State Spaces & Partial-Order Planning

Material from Dr. Marie desJardin, Some material adopted from notes by Andreas Geyer-Schulz and Chuck Dyer

Overview

- What is planning?
- Approaches to planning
  - GPS / STRIPS
  - Situation calculus formalism [revisited]
  - Partial-order planning

Planning Problem

- What is the planning problem?
- Find a **sequence of actions** that achieves a **goal** when executed from an **initial state**.
- That is, given
  - A set of operators (possible actions)
  - An initial state description
  - A goal (description or conjunction of predicates)
- Compute a sequence of operations: a **plan**.

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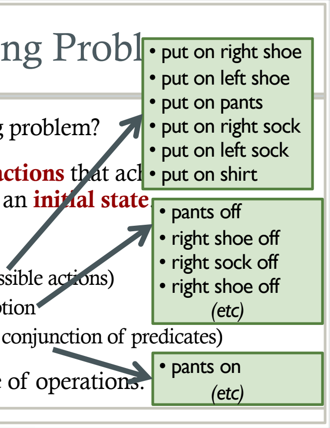


- put on right shoe
- put on left shoe
- put on pants
- put on right sock
- put on left sock
- put on shirt

- pants off
- right shoe off
- right sock off
- right shoe off
- *(etc)*

- pants on
- *(etc)*

Some example domains

- We'll use some simple problems to illustrate planning problems and algorithms
- Putting on your socks and shoes in the morning
  - Actions like put-on-left-sock, put-on-right-shoe
- Planning a shopping trip involving buying several kinds of items
  - Actions like go(X), buy(Y)

Typical Assumptions (1)

- **Atomic time**: Each action is indivisible
  - Can't be interrupted halfway through putting on pants
- **No concurrent actions** allowed
  - Can't put on socks at the same time
- **Deterministic actions**
  - The result of actions are completely known – no uncertainty

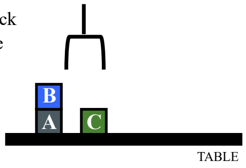
Typical Assumptions

- Agent is the **sole cause of change** in the world
  - Nobody else is putting on your socks
- Agent is **omniscient:**
  - Has complete knowledge of the state of the world
- **Closed world assumption**:
  - Everything known-true about the world is in the *state description*
  - Anything not known-true is known-false

Blocks World

The **blocks world** consists of a table, set of blocks, and a robot gripper

Some domain constraints:
- Only one block on another block
- Any number of blocks on table
- Hand can only hold one block

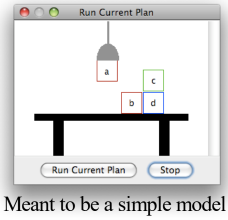


Typical representation:

ontable(a)	handempty
ontable(c)	on(b,a)
clear(b)	clear(c)

Blocks world

- A micro-world
- Some domain constraints:
  - Only one block can be on another block
  - Any number of blocks can be on the table
  - The hand can only hold one block

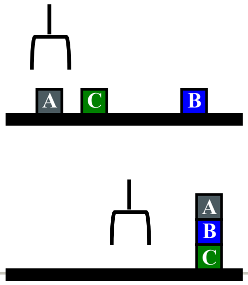


Meant to be a simple model!
Try demo at:
http://aispace.org/planning/

Typical BW planning problem

Initial state:
- clear(a)
- clear(b)
- clear(c)
- ontable(a)
- ontable(b)
- ontable(c)
- handempty

Goal state:
- on(b,c)
- on(a,b)
- ontable(c)



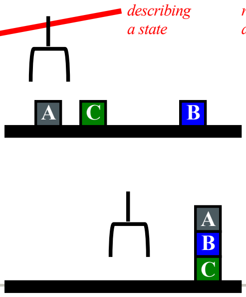
Typical BW planning problem

Initial state:
- clear(a)
- clear(b)
- clear(c)
- ontable(a)
- ontable(b)
- ontable(c)
- handempty

Goal state:
- on(b,c)
- on(a,b)
- ontable(c)

assertions describing a state

atomic robot actions



Plan:
- pickup(b)
- stack(b,c)
- pickup(a)
- stack(a,b)

Major Approaches

- GPS / STRIPS
- **Situation calculus**
- **Partial order planning**
- Hierarchical decomposition (HTN planning)
- Planning with constraints (SATplan, Graphplan)
- ***Reactive planning***

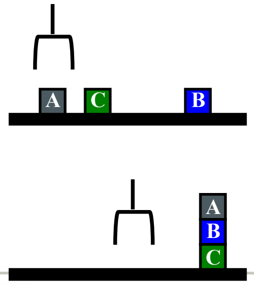
Planning vs. problem solving

- Planning *vs.* problem solving: can often solve similar problems
- Planning is more powerful and efficient because of the representations and methods used
- States, goals, and actions are decomposed into sets of sentences (usually in first-order logic)
- Search often proceeds through *plan space* rather than *state space* (though there are also state-space planners)
- Sub-goals can be planned independently, reducing the complexity of the planning problem

Another BW planning problem

Initial state:
clear(a)
clear(b)
clear(c)
ontable(a)
ontable(b)
ontable(c)
handempty

Goal:
on(a,b)
on(b,c)
ontable(c)

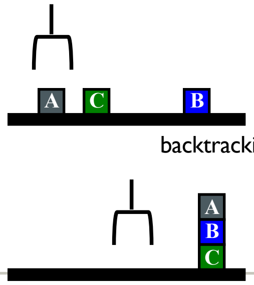


A plan
pickup(a)
stack(a,b)
unstack(a,b)
putdown(a)
pickup(b)
stack(b,c)
pickup(a)
stack(a,b)

Yet Another BW planning problem

Initial state:
clear(c)
ontable(a)
on(b,a)
on(c,b)
handempty

Goal:
on(a,b)
on(b,c)
ontable(c)



backtracking

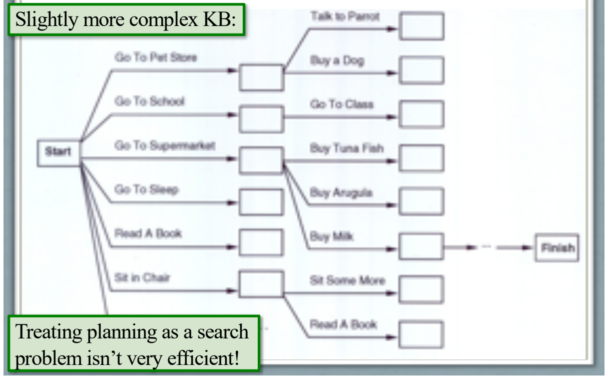
Plan:
unstack(c,b)
putdown(c)
unstack(b,a)
putdown(b)
putdown(b)
pickup(a)
stack(a,b)
unstack(a,b)
putdown(a)
pickup(b)
stack(b,c)
pickup(a)
stack(a,b)

Planning as Search

- Can think of planning as a search problem
  - **Actions:** generate successor states
  - **States:** completely described & only used for successor generation, heuristic fn. evaluation & goal testing
  - **Goals:** represented as a goal test and using a heuristic function
  - **Plan representation:** unbroken sequences of actions forward from initial states or backward from goal state

“Get a quart of milk, a bunch of bananas and a variable-speed cordless drill.”

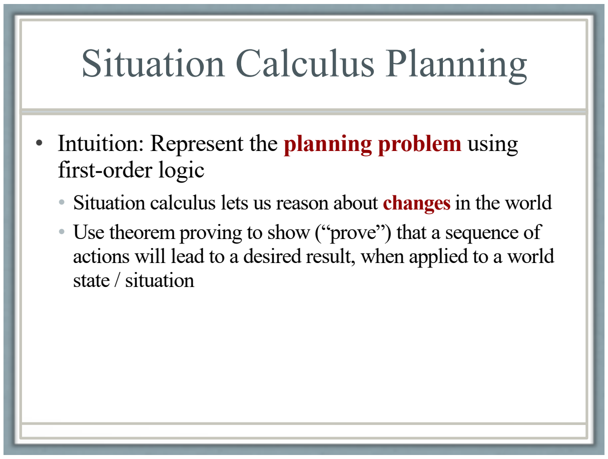
Slightly more complex KB:



Treating planning as a search problem isn’t very efficient!

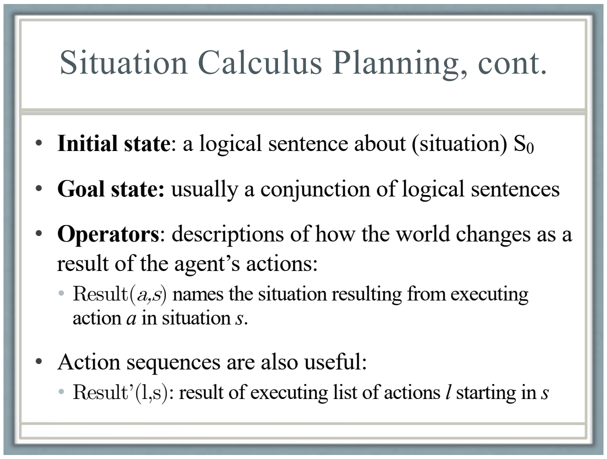
General Problem Solver

- The **General Problem Solver (GPS)** system
  - An early planner (Newell, Shaw, and Simon)
- Generate actions that *reduce difference* between current state and goal state
- Uses *Means-Ends Analysis*
  - Compare what is **given** or **known** with what is desired
  - Select a reasonable thing to do next
  - Use a **table of differences** to identify procedures to reduce differences
- GPS is a **state space planner**
  - Operates on state space problems specified by an initial state, some goal states, and a set of operations



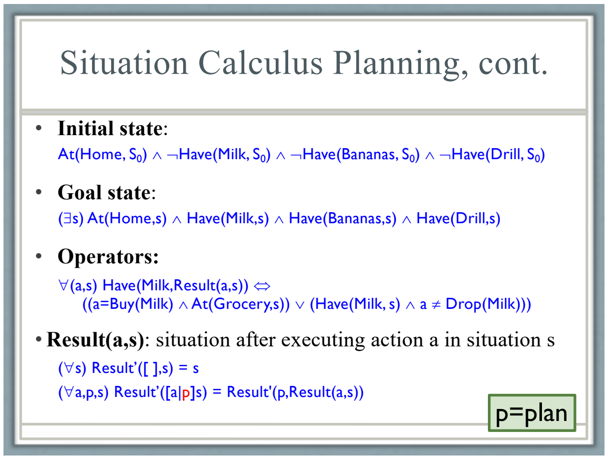
Situation Calculus Planning

- Intuition: Represent the **planning problem** using first-order logic
  - Situation calculus lets us reason about **changes** in the world
  - Use theorem proving to show (“prove”) that a sequence of actions will lead to a desired result, when applied to a world state / situation



Situation Calculus Planning, cont.

- **Initial state**: a logical sentence about (situation) S_0
- **Goal state:** usually a conjunction of logical sentences
- **Operators**: descriptions of how the world changes as a result of the agent’s actions:
  - Result(*a,s*) names the situation resulting from executing action *a* in situation *s*.
- Action sequences are also useful:
  - Result’(l,s): result of executing list of actions *l* starting in *s*



Situation Calculus Planning, cont.

- **Initial state**:

  $At(Home, S_0) \wedge \neg Have(Milk, S_0) \wedge \neg Have(Bananas, S_0) \wedge \neg Have(Drill, S_0)$

- **Goal state**:

  $(\exists s)\ At(Home,s) \wedge Have(Milk,s) \wedge Have(Bananas,s) \wedge Have(Drill,s)$

- **Operators:**

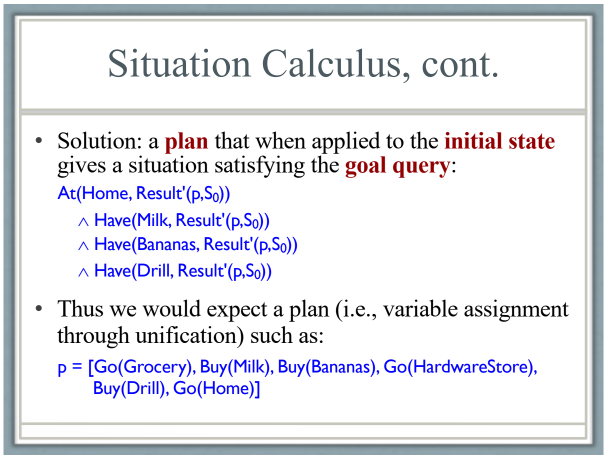
  $\forall(a,s)\ Have(Milk,Result(a,s)) \Leftrightarrow$
  $((a=Buy(Milk) \wedge At(Grocery,s)) \vee (Have(Milk, s) \wedge a \neq Drop(Milk)))$

- **Result(a,s)**: situation after executing action a in situation s

  $(\forall s)\ Result'([\ ],s) = s$

  $(\forall a,p,s)\ Result'([a|p]s) = Result'(p,Result(a,s))$

p=plan



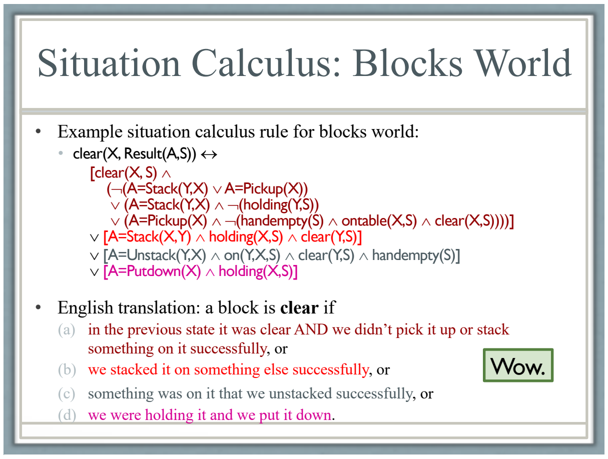
Situation Calculus, cont.

- Solution: a **plan** that when applied to the **initial state** gives a situation satisfying the **goal query**:

  $At(Home, Result'(p,S_0))$
  $\wedge\ Have(Milk, Result'(p,S_0))$
  $\wedge\ Have(Bananas, Result'(p,S_0))$
  $\wedge\ Have(Drill, Result'(p,S_0))$

- Thus we would expect a plan (i.e., variable assignment through unification) such as:

  p = [Go(Grocery), Buy(Milk), Buy(Bananas), Go(HardwareStore), Buy(Drill), Go(Home)]



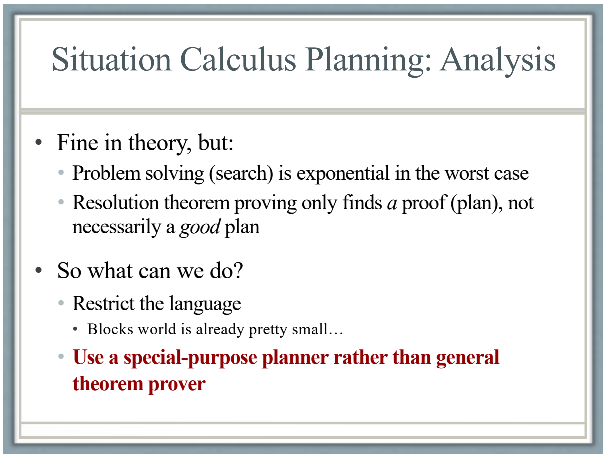
Situation Calculus: Blocks World

- Example situation calculus rule for blocks world:
  - $clear(X, Result(A,S)) \leftrightarrow$

$$
\begin{aligned}
&[clear(X, S) \wedge \\
&\quad(\neg(A=Stack(Y,X) \vee A=Pickup(X)) \\
&\quad\ \vee (A=Stack(Y,X) \wedge \neg(holding(Y,S)) \\
&\quad\ \vee (A=Pickup(X) \wedge \neg(handempty(S) \wedge ontable(X,S) \wedge clear(X,S))))] \\
&\vee [A=Stack(X,Y) \wedge holding(X,S) \wedge clear(Y,S)] \\
&\vee [A=Unstack(Y,X) \wedge on(Y,X,S) \wedge clear(Y,S) \wedge handempty(S)] \\
&\vee [A=Putdown(X) \wedge holding(X,S)]
\end{aligned}
$$

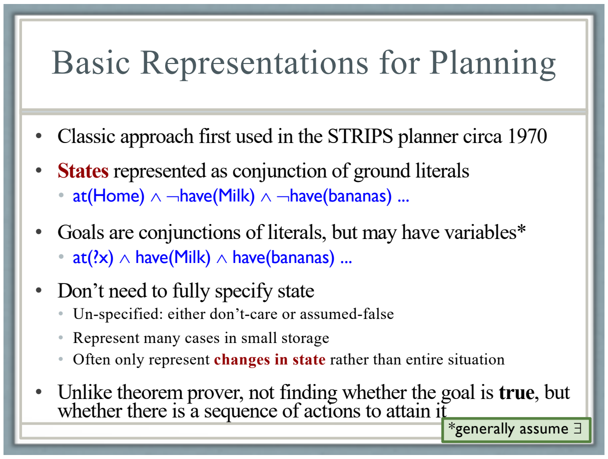
- English translation: a block is **clear** if
  (a) in the previous state it was clear AND we didn’t pick it up or stack something on it successfully, or
  (b) we stacked it on something else successfully, or
  (c) something was on it that we unstacked successfully, or
  (d) we were holding it and we put it down.

Wow.



Situation Calculus Planning: Analysis

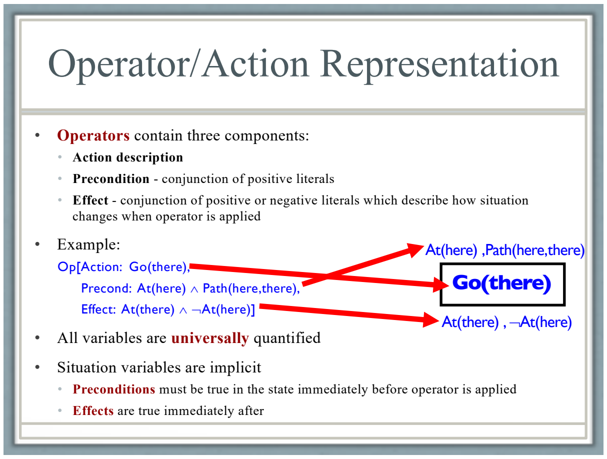
- Fine in theory, but:
  - Problem solving (search) is exponential in the worst case
  - Resolution theorem proving only finds *a* proof (plan), not necessarily a *good* plan
- So what can we do?
  - Restrict the language
    - Blocks world is already pretty small…
  - **Use a special-purpose planner rather than general theorem prover**



Basic Representations for Planning

- Classic approach first used in the STRIPS planner circa 1970
- **States** represented as conjunction of ground literals
  - at(Home) ∧ ¬have(Milk) ∧ ¬have(bananas) ...
- Goals are conjunctions of literals, but may have variables*
  - at(?x) ∧ have(Milk) ∧ have(bananas) ...
- Don't need to fully specify state
  - Un-specified: either don't-care or assumed-false
  - Represent many cases in small storage
  - Often only represent **changes in state** rather than entire situation
- Unlike theorem prover, not finding whether the goal is **true**, but whether there is a sequence of actions to attain it

*generally assume ∃

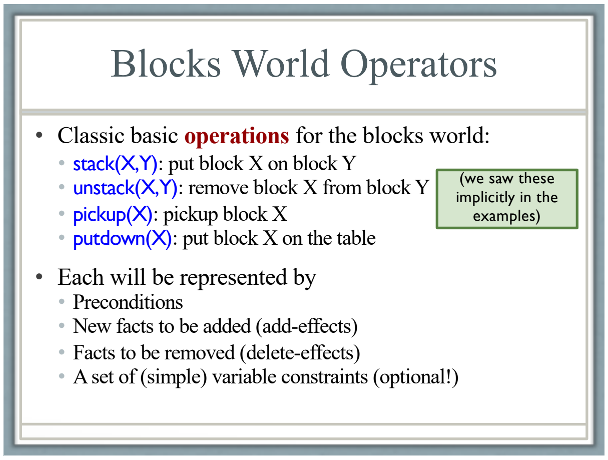


Operator/Action Representation

- **Operators** contain three components:
  - **Action description**
  - **Precondition** - conjunction of positive literals
  - **Effect** - conjunction of positive or negative literals which describe how situation changes when operator is applied
- Example:

Op[Action: Go(there),
Precond: At(here) ∧ Path(here,there),
Effect: At(there) ∧ ¬At(here)]

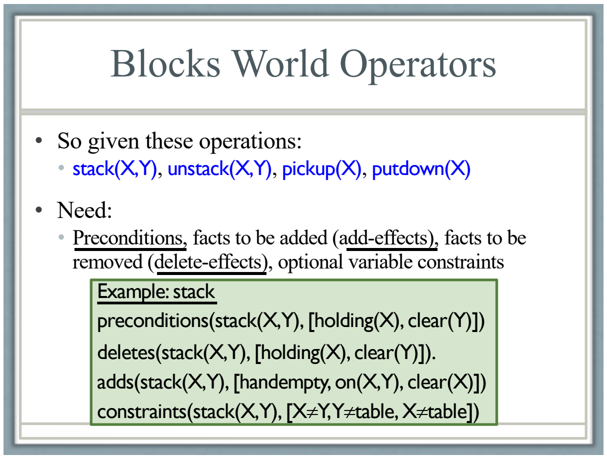
- All variables are **universally** quantified
- Situation variables are implicit
  - **Preconditions** must be true in the state immediately before operator is applied
  - **Effects** are true immediately after



Blocks World Operators

- Classic basic **operations** for the blocks world:
  - stack(X,Y): put block X on block Y
  - unstack(X,Y): remove block X from block Y
  - pickup(X): pickup block X
  - putdown(X): put block X on the table
- Each will be represented by
  - Preconditions
  - New facts to be added (add-effects)
  - Facts to be removed (delete-effects)
  - A set of (simple) variable constraints (optional!)

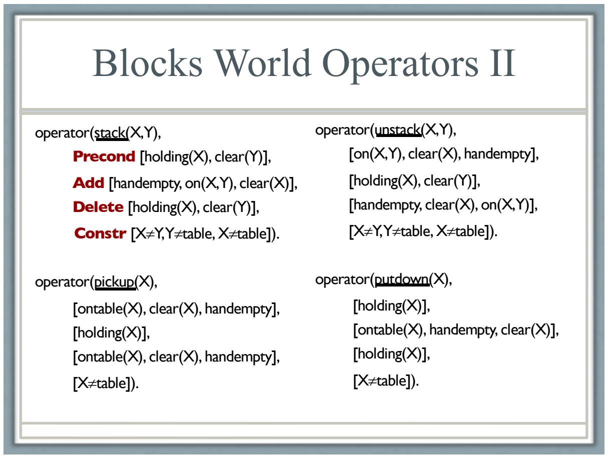
(we saw these implicitly in the examples)



Blocks World Operators

- So given these operations:
  - stack(X,Y), unstack(X,Y), pickup(X), putdown(X)
- Need:
  - Preconditions, facts to be added (add-effects), facts to be removed (delete-effects), optional variable constraints

Example: stack
preconditions(stack(X,Y), [holding(X), clear(Y)])
deletes(stack(X,Y), [holding(X), clear(Y)]).
adds(stack(X,Y), [handempty, on(X,Y), clear(X)])
constraints(stack(X,Y), [X≠Y, Y≠table, X≠table])



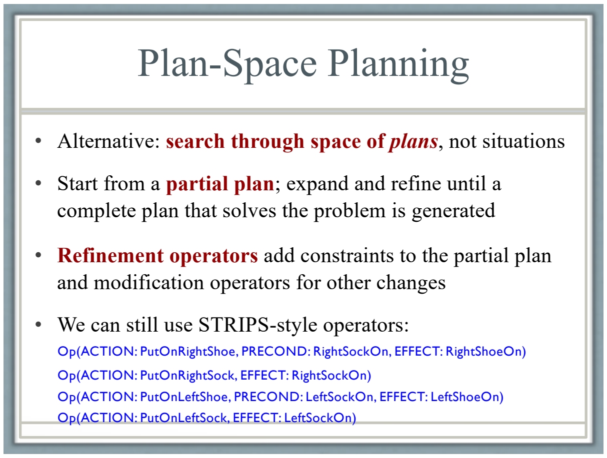
Blocks World Operators II

operator(stack(X,Y),
Precond [holding(X), clear(Y)],
Add [handempty, on(X,Y), clear(X)],
Delete [holding(X), clear(Y)],
Constr [X≠Y, Y≠table, X≠table]).

operator(unstack(X,Y),
[on(X,Y), clear(X), handempty],
[holding(X), clear(Y)],
[handempty, clear(X), on(X,Y)],
[X≠Y, Y≠table, X≠table]).

operator(pickup(X),
[ontable(X), clear(X), handempty],
[holding(X)],
[ontable(X), clear(X), handempty],
[X≠table]).

operator(putdown(X),
[holding(X)],
[ontable(X), handempty, clear(X)],
[holding(X)],
[X≠table]).



Plan-Space Planning

- Alternative: **search through space of *plans***, not situations
- Start from a **partial plan**; expand and refine until a complete plan that solves the problem is generated
- **Refinement operators** add constraints to the partial plan and modification operators for other changes
- We can still use STRIPS-style operators:

Op(ACTION: PutOnRightShoe, PRECOND: RightSockOn, EFFECT: RightShoeOn)
Op(ACTION: PutOnRightSock, EFFECT: RightSockOn)
Op(ACTION: PutOnLeftShoe, PRECOND: LeftSockOn, EFFECT: LeftShoeOn)
Op(ACTION: PutOnLeftSock, EFFECT: LeftSockOn)

Partial-Order Planning

Partial-Order Planning

- The big idea: Don't specify the order of steps if you don't have to.

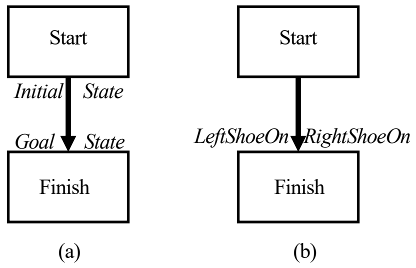
… → PutOnRightSock → PutOnLeftSock → …

vs.

… → PutOnLeftSock → PutOnRightSock → …

- Doesn't matter, but a regular planner has to consider and specify all the options.

A simple graphical notation



Partial-Order Planning

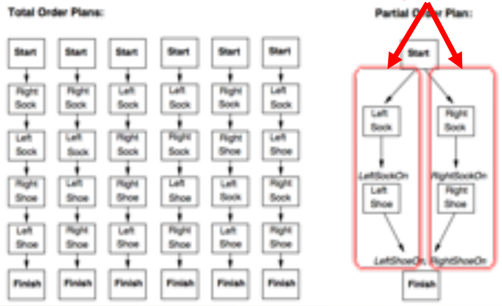
- A **linear planner** builds a plan as a **totally ordered sequence** of plan steps
- A **non-linear planner (aka partial-order planner)** builds up a plan as a set of steps with some temporal constraints
  - E.g., S1<S2 (step S1 must come before S2)

PutOnRightSock < PutOnRightShoe

The order here *does* matter, so the planner has to know that.

- Partially ordered plan (POP) **refined** by either:
  - adding a new **plan step**, or
  - adding a new **constraint** to the steps already in the plan.
- A POP can be linearized by topological sorting – R&N 223

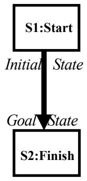
Linear *vs.* POP: Shoes



Do these sequences in any order

The Initial Plan

Every plan starts the same way



Least Commitment

- Non-linear planners embody the principle of **least commitment**
  - Only choose actions, orderings and variable bindings absolutely necessary, postponing other decisions
  - Avoid early commitment to decisions that don't really matter
- Linear planners always choose to add a plan step in a particular place in the sequence
- Non-linear planners choose to add a step and possibly some temporal constraints

Non-Linear Plan Components

1) A set of **steps** {S_1, S_2, S_3, S_4…}
   - Each step has an **operator description**, **preconditions** and **post-conditions**
   - ACTION: LeftShoe, PRECOND: LeftSockOn, EFFECT: LeftShoeOn
2) A set of **causal links** { … (S_i,C,S_j) …}
   - (One) goal of step S_i is to achieve precondition C of step S_j
   - (PutOnLeftShoe, LeftShoeOn, Finish)
     - This says: No action that undoes LeftShoeOn is allowed to happen after PutOnLeftShoe and before Finish. Any action that undoes LeftShoeOn must either be before PutOnLeftShoe or after Finish.
3) A set of **ordering constraints** { … S_i<S_j … }
   - If step S_i must come before step S_j
   - PutOnSock < Finish

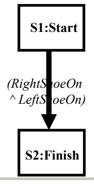
Non-Linear Plan: Completeness

- A non-linear plan consists of
  - (1) A set of **steps** {S_1, S_2, S_3, S_4…}
  - (2) A set of **causal links** { … (S_i,C,S_j) …}
  - (3) A set of **ordering constraints** { … S_i<S_j … }
- A non-linear plan is **complete** iff
  - Every step mentioned in (2) and (3) is in (1)
  - If S_j has prerequisite C, then there exists a causal link in (2) of the form (S_i,C,S_j) for some S_i
  - If (S_i,C,S_j) is in (2) and step S_k is in (1), and S_k threatens (S_i,C,S_j) (makes C false), then (3) contains either S_k<S_i or S_j<S_k

Trivial Example

Operators:

Op(ACTION: RightShoe, PRECOND: RightSockOn, EFFECT: RightShoeOn)
Op(ACTION: RightSock, EFFECT: RightSockOn)
Op(ACTION: LeftShoe, PRECOND: LeftSockOn, EFFECT: LeftShoeOn)
Op(ACTION: LeftSock, EFFECT: leftSockOn)

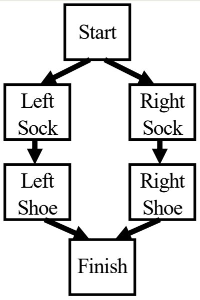


Steps: {S1:[Op(Action:Start)],
S2:[Op(Action:Finish,
Pre: RightShoeOn^LeftShoeOn)]}

Links: {}

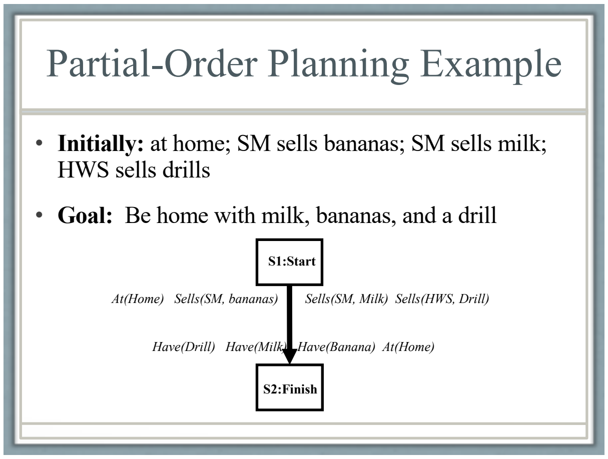
Orderings: {S1<S2}

Solution



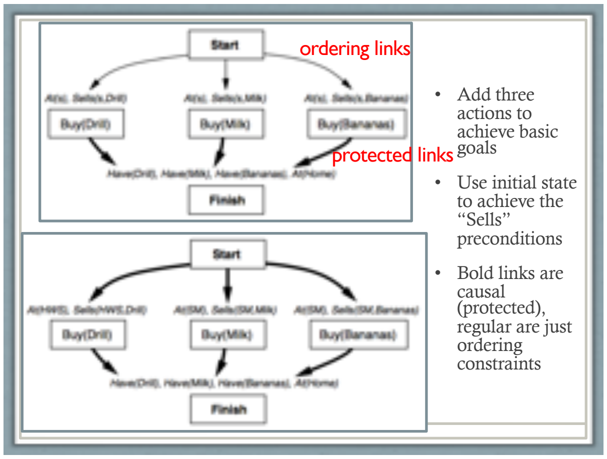
POP Constraints and Search Heuristics

- Only add steps that reach a not-yet-achieved precondition
- Use a least-commitment approach:
  - Don't order steps unless they need to be ordered
- Honor causal links $S_1 \xrightarrow[c]{} S_2$ that **protect** a condition *c*:
  - Never add an intervening step S_3 that violates *c*
  - If a parallel action **threatens** *c* (i.e., has the effect of negating or **clobbering** *c*), resolve that threat by adding ordering links:
    - Order S_3 before S_1 (**demotion**)
    - Order S_3 after S_2 (**promotion**)

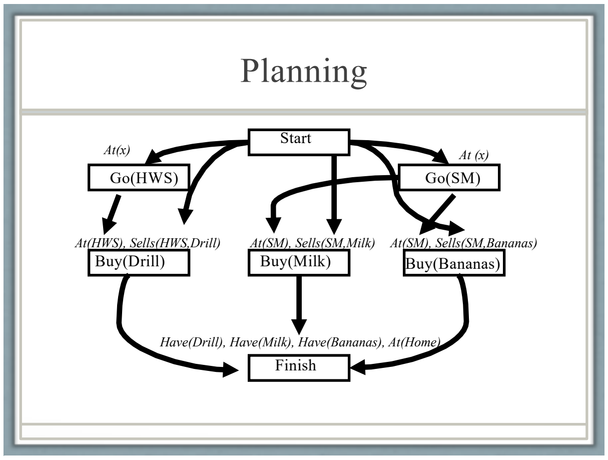


Partial-Order Planning Example

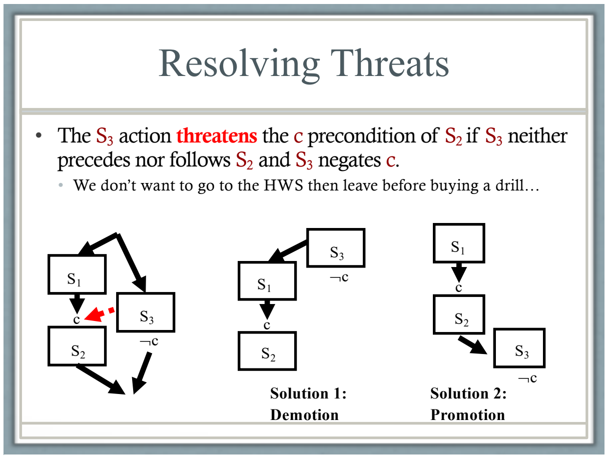
- **Initially:** at home; SM sells bananas; SM sells milk; HWS sells drills
- **Goal:** Be home with milk, bananas, and a drill



- Add three actions to achieve basic goals
- Use initial state to achieve the "Sells" preconditions
- Bold links are causal (protected), regular are just ordering constraints



Planning



Resolving Threats

- The S_3 action **threatens** the c precondition of S_2 if S_3 neither precedes nor follows S_2 and S_3 negates c.
  - We don't want to go to the HWS then leave before buying a drill…

Solution 1: Demotion

Solution 2: Promotion

Real-World Planning Domains

- Real-world domains are complex
- Don't satisfy assumptions of STRIPS or partial-order planning methods
- Some of the characteristics we may need to deal with:
  - Modeling and reasoning about resources
  - Representing and reasoning about time } Scheduling
  - Planning at different levels of abstractions
  - Conditional outcomes of actions } Planning under uncertainty
  - Uncertain outcomes of actions
  - Exogenous events
  - Incremental plan development } HTN planning
  - Dynamic real-time replanning

Hierarchical Planning

Hierarchical Decomposition

- The big idea: **Plan over high-level actions (HLAs), then figure out the steps to accomplish those.**
- Reduces complexity of planning space
  - Consider plan made of HLAs
  - **Then** make a plan for steps within each
  - Don't consider silly orderings that violate high-level concepts
- Can nest more than one level

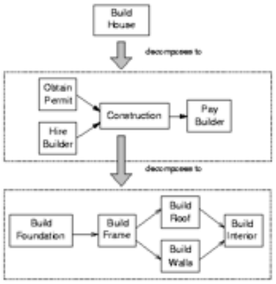
Hierarchical Decomposition: Example

- If we want to go to Hawaii (and we do)
  - Operators, unordered (because we haven't planned yet): DriveToAirport, TaxiToHotel, PutClothesInSuitcase, BuySunscreen, BoardPlane, BuySwimsuit, FindPassport, PutPassportInCarryon, DisembarkFromPlane, BookHotel, …
- High-Level Actions (HLAs): "Get to island" "Prepare for trip"
  - Order HLAs first: PrepareForTrip → GetToIsland
  - THEN order the subgoals within them
  - Don't have to consider "disembark" ←→ "find passport" ordering
- Nest as as needed
  - PrepareForTrip can include ShopForTrip, which includes …

Hierarchical Decomposition

- Hierarchical decomposition, or hierarchical task network (**HTN**) planning, uses **abstract operators** to **incrementally** decompose a planning problem from a **high-level goal** statement to a **primitive plan network**
- **Primitive operators** represent actions that are **executable**, and can appear in the final plan
- **Non-primitive operators** represent **goals** (equivalently, **abstract actions**) that require further decomposition (or *operationalization*) to be executed
- There is no "right" set of primitive actions: One agent's goals are another agent's actions!

HTN Planning: Example



HTN Operator: Example

```
OPERATOR decompose
PURPOSE: Construction
CONSTRAINTS:
    Length (Frame) <= Length (Foundation),
    Strength (Foundation) > Wt(Frame) + Wt(Roof)
      + Wt(Walls) + Wt(Interior) + Wt(Contents)
PLOT: Build (Foundation)
      Build (Frame)
      PARALLEL
            Build (Roof)
            Build (Walls)
      END PARALLEL
      Build (Interior)
```

HTN Operator Representation

- Russell & Norvig explicitly represent causal links
  - Can also be computed dynamically by using a model of preconditions and effects
  - Dynamically computing causal links means that actions from one operator can safely be interleaved with other operators, and subactions can safely be removed or replaced during plan repair
- R&N representation only includes variable bindings
  - Can actually introduce a wide array of variable constraints

Truth Criterion

- Determining whether a **formula is true** at a particular point in a partially ordered plan is, in the general case, NP-hard
- Intuition: there are exponentially many ways to **linearize** a partially ordered plan
- In the worst case, if there are N actions unordered with respect to each other, there are N! linearizations
- Ensuring soundness of truth criterion requires checking the formula under all possible linearizations
- Use heuristic methods instead to make planning feasible
- Check later to be sure no constraints have been violated

Truth Criterion in HTN Planners

- Heuristic:
  1. Prove that there exists *one* possible ordering of the actions that makes the formula true
  2. But don't insert ordering links to enforce that order
- Such a proof is efficient
  - Suppose you have an action A1 with a precondition P
  - Find an action A2 that achieves P (A2 can be initial world state)
  - Make sure there is no action *necessarily* between A2 and A1 that negates P
- Applying this heuristic for all preconditions in the plan can result in infeasible plans

Increasing Expressivity

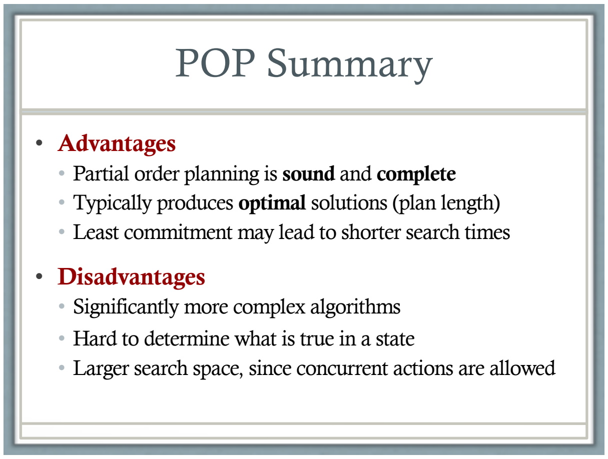
- Conditional effects
  - Instead of different operators for different conditions, use a single operator with conditional effects
  - Move (block1, from, to) and MoveToTable (block1, from) collapse into one Move (block1, from, to):
    - Op(ACTION: Move(block1, from, to),
      PRECOND: On (block1, from) ^ Clear (block1) ^ Clear (to)
      EFFECT: On (block1, to) ^ Clear (from) ^ ~On(block1, from) ^
      ~Clear(to) when to<>Table
    - There's a problem with this operator: can you spot it?
- Negated and disjunctive goals
- Universally quantified preconditions and effects

Reasoning About Resources

- What if I only have so much money for bananas and drills?
  - It suddenly matters that I don't introduce, e.g., BuyGrapes
- Introduce numeric variables that can be used as *measures*
- These variables represent resource quantities, and change over the course of the plan
- Certain actions **produce** (increase the quantity of) resources
- Other actions **consume** (decrease the quantity of) resources
- More generally, may want different types of resources
  - Continuous vs. discrete
  - Sharable vs. nonsharable
  - Reusable vs. consumable vs. self-replenishing

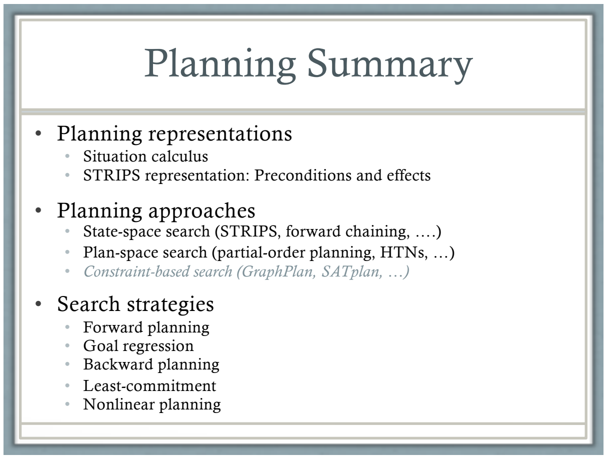
Other Real-World Planning Issues

- Conditional planning
- Partial observability
- Information gathering actions
- Execution monitoring and replanning
- Continuous planning
- Multi-agent (cooperative or adversarial) planning



POP Summary

- **Advantages**
  - Partial order planning is **sound** and **complete**
  - Typically produces **optimal** solutions (plan length)
  - Least commitment may lead to shorter search times
- **Disadvantages**
  - Significantly more complex algorithms
  - Hard to determine what is true in a state
  - Larger search space, since concurrent actions are allowed



Planning Summary

- Planning representations
  - Situation calculus
  - STRIPS representation: Preconditions and effects
- Planning approaches
  - State-space search (STRIPS, forward chaining, ….)
  - Plan-space search (partial-order planning, HTNs, …)
  - *Constraint-based search (GraphPlan, SATplan, …)*
- Search strategies
  - Forward planning
  - Goal regression
  - Backward planning
  - Least-commitment
  - Nonlinear planning