Local Search

AI Class 6 (Ch. 4.1-4.2)

Based on slides by Dr. Marie desJardin. Some material also adapted from slides by Dr. Matuszek @ Villanova University, which are based on Hwee Tou Ng at Berkeley, which are based on Russell at Berkeley. Some diagrams are based on AIMA.

Cynthia Matuszek – CMSC 671

Today's Class

- Local Search
- Iterative improvement methods
- Hill climbing
- Simulated annealing
- Local beam search
- Genetic algorithms
- Online search

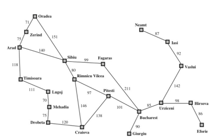
"If the **path** to the goal does not matter… [we can use] a single **current node** and move to neighbors of that node."

– R&N pg. 121

3

Admissibility

- Admissibility is a property of **heuristics**
  - They are *optimistic* – think goal is closer than it is
  - (Or, exactly right)
- Is $h(n)$: "1 kilometer" admissible?
- Admissible algorithms can be pretty bad!
- Using admissible heuristics guarantees that the first solution found will be optimal, **for some algorithms** (A*).



4

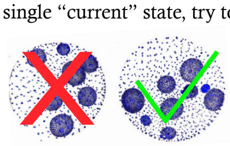
Admissibility and Optimality

- Intuitively:
  - When A* finds a path of length k, it has already tried **every other path which can have length ≤ k**
  - Because all frontier nodes have been sorted in ascending order of $f(n)=g(n)+h(n)$
- Does an admissible heuristic guarantee optimality for greedy search?
  - Reminder: $f(n) = h(n)$, always choose node "nearest" goal
  - No sorting beyond that

5

Local Search Algorithms

- Sometimes the path to the goal is irrelevant
  - Goal state itself is the solution
  - ∃ an **objective function** to evaluate states
- In such cases, we can use local search algorithms
- Keep a single "current" state, try to improve it



6

Local Search Algorithms

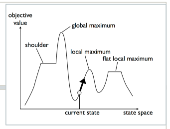
- Sometimes the path to the goal is irrelevant
  - Goal state itself is the solution
  - ∃ an **objective function** to evaluate states
- State space = set of "complete" configurations
  - That is, all elements of a solution are present
  - Find configuration satisfying constraints
  - Example?
- In such cases, we can use local search algorithms
- Keep a single "current" state, try to improve it

Very efficient!

Why?

7

Landscapes

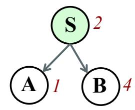


- **Search graph = landscape**
- Each node has **successor(s)** it can reach (called *s*)
  - Its children, unless there are loops
- Each successor has some “goodness” (desirability) according to the **objective** function
- $h(n) - h(s)$ is a positive, negative, or 0
- Positive is “uphill” (moving to a more desirable state)

Minor hassle:
Sometimes maximizing,
sometimes minimizing.

8

State Space (Landscape)

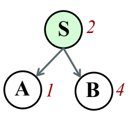


$f(S) = 2$

$f(A) = 1$

$f(B) = 4$

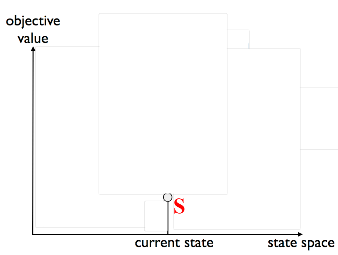
State Space (Landscape)



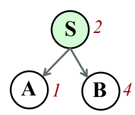
$f(S) = 2$

$f(A) = 1$

$f(B) = 4$



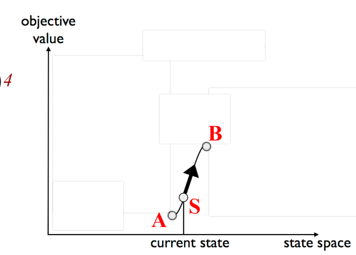
State Space (Landscape)



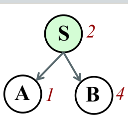
$f(S) = 2$

$f(A) = 1$

$f(B) = 4$



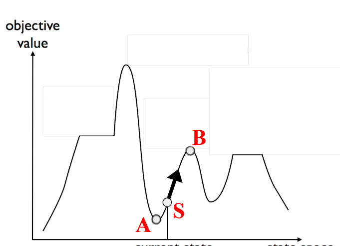
State Space (Landscape)



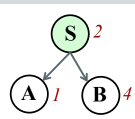
$f(S) = 2$

$f(A) = 1$

$f(B) = 4$



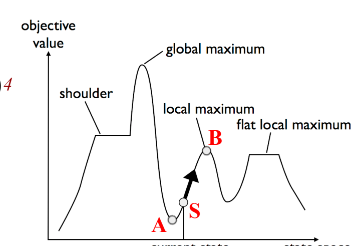
State Space (Landscape)



$f(S) = 2$

$f(A) = 1$

$f(B) = 4$

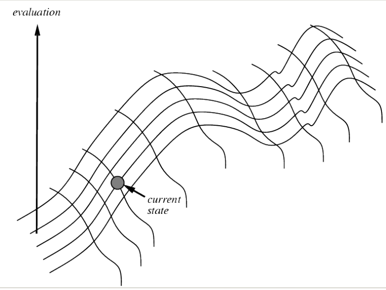


Iterative Improvement Search

- Start with an initial guess
- Gradually improve it until it is legal or optimal
- Some examples:
  - Hill climbing
  - Simulated annealing
  - Constraint satisfaction

Hill Climbing on State Surface

- Concept: trying to reach the "highest" (most desirable) point (state)
- "Height" Defined by Evaluation Function

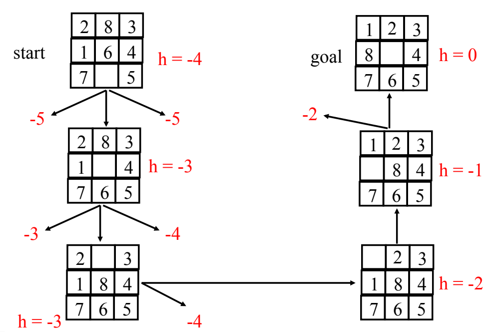


Hill Climbing Search

- If there exists a successor *s* for the current state *n* such that
  - $h(s) > h(n)$
  - $h(s) >= h(t)$ for all the successors *t* of *n*,

  then move from *n* to *s*. Otherwise, halt at *n*.
- Look one step ahead to determine if any successor is "better" than current state
  - If so, move to the best successor
- A kind of Greedy search in that it uses *h*
  - But, does not allow backtracking or jumping to an alternative path
  - Doesn't "remember" where it has been.
- Not *complete*
  - Search will terminate at local minima, plateaux, ridges.

Hill Climbing Example



f(n) = -(number of tiles out of place)

Exploring the Landscape

- **Local Maxima**:
  - Peaks that aren't the highest point in the space
- **Plateaus:**
  - A broad flat region that gives the search algorithm no direction (random walk)
- **Ridges:**
  - Flat like a plateau, but with drop-offs to the sides; steps to the North, East, South and West may go down, but a step to the NW may go up.

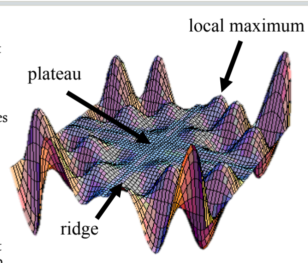
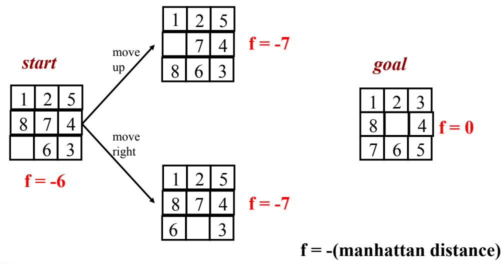


Image from: http://classes.yale.edu/fractals/CA/GA/Fitness/Fitness.html

Drawbacks of Hill Climbing

- Problems: local maxima, plateaus, ridges
- Remedies:
  - **Random restart:** keep restarting the search from random locations until a goal is found.
  - **Problem reformulation:** reformulate the search space to eliminate these problematic features
- Some problem spaces are great for hill climbing; others are terrible

Example of a Local Optimum



Some Extensions of Hill Climbing

- Simulated Annealing
  - Escape local maxima by allowing *some* "bad" moves but gradually decreasing their frequency
- Local Beam Search
  - Keep track of *k* states rather than just one
  - At each iteration:
    - All successors of the *k* states are generated and evaluated
    - Best *k* are chosen for the next iteration

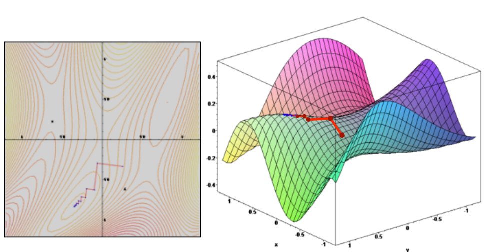
Some Extensions of Hill Climbing

- Stochastic Beam Search
  - Chooses semi-randomly from "uphill" possibilities
  - "Steeper" moves have a higher probability of being chosen
- Random-Restart Climbing
  - Can actually be applied to any form of search
  - Pick random starting points until one leads to a solution
- Genetic Algorithms
  - Each successor is generated from two predecessor (parent) states

Gradient Descent (or Ascent)

- Downward "steps" whose length is proportional to negative of the gradient (slope) at the current state.
  - "Steepest descent" → long "steps"
  - Jump to a node that is "farther away" if $f(\cdot)$ difference is large
- Gradient descent procedure for finding the $arg_x \; min \; f(x)$
  - choose initial x_0 randomly
  - repeat: $x_{i+1} \leftarrow x_i - \eta f'(x_i)$
  - until the sequence $x_0, x_1, \ldots, x_i, x_{i+1}$ converges
- Step size η (eta) is small (~0.1–0.05)
- Good for **differentiable, continuous** spaces

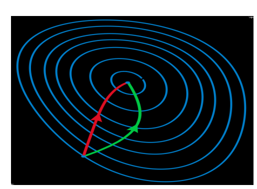
Gradient Ascent / Descent



 Images from http://en.wikipedia.org/wiki/Gradient_descent

Gradient Methods vs. Newton's Method

- A reminder of Newton's method from Calculus:
  $x_{i+1} \leftarrow x_i - \eta \; f'(x_i) \,/\, f''(x_i)$
- Newton's method uses 2nd order information (the second derivative, or, **curvature**) to take a more direct route to the minimum.
- The second-order information is more expensive to compute, but converges more quickly.



Contour lines of a function (blue)
• Gradient descent (green)
• Newton's method (red)

Images from http://en.wikipedia.org/wiki/Newton's_method_in_optimization

Simulated Annealing

- Simulated annealing (SA): analogy between the way metal cools into a minimum-energy crystalline structure and the search for a minimum generally
  - In very hot metal, molecules can move fairly freely
  - But, they are slightly less likely to move out of a stable structure
  - As you slowly cool the metal, more molecules are "trapped" in place
- Conceptually: Escape local maxima by allowing some "bad" (locally counterproductive) moves but gradually decreasing their frequency

Simulated Annealing (II)

- Can avoid becoming trapped at local minima.
- Uses a random local search that:
  - Accepts changes that increase objective function f
  - **As well as** some that **decrease** it
- Uses a control parameter T
  - By analogy with the original application
  - Is known as the system "**temperature**"

freedom to make "bad" moves

- T starts out high and gradually decreases toward 0

Simulated Annealing (IV)

- $f(s)$ represents the quality of state n (high is good)
- A "bad" move from A to B is accepted with probability
  $P(\text{move}_{A\to B}) \approx e^{(f(B) - f(A))\,/\,T}$
  - (Note that $f(B) - f(A)$ will be negative, so bad moves always have a relative probability less than one. Good moves, for which $f(B) - f(A)$ is positive, have a relative probability greater than one.)
- Temperature
  - Higher temperature = more likely to make a "bad" move
  - As T tends to zero, this probability tends to zero
    - SA becomes more like hill climbing
  - If T is lowered slowly enough, SA is complete and admissible.
    - domain-specific
    - sometimes hard to determine

The Simulated Annealing Algorithm

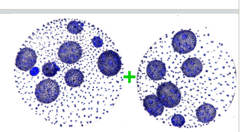
```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
  inputs: problem, a problem
          schedule, a mapping from time to "temperature"
  static: current, a node
          next, a node
          T, a "temperature" controlling the probability of downward steps

  current ← MAKE-NODE(INITIAL-STATE[problem])
  for t ← 1 to ∞ do
      T ← schedule[t]
      if T=0 then return current
      next ← a randomly selected successor of current
      ΔE ← VALUE[next] – VALUE[current]
      if ΔE > 0 then current ← next
      else current ← next only with probability e^(ΔE/T)
```


Local Beam Search

- Begin with k random states
  - k, instead of one, current state(s)
- Generate all successors of these states
- Keep the k best states across **all** successors
- Stochastic beam search
  - Probability of keeping a state is a **function** of its heuristic value
  - More likely to keep "better" successors

Genetic Algorithms

- The Idea:
  - New states are generated by "mutating" a single state or "reproducing" (somehow combining) two parent states
  - Selected according to their **fitness**



- Similar to stochastic beam search
- Start with k random states (the **initial population**)
  - Encoding used for the "genome" of an individual strongly affects the behavior of the search
  - Must have some combinable representation of state spaces
  - Genetic algorithms / genetic programming are a large and active area of research

Tabu Search

- Problem: Hill climbing can get stuck on local maxima
- Solution: Maintain a list of *k* previously visited states, and prevent the search from revisiting them
- Why not always do this?

Online Search

- Interleave computation and action (search some, act some)
  - Exploration: Can't infer outcomes of actions; must actually perform them to learn what will happen
- Competitive ratio = Path cost found* / Path cost that could be found**
  * On average, or in an adversarial scenario (worst case)
  ** If the agent knew the nature of the space, and could use offline search
- Relatively easy if actions are reversible
- LRTA* (Learning Real-Time A*): Update *h*(*s*) (in state table) based on experience
- More about online search and nondeterministic actions next time…

Summary: Local Search (I)

- State space can be treated as a "landscape" of movement on quality of states where we are trying to find "high" points
- **Best-first search** is a general class of search algorithms where the minimum-cost nodes are expanded first
- **Greedy search** uses minimal estimated cost *h*(n) to the goal state as measure of goodness
  - Reduces search time, but is neither complete nor optimal

Summary: Local Search (II)

- **Hill-climbing algorithms** keep only a single state in memory, but can get stuck on local optima.
- **Simulated annealing** escapes local optima, and is complete and optimal given a "long enough" cooling schedule.
- **Genetic algorithms** search a space by modeling biological evolution.
- **Online search** algorithms are useful in state spaces with partial/no information.

Questions?

Class Exercise:
Local Search for N-Queens

Q					
	Q				
		Q			
			Q		
				Q	
					Q

(more on constraint satisfaction heuristics next time...)