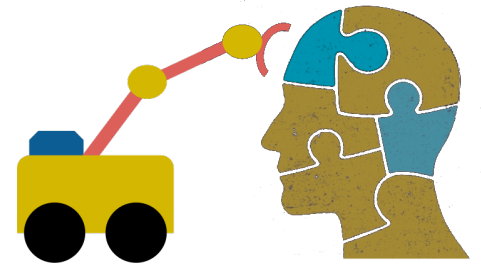


9.3.2



First-Order Logic (FOL) part 2

Overview

- We'll first give some examples of how to translate between FOL and English
- Then look at modelling family relations in FOL
- And finally touch on a few other topics

Translating English to FOL

Every gardener likes the sun

$$\forall x \text{ gardener}(x) \rightarrow \text{likes}(x, \text{Sun})$$

All purple mushrooms are poisonous

$$\forall x (\text{mushroom}(x) \wedge \text{purple}(x)) \rightarrow \text{poisonous}(x)$$

No purple mushroom is poisonous (two ways)

$$\neg \exists x \text{ purple}(x) \wedge \text{mushroom}(x) \wedge \text{poisonous}(x)$$

$$\forall x (\text{mushroom}(x) \wedge \text{purple}(x)) \rightarrow \neg \text{poisonous}(x)$$

English to FOL: Counting



Use = predicate to identify different individuals

There are at least two purple mushrooms

$$\exists x \exists y \text{ mushroom}(x) \wedge \text{purple}(x) \wedge \text{mushroom}(y) \wedge \text{purple}(y) \wedge \neg(x=y)$$

This says that there exist an x and a y such that

- “ x is a purple mushroom” and
- “ y is a purple mushroom” and
- “ x and y are not the same objects”

English to FOL: Counting



There are exactly two purple mushrooms

$$\begin{aligned} & \exists x \exists y \text{ mushroom}(x) \wedge \text{purple}(x) \wedge \text{mushroom}(y) \wedge \\ & \text{purple}(y) \wedge \neg(\mathbf{x=y}) \wedge \\ & \forall z (\text{mushroom}(z) \wedge \text{purple}(z)) \rightarrow ((\mathbf{x=z}) \vee (\mathbf{y=z})) \end{aligned}$$

This says that

- “x is a purple mushroom” and
- “y is a purple mushroom” and
- “x and y are not the same objects”
- If there’s a purple mushroom z, then either z=x or z=y

Saying there are 802 different Pokemon is hard!

Direct use of FOL is not for everything!

Translating English to FOL



What do these mean?

- You can fool *some of* the people *all of* the time
- You can fool *all of* the people *some of* the time

Translating English to FOL



What do these mean?

Both English statements are ambiguous

- **You can fool *some of the people all of the time***

#1 There is a nonempty subset of people so easily fooled that you can fool that subset every time*

#2 For any given time, there is a non-empty subset at that time that you can fool

- **You can fool *all of the people some of the time***

#1 There are one or more times when it's possible to fool everyone*

#2 Each individual can be fooled at some point in time

* Most common interpretation, I think

Some terms we will need



- **person(x)**: True iff x is a person
- **time(t)**: True iff t is a point in time
- **canFool(x, t)**: True iff x can be fooled at time t

Note: *iff* = *if and only if* = \leftrightarrow

Translating English to FOL



You can fool *some of* the people *all of* the time

#1 There is a nonempty group of people so easily fooled that you can fool that group every time*

≡ There's (at least) one person you can fool every time

$\exists x \forall t \text{ person}(x) \wedge \text{time}(t) \rightarrow \text{canFool}(x, t)$

#2 For any given time, there is a non-empty group at that time that you can fool

≡ For every time, there's a person at that time that you can fool

$\forall t \exists x \text{ person}(x) \wedge \text{time}(t) \rightarrow \text{canFool}(x, t)$

* Most common interpretation, I think

Translating English to FOL



You can fool *all of* the people *some of* the time

#1 There's at least one time when you can fool everyone*

$\exists t \forall x \text{ time}(t) \wedge \text{person}(x) \rightarrow \text{canFool}(x, t)$

#2 Everybody can be fooled at some point in time

$\forall x \exists t \text{ person}(x) \wedge \text{time}(t) \rightarrow \text{canFool}(x, t)$

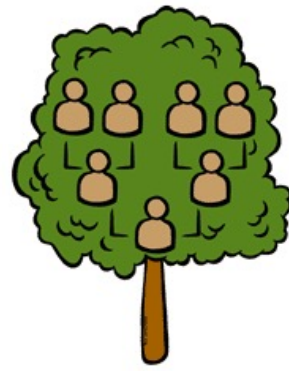
* Most common interpretation, I think

Representation Design



- Many options for representing even a simple fact, e.g., something's color as red, green or blue, e.g.:
 - green(kermit)
 - color(kermit, green)
 - hasProperty(kermit, color, green)
- Choice can influence how easy it is to use
- Last option of representing properties & relations as triples used by modern knowledge graphs
 - Easy to ask: What color is Kermit? What are Kermit's properties?, What green things are there? What properties are there? What thinks Tell me everything you know, ...

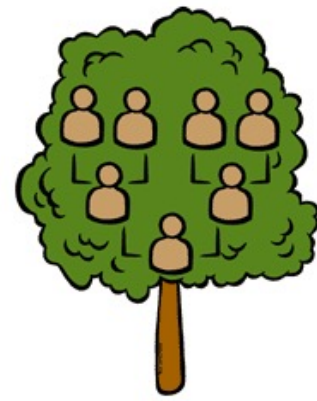
Simple genealogy KB in FOL



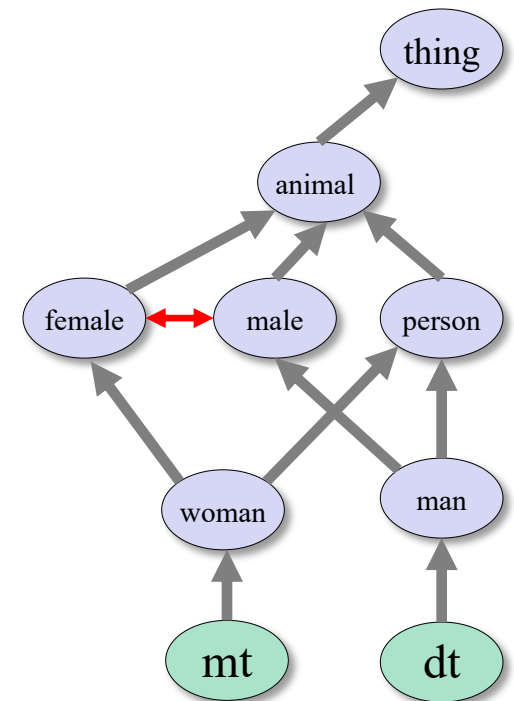
Design a knowledge base using FOL that

- Has facts of immediate family relations, e.g., spouses, parents, etc.
- Defines more complex relations (ancestors, relatives)
- Detect inconsistencies, e.g., a person is her own parent
- Infers relations, e.g., grandparent from parent
- Answers queries about relationships between people

How do we approach this?



- Design an initial ontology of types, e.g.
 - person, animal, man, woman, ...
- Types form a taxonomy or lattice*, e.g.
 - $\text{person}(X) \Leftrightarrow \text{man}(X) \vee \text{woman}(Y)$
 - $\text{man}(X) \Leftrightarrow \text{person}(X) \wedge \text{male}(X)$
 - $\text{woman}(X) \Leftrightarrow \text{person}(X) \wedge \text{female}(X)$
 - $\text{female}(X) \Leftrightarrow \sim \text{male}(X)$
- Make assertions about individuals, e.g.
 - $\text{man}(\text{donaldTrump})$
 - $\text{woman}(\text{melaniaTrump})$



* In a lattice, objects can have multiple immediate types

Extend with relations and constraints

- Simple two argument relations, e.g.
 - spouse, has_child, has_parent
- Add general constraints to relations, e.g.
 - $\text{spouse}(X,Y) \Rightarrow \sim (X = Y)$
 - $\text{spouse}(X,Y) \Rightarrow \text{person}(X) \wedge \text{person}(Y)$
 - $\text{spouse}(X,Y) \Rightarrow (\text{man}(X) \wedge \text{woman}(Y)) \vee (\text{woman}(X) \wedge \text{man}(Y))^*$
- Add FOL sentences for inference, e.g.
 - $\text{spouse}(X,Y) \Leftrightarrow \text{spouse}(Y,X)$
- Add instance data
 - e.g., $\text{spouse}(\text{djt}, \text{mt})$

** Note this constraint is a traditional one than no longer holds*

Example: A simple genealogy KB in FOL

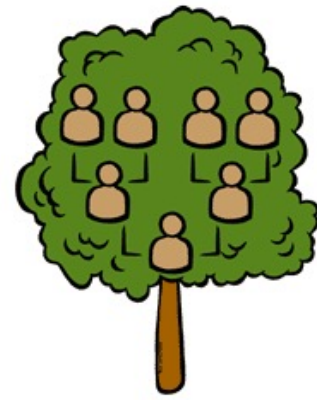
Predicates:

- parent(X, Y), child(X, Y), father(X, Y), daughter(X, Y), etc.
- spouse(X, Y), husband(X, Y), wife(X, Y)
- ancestor(X, Y), descendant(X, Y)
- male(X), female(Y)
- relative(X, Y)

Facts:

- husband(joe, mary), son(fred, joe)
- spouse(john, nancy), male(john), son(mark, nancy)
- father(jack, nancy), daughter(linda, jack)
- daughter(liz, linda)
- etc.

Example Axioms



$(\forall X, Y) \text{ parent}(X, Y) \leftrightarrow \text{child}(Y, X)$

$(\forall X, Y) \text{ father}(X, Y) \leftrightarrow \text{parent}(X, Y) \wedge \text{male}(X)$

$(\forall X, Y) \text{ mother}(X, Y) \leftrightarrow \text{parent}(X, Y) \wedge \text{female}(X)$

$(\forall X, Y) \text{ daughter}(X, Y) \leftrightarrow \text{child}(X, Y) \wedge \text{female}(X)$

$(\forall X, Y) \text{ son}(X, Y) \leftrightarrow \text{child}(X, Y) \wedge \text{male}(X)$

$(\forall X, Y) \text{ husband}(X, Y) \leftrightarrow \text{spouse}(X, Y) \wedge \text{male}(X)$

$(\forall X, Y) \text{ spouse}(X, Y) \leftrightarrow \text{spouse}(Y, X)$

...

Axioms, definitions and theorems

- **Axioms**: facts and rules that capture (important) facts & concepts in a domain; used to prove **theorems**
 - Mathematicians dislike unnecessary (dependent) axioms, i.e., ones that can be derived from others
 - Dependent axioms can make reasoning faster, however
 - Choosing a good set of axioms is a design problem
- A **definition** of a predicate is of the form “ $p(X) \leftrightarrow \dots$ ” and can be decomposed into two parts
 - **Necessary** description: “ $p(X) \rightarrow \dots$ ”
 - **Sufficient** description “ $p(X) \leftarrow \dots$ ”
 - Some concepts have definitions (e.g., triangle) and some don't (e.g., person)

More on definitions

Example: define $\text{father}(x, Y)$ by $\text{parent}(X, Y)$ & $\text{male}(X)$

- **$\text{parent}(X, Y)$** is a **necessary** (but not sufficient) description of $\text{father}(X, Y)$

$$\text{father}(X, Y) \rightarrow \text{parent}(X, Y)$$

- **$\text{parent}(X, Y) \wedge \text{male}(X) \wedge \text{age}(X, 35)$** is a **sufficient** (but not necessary) description of $\text{father}(X, Y)$:

$$\text{father}(X, Y) \leftarrow \text{parent}(X, Y) \wedge \text{male}(X) \wedge \text{age}(X, 35)$$

- **$\text{parent}(X, Y) \wedge \text{male}(X)$** is a **necessary and sufficient** description of $\text{father}(X, Y)$

$$\text{parent}(X, Y) \wedge \text{male}(X) \leftrightarrow \text{father}(X, Y)$$

Necessary and sufficient descriptions are definitions

Higher-order logic

- FOL only lets us quantify over variables, and **variables can only range over objects**
- HOL allows us to quantify over relations, e.g.
“two functions are equal iff they produce the same value for all arguments”

$$\forall f \forall g (f = g) \leftrightarrow (\forall x f(x) = g(x))$$

- E.g.: (quantify over predicates)

$$\forall r \text{transitive}(r) \rightarrow (\forall xyz) r(x,y) \wedge r(y,z) \rightarrow r(x,z))$$

- More expressive, but reasoning is undecidable, in general



Examples of FOL in use

- Semantics of W3C's [Semantic Web](#) stack (RDF, RDFS, OWL) is defined in FOL
- [OWL](#) Full is equivalent to FOL
- Other OWL profiles support a subset of FOL and are more efficient
- FOL oriented knowledge representation systems have many user friendly tools
- E.g.: Protégé for creating, editing and exploring OWL ontologies



schema.org

Examples of FOL in use



Many practical approaches embrace the approach that “some data is better than none”

- The semantics of schema.org is only defined in natural language text
- [Wikidata](https://www.wikidata.org)'s knowledge graph has a rich schema
 - Many constraint/logical violations are flagged with warnings
 - However, not all, see this [Wikidata query](#) that finds people who are their own grandfather



Wikidata knowledge graph

- Community knowledge graph with ~1B statements about ~100M items
- Fine-grained **ontology** has ~2M types & ~9K properties
- Multilingual: all text values tagged with language id
- Has both a human and query interface
- Many community tools for editing, search, visualization, update

The screenshot shows the Wikidata web interface for the entity 'University of Maryland, Baltimore County' (Q735049). The page includes the Wikidata logo, navigation links, and a list of statements. The statements section shows three entries: 'instance of university', 'public educational institution of the United States', and 'research university'. Each entry has an 'edit' button and a '1 reference' link. Below the statements, there is a 'logo image' section showing the UMBC logo with a '1 reference' link and an 'add value' button. The page also features a sidebar with various navigation and tool options.

Huge Ontology

How can we understand an ontology with more than two million types?

wdtaxonomy is a useful tool for exploring the ontology

Given a type (e.g, Q3918, university) you can quickly see

Subtypes or supertypes (immediate or inferred),

Number of instances (immediate or inferred),

Direct instances

Number Wikimedia sites it's in

Implemented in javascript with a command line script

```
$$ wdtaxonomy Q3918 -c -t
university (Q3918) •163 x15380 ↑↑
├─Universities in Germany (Q212462) •2
├─national university (Q265662) •11 x73
├─National University (Q366354) •5
├─Imperial universities of Japan (Q562092) •12
├─Byzantine university (Q622870) •4
├─college and university rankings (Q847843) •23 x45 ↑
├─public university (Q875538) •39 x974 ↑
├─private university (Q902104) •32 x846 ↑
├─new university (Q987075) •4 x1
├─Red brick university (Q1202123) •11
├─??? (Q1305046) •2
├─institute of technology (Q1371037) •20 x325
├─veterinary medicine school (Q1384955) •5 x28
├─online university (Q1407393) •4 x10 ↑
├─virtual university (Q1755248) •8 x11
├─┬─online university (Q1407393) •4 x10 ↑ ...
├─comprehensive university (Q1767829) •2 x6
├─plate glass university (Q1902446) •8
├─medical university (Q1916585) •1 x9 ↑
├─??? (Q2073922) •1
├─pontifical university (Q2120466) •18 x37 ↑↑
├─Corporate university (Q2278672) •6
├─ancient university (Q2667285) •9 x1
├─central university (Q3351682) •12 x2
├─collegiate university (Q3354859) •9 x12
├─deemed university (Q3520135) •6 x16
├─university in France (Q3551775) •3 x75 ↑
├─Istituto superiore per le industrie artistiche (Q3803831) •2 x4
├─??? (Q3803846) •1 x2
├─Smolny Institute for Noble Maidens (Q4432880) •1
├─??? (Q4475845) •2
├─federal university (Q4481793) •3 x3
├─ecclesiastical university (Q5332280) •6 x2
├─labor universities (Q5690751) •1 x6
├─open university (Q6755402) •4 x1
├─Urban university (Q7900184) •2
├─??? (Q10387922) •1
├─international university (Q10829188) •3 x9
├─autonomous university (Q11057861) •2 x1
├─research university (Q15936437) •9 x224
├─Italian universities (Q20009854) •2
├─??? (Q20052016) •1 x2
├─Canciller de Universidad (Q21547263)
├─imperial university of the Russian Empire (Q28667313) •2 x12
├─universities in China (Q28700403) •1
├─Institute of National Importance (Q47531586) x1 ↑
├─campusuniversity (Q59537665) x3
├─Indiana University Bloomington Department of French and Italian (Q63441027)
├─Indiana University Department of French and Italian (Q63441251)
├─Indiana University Bloomington Department of History (Q63441447)
```

Virtual assistants and Infoboxes



- Web search engines and virtual assistants like Alexa use custom **knowledge graphs** to
 - help understand queries and content of web pages & documents
 - Answer questions
 - Show infoboxes

- Wikidata shares roots with these

- All draw on the similar knowledge, like the ~300 Wikipedia & Wikimedia sites

Google search results for "what did marie curie discover". The search bar shows the query and a search icon. Below the search bar, there are navigation options: All, News, Images, Books, Videos, and More. The search results show an infobox for Marie Curie/Discovered, with images of Radium and Polonium. Below the infobox, there is a link to "Marie Curie - Facts - NobelPrize.org" and a snippet of text: "1911 Prize: After Marie and Pierre Curie first discovered the radioactive elements polonium and radium, Marie continued to investigate their properties." Below this, there is a "People also ask" section with questions like "What is Marie Curie most famous for?", "What 3 things did Marie Curie discover?", "Did Marie Curie discover penicillin?", and "What did Marie Curie get the Nobel Prize for?". To the right of the search results, there is a detailed infobox for Marie Curie, including a portrait, a "More images" button, and biographical information: "Marie Salomea Sklodowska Curie, was a Polish and naturalized-French physicist and chemist who conducted pioneering research on radioactivity. Wikipedia", "Born: November 7, 1867, Warsaw, Poland", "Died: July 4, 1934, Passy, France", and "Spouse: Pierre Curie (m. 1895-1906)".



Virtual assistants & search engines



Google **question** Sign in

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Marie Curie / Discovered **answer**

 Radium  Polonium **Infobox**

<https://www.nobelprize.org/prizes/physics/facts> **Marie Curie - Facts - NobelPrize.org**
1911 Prize: After Marie and Pierre Curie first discovered the radioactive elements polonium and radium, Marie continued to investigate their properties.
Date of death: July 4, 1934 Born: November 7, 1867, Warsaw
[Questions and answers](#) · [Nobel Prize in Chemistry](#) · [Biographical](#)







<https://www.nobelprize.org/stories/marie-curie> **Women who changed science | Marie Curie - The Nobel Prize**
Indefatigable despite a career of physically demanding and ultimately fatal work, she discovered polonium and radium, championed the use of radiation in ...

People also ask

- What is Marie Curie most famous for? ▾
- What 3 things did Marie Curie discover? ▾
- Did Marie Curie discover penicillin? ▾
- What did Marie Curie get the Nobel Prize for? ▾

[Feedback](#)

[Feedback](#)

[More images](#)

Marie Curie Polish-French physicist

Marie Salomea Skłodowska Curie, was a Polish and naturalized-French physicist and chemist who conducted pioneering research on radioactivity.
[Wikipedia](#)

Born: November 7, 1867, Warsaw, Poland
Died: July 4, 1934, Passy, France
Spouse: Pierre Curie (m. 1895–1906)

FOL Summary

- First order logic (FOL) introduces predicates, functions and quantifiers
- More expressive, but reasoning more complex
 - Reasoning in propositional logic is NP hard, FOL is semi-decidable
- Common AI knowledge representation language
 - Other KR languages (e.g., [OWL](#)) are often defined by mapping them to FOL
- FOL variables range over objects
 - HOL variables range over functions, predicates or sentences
- Some practical systems avoid enforcing rigid FOL constraints due to having noisy data

Fín