×45 ↑
├──public university (Q875538) •39 ×974 ↑
├──private university (Q902104) •32 ×846 ↑
├──new university (Q987075) •4 ×1
├──Red brick university (Q1202123) •11
├──??? (Q1305046) •2
├──institute of technology (Q1371037) •20 ×325
├──veterinary medicine school (Q1384955) •5 ×28
├──online university (Q1407393) •4 ×10 ↑
├──virtual university (Q1755248) •8 ×11
│ └──online university (Q1407393) •4 ×10 ↑ …
├──comprehensive university (Q1767829) •2 ×6
├──plate glass university (Q1902446) •8
├──medical university (Q1916585) •1 ×9 ↑
├──??? (Q2073922) •1
├──pontifical university (Q2120466) •18 ×37 ↑↑
├──Corporate university (Q2278672) •6
├──ancient university (Q2667285) •9 ×1
├──central university (Q3351682) •12 ×2
├──collegiate university (Q3354859) •9 ×12
├──deemed university (Q3520135) •6 ×16
├──university in France (Q3551775) •3 ×75 ↑
├──Istituto superiore per le industrie artistiche (Q3803831) •2 ×4
├──??? (Q3803846) •1 ×2
├──Smolny Institute for Noble Maidens (Q4432880) •1
├──??? (Q4475845) •2
├──federal university (Q4481793) •3 ×3
├──ecclesiastical university (Q5332280) •6 ×2
├──labor universities (Q5690751) •1 ×6
├──open university (Q6755402) •4 ×1
├──Urban university (Q7900184) •2
├──??? (Q10387922) •1
├──international university (Q10829188) •3 ×9
├──autonomous university (Q11057861) •2 ×1
├──research university (Q15936437) •9 ×224
├──Italian universities (Q20009854) •2
├──??? (Q20052016) •1 ×2
├──Canciller de Universidad (Q21547263)
├──imperial university of the Russian Empire (Q28667313) •2 ×12
├──universities in China (Q28700403) •1
├──Institute of National Importance (Q47531586) ×1 ↑
├──campusuniversity (Q59537665) ×3
├──Indiana University Bloomington Department of French and Italian (Q63441027)
├──Indiana University Department of French and Italian (Q63441251)
└──Indiana University Bloomington Department of History (Q63441447)
```

Virtual assistants and Infoboxes

- Web search engines and virtual assistants like Alexa use custom **knowledge graphs** to
  – help understand queries and content of web pages & documents
  – Answer questions
  – Show infoboxes
- Wikidata shares roots with these
- All draw on the similar knowledge, like the ~300 Wikipedia & Wikimedia sites



Virtual assistants & search engines



FOL Summary

- First order logic (FOL) introduces predicates, functions and quantifiers
- More expressive, but reasoning more complex
  - Reasoning in propositional logic is NP hard, FOL is semi-decidable
- Common AI knowledge representation language
  - Other KR languages (e.g., OWL) are often defined by mapping them to FOL
- FOL variables range over objects
  - HOL variables range over functions, predicates or sentences
- Some practical systems avoid enforcing rigid FOL constraints due to having noisy data

Fin