



Logical Inference 1 introduction

Chapter 9

Overview

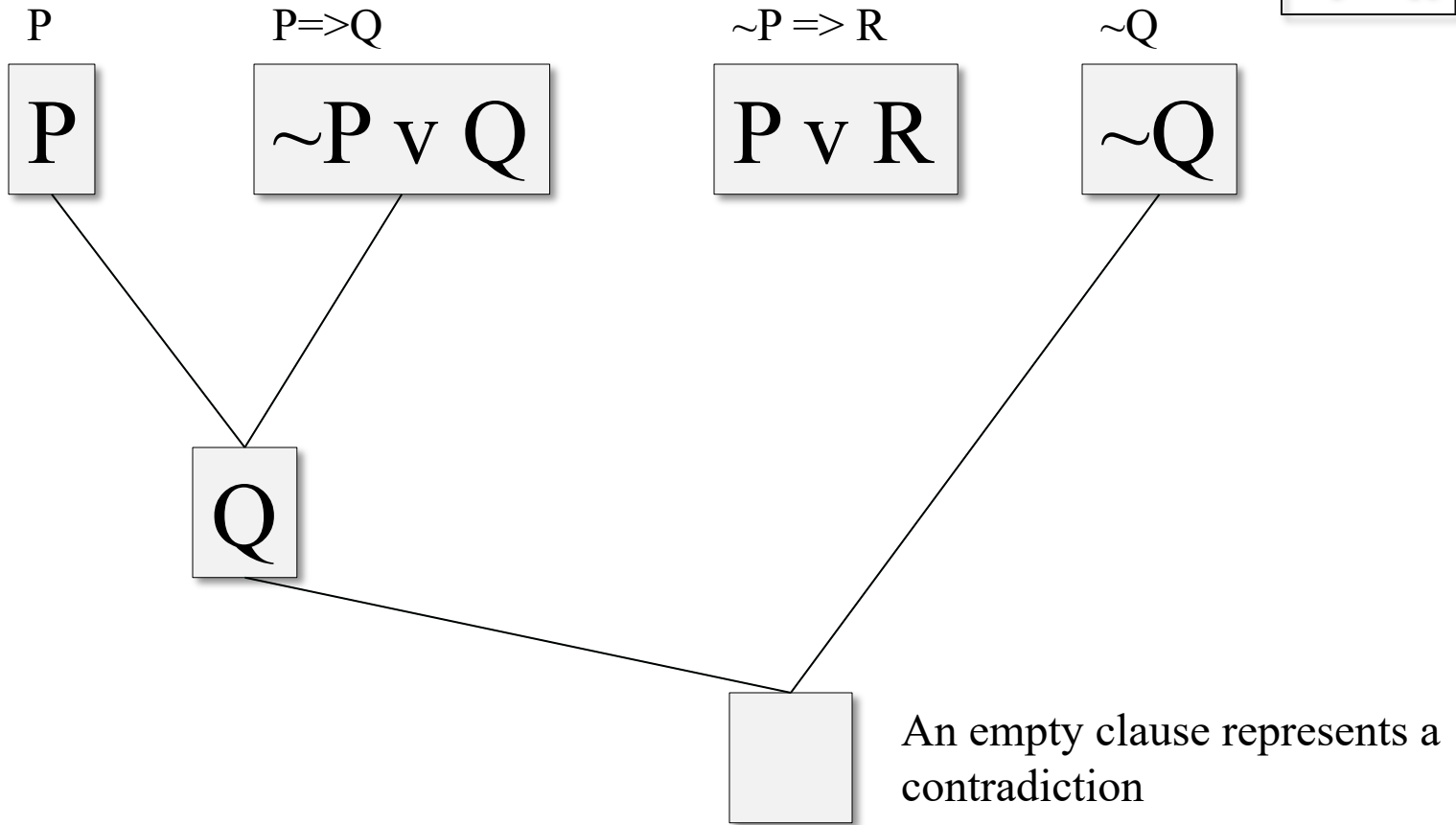
- A: Model checking for propositional logic
- Rule based reasoning in first-order logic
 - Inference rules and generalized modes ponens
 - Forward chaining
 - Backward chaining
- Resolution-based reasoning in first-order logic
 - Clausal form
 - Unification
 - Resolution as search
- Inference wrap up

From Satisfiability to Proof

- To see if a satisfiable KB entails sentence S , see if $KB \wedge \neg S$ is satisfiable
 - If it is not, then the KB entails S
 - If it is, then the KB does not entail S
 - This is a refutation proof
- Consider the KB with $(P, P \Rightarrow Q, \sim P \Rightarrow R)$
 - Does the KB entail Q ? R ?

Does the KB entail Q?

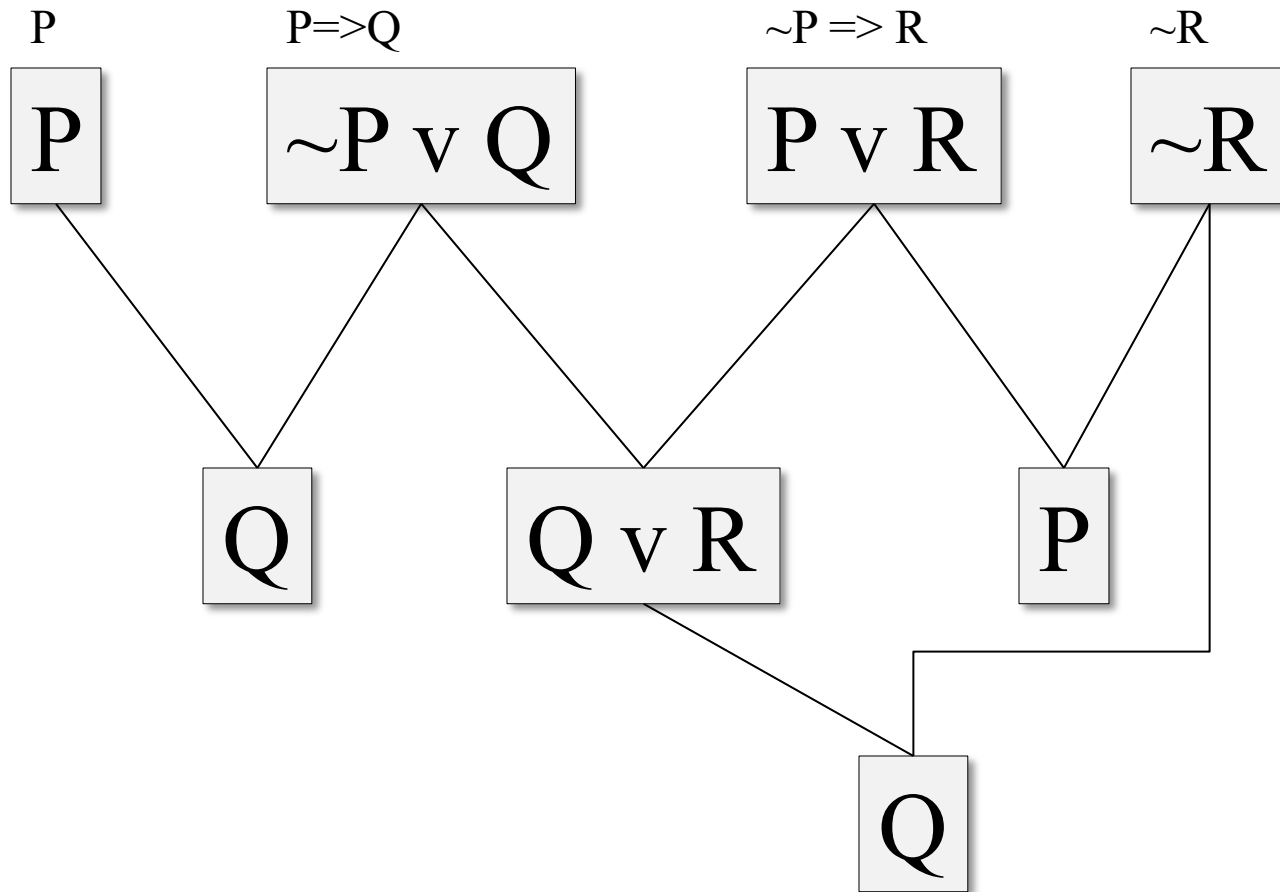
KB
P
P= \Rightarrow Q
 \sim P= \Rightarrow R



We assume that every sentence in the KB is true. Adding $\sim Q$ to the KB yields a contradiction, so $\sim Q$ must be false, so Q must be true.

Does the KB entail R?

KB
P
$P \Rightarrow Q$
$\sim P \Rightarrow R$



Adding $\sim R$ to KB does not produce a contradiction after drawing all possible conclusions, so it could be False, so KB doesn't entail R.

Propositional logic model checking

- Given KB, does a sentence S hold?
 - All the variables in S must be in the KB
 - A candidate model is just an assignment of T|F to every variable in the KB
- Basically generate and test:
 - Consider candidate models M for the KB
 - If $\forall M S$ is true, then S is **provably true**
 - If $\forall M \neg S$, then S is **provably false**
 - Otherwise ($\exists M1 S \wedge \exists M2 \neg S$): S is **satisfiable** but neither provably true or provably false

Efficient PL model checking (1)

Davis-Putnam algorithm (DPLL) is generate-and-test model checking with several optimizations:

- *Early termination*: short-circuiting of disjunction or conjunction sentences
- *Pure symbol heuristic*: symbols appearing only negated or un-negated must be FALSE/TRUE respectively
e.g., in $[(A \vee \neg B), (\neg B \vee \neg C), (C \vee A)]$ A & B are pure, C impure.
Make pure symbol literal true: if there's a model for S, making pure symbol true is also a model
- *Unit clause heuristic*: Symbols in a clause by itself can immediately be set to TRUE or FALSE

Using the AIMA Code

```
python> python
Python ...
>>> from logic import *
>>> expr('P & P==>Q & ~P==>R')
((P & (P >> Q)) & (~P >> R))
>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R'))
{R: True, P: True, Q: True}
>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R & ~R'))
{R: False, P: True, Q: True}
>>> dpll_satisfiable(expr('P & P==>Q & ~P==>R & ~Q'))
False
>>>
```

expr parses a string, and returns a logical expression

dpll_satisfiable returns a model if satisfiable else False

The KB entails Q but does not entail R

Efficient PL model checking (2)

- [WalkSAT](#): a local search for satisfiability: Pick a symbol to flip (toggle TRUE/FALSE), either using min-conflicts *or* choosing randomly
- ...or use *any* local or global search algorithm
- Many model checking algorithms & systems:
 - E.g.: [MiniSat](#): minimalistic, open-source SAT solver developed to help researchers & developers use SAT”
 - E.g.: [International SAT Competition](#) (2002...2020): identify new challenging **benchmarks** to promote new **solvers** for Boolean SAT”

AIMA KB Class

```
>>> kb1 = PropKB()
>>> kb1.clauses
[]
>>> kb1.tell(expr('P==>Q & ~P==>R'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
>>> kb1.tell(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P), P]
>>> kb1.ask(expr('Q'))
{}
>>> kb1.retract(expr('P'))
>>> kb1.clauses
[(Q | ~P), (R | P)]
>>> kb1.ask(expr('Q'))
False
```

PropKB is a subclass

A sentence is converted to CNF and the clauses added

The KB does not entail Q

After adding P the KB does entail Q

Retracting P removes it and the KB no longer entails Q

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