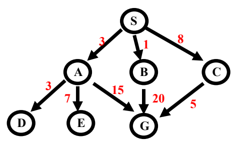
Artificial Intelligence

Class 4: Uninformed Search (Ch. 3.4)



Some material adapted from slides by Gang Hua of Stevens Institute of Technology
Some material adapted from slides by Charles R. Dyer, University of Wisconsin-Madison

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Slides adapted with thanks from: Dr. Marie desJardin

Today's Class

- Q&A
- Sudoku example
- Formalizing search
- Uninformed search
  - What does that mean?
- Specific algorithms
  - Breadth-first search
  - Depth-first search
  - Uniform cost search
  - Depth-first iterative deepening
- Example search problems revisited

"This is the essence of search—following up one option now and putting the others aside for later, in case the first choice does not lead to a solution."

– R&N pg. 75

Questions?

- Bread-first, depth-first, and uniform cost search
- Heuristic functions
- Admissibility
- Generation and expansion
- Goal tests
- Queueing function
- Complexity, completeness, and optimality

Sudoku, Naïvely

- **State space:** 4x4 matrix, divided into four 2x2 matrices: A, B, C, D, cells containing values [1-4]
- **Operators:**
  - Put a **2** in square <x,y>
  - **Preconditions:**
    - <x,y> is empty — 1
    - <x, (y±1)> ≠ 2; <x, (y±2)> ≠ 2; … — 3
    - <(x±1), y> ≠ 2; … <(x±3), y> ≠ 2 — 3
    - **if** <x,y> **in** A, **then** 3 ∉ A; … — 4

  × 4

	3		
			1
3			
		2	

- How many operators is that? How many preconditions?
- **Goal:** all blocks are filled

Formalizing Search III

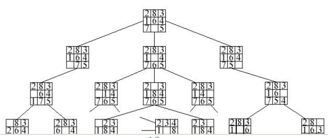
- A **solution** is a sequence of operators that is associated with a path in a state space from a start node to a goal node.
- The **cost of a solution** is the sum of the arc costs on the solution path.
  - If all arcs have the same cost, then the solution cost = the length of the solution (number of steps / state transitions)

Formalizing Search IV

- **State-space search:** searching through a state space for a solution by **making explicit** a sufficient portion of an **implicit** state-space graph to find a goal node
  - Initially V={S}, where S is the start node
  - When S is **expanded**, its successors are **generated**; those nodes are added to V and the arcs are added to E
  - This process continues until a goal node is found
- It isn't usually practical to represent entire space

Formalizing Search V

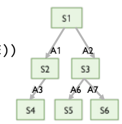
- Each node implicitly or explicitly represents a **partial solution path** (and its cost) from start node to given node.
  - In general, from a node there are many possible paths (and therefore solutions) that have this partial path as a prefix



State-Space Search Algorithm

```
function general-search (problem, QUEUEING-FUNCTION)
;; problem describes start state, operators, goal test,
;;    and operator costs
;; queueing-function is a comparator function that
;;    ranks two states
;; returns either a goal node or failure

nodes = MAKE-QUEUE(MAKE-NODE(problem.INITIAL-STATE))
  loop
     if EMPTY(nodes) then return "failure"
     node = REMOVE-FRONT(nodes)
     if problem.GOAL-TEST(node.STATE) succeeds
        then return node
     nodes = QUEUEING-FUNCTION(nodes, EXPAND(node,
             problem.OPERATORS))
 end
     ;; Note: The goal test is NOT done when nodes are generated
     ;; Note: This algorithm does not detect loops
```



Uninformed vs. Informed Search

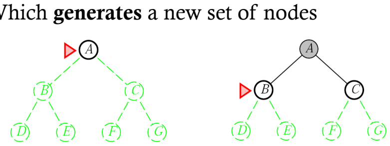
- Uninformed search strategies
  - Use no information about the "direction" of the goal node(s)
  - Also known as "blind search"
  - Methods: Breadth-first, depth-first, depth-limited, uniform-cost, depth-first iterative deepening, bidirectional
- Informed search strategies (**next** class...)
  - Use information about the domain to (try to) (usually) head in the general direction of the goal node(s)
  - Also known as "heuristic search"
  - Methods: Hill climbing, best-first, greedy search, beam search, A, A*

Key Procedures to Define

- `EXPAND`
  - Generate all successor nodes of a given node
- `GOAL-TEST`
  - Test if state satisfies all goal conditions
- `QUEUEING-FUNCTION`
  - Used to maintain a ranked list of nodes that are candidates for expansion

Generation vs. Expansion

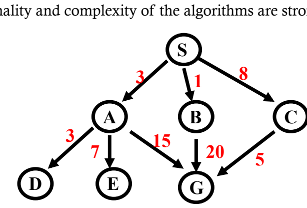
- **Selecting** a state means making that node current
- **Expanding** the current state means applying every legal action to the current state
- Which **generates** a new set of nodes



R&N pg. 68, 80

Why Apply Goal Test Late?

- **Why does it matter when the goal test is applied (expansion time vs. generation time)?**
- Optimality and complexity of the algorithms are strongly affected!



Review: Characteristics

- **Completeness:** Is the algorithm guaranteed to find a solution (if one exists)?
- **Optimality:** Does it find the optimal solution?
  - (The solution with the lowest path cost of all possible solutions)
- **Time complexity:** How long does it take to find a solution?
- **Space complexity:** How much memory is needed to perform the search?

R&N pg. 68, 80

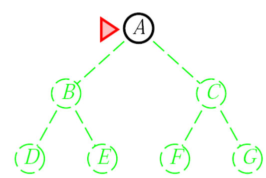
Admissibility

- A heuristic function IS **admissible** if it never *overestimates* the cost of reaching the goal
- The *estimated cost* it estimates is not higher than the lowest possible cost from the current point in the path

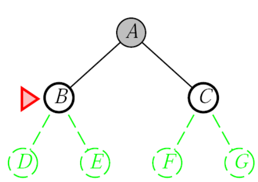
Breadth-First

- Enqueue nodes in **FIFO** (first-in, first-out) order
- Characteristics:
  - **Complete** (meaning?)
  - **Optimal** (i.e., admissible) if all operators have the same cost
  - Otherwise, not optimal but finds solution with shortest path length
  - **Exponential time and space complexity**, $O(b^d)$, where:
    - d is the depth of the solution
    - b is the branching factor (number of children) at each node
- Takes a **long time to find long-path solutions**

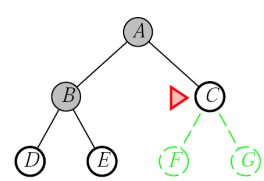
BFS



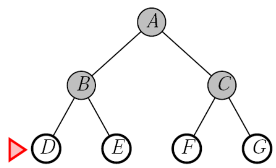
BFS



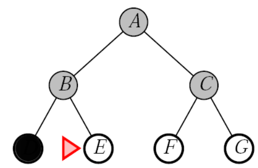
BFS



BFS



BFS



Breadth-First: Analysis

- Takes a **long time to find long-path solutions**
  - Must look at all shorter length possibilities first
  - A complete search tree of depth d where each non-leaf node has b children:

    $1 + b + b^2 + \ldots + b^d = (b^{d+1} - 1)/(b-1)$ nodes

  - What if we expand nodes when they are selected?

Breadth-First: O(Example)

$1 + b + b^2 + \ldots + b^d = (b^{d+1} - 1)/(b-1)$ nodes

- Tree where: d=12
- Every node at depths 0, ..., 11 has 10 children (b=10)
- Every node at depth 12 has 0 children
- $1 + 10 + 100 + 1000 + \ldots + 10^{12} = (10^{13} - 1)/9 = O(10^{12})$ nodes in the complete search tree
- If BFS expands 1000 nodes/sec and each node uses 100 bytes of storage
- Will take 35 years to run in the worst case
- Will use 111 terabytes of memory

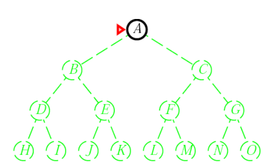
Depth-First (DFS)

- Enqueue nodes on nodes in **LIFO** (last-in, first-out) order
  - That is, nodes used as a stack data structure to order nodes
- Characteristics:
  - **Might not terminate** without a "depth bound"
    - I.e., cutting off search below a fixed depth D ("depth-limited search")
  - **Not complete**
    - With or without cycle detection, and with or without a cutoff depth
  - **Exponential time**, $O(b^d)$, but only **linear space**, O(bd)
    - Can find **long solutions quickly** if lucky
    - And **short solutions slowly** if unlucky

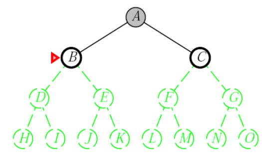
Loops?

Infinite search spaces?

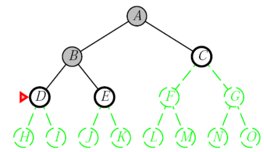
DFS



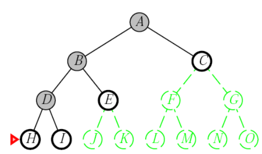
DFS



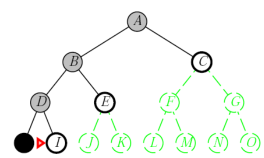
DFS



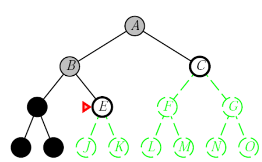
DFS



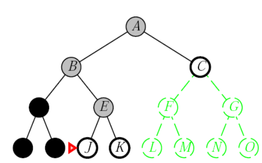
DFS



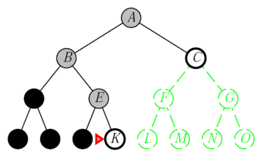
DFS



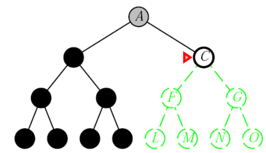
DFS



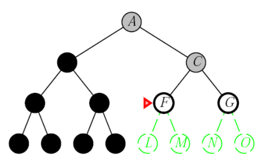
DFS



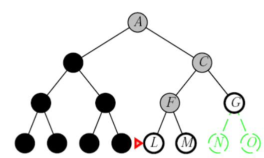
DFS



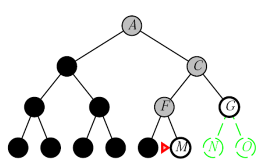
DFS



DFS



DFS



Depth-First (DFS): Analysis

- DFS:
  - Can find **long solutions quickly** if lucky
  - And **short solutions slowly** if unlucky
- When search hits a dead end
  - Can only back up one level at a time*
  - Even if the "problem" occurs because of a bad operator choice near the top of the tree
  - Hence, only does "chronological backtracking"

* Why?

Uniform-Cost (UCS)

- Enqueue nodes by **path cost**:
  - Let g(n) = cost of path from start node to current node n
  - Sort nodes by increasing value of g
  - Identical to breadth-first search **if** all operators have equal cost
- *"Dijkstra's Algorithm"* in algorithms literature
- *"Branch and Bound Algorithm"* in operations research literature
- **Complete** (*)
- **Optimal/Admissible** (*)
  - Admissibility depends on the goal test being applied *when a node is removed from the nodes list*, not when its parent node is expanded and the node is first generated
- **Exponential time and space complexity**, $O(b^d)$

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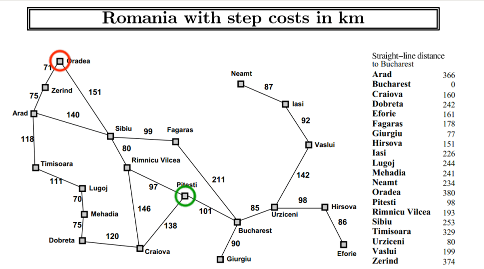
Example: Path Costs



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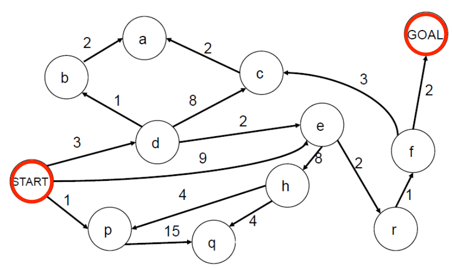
UCS Implementation

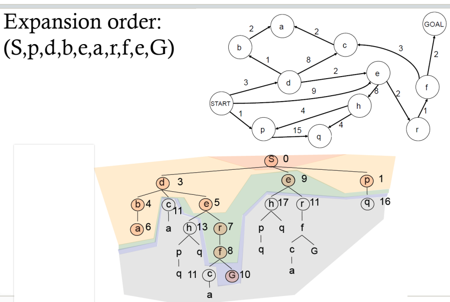
- For each frontier node, save the total cost of the path from the initial state to that node
- Expand the frontier node with the lowest path cost
- Equivalent to breadth-first if step costs all equal
- Equivalent to Dijkstra's algorithm in general

Uniform-cost search example

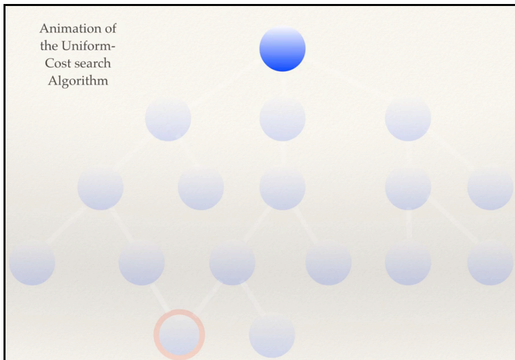


Uniform-cost search example

- Expansion order: (S,p,d,b,e,a,r,f,e,G)



Animation of the Uniform-Cost search Algorithm



Shaul Markovitch © 2018

https://www.youtube.com/watch?v=XyoucHYKYSE

Depth-First Iterative Deepening (DFID)

1. DFS to depth 0 (i.e., treat start node as having no successors)
2. Iff no solution found, do DFS to depth 1

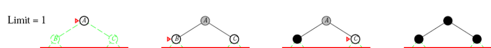
until solution found do:
DFS with depth cutoff c;
c = c+1

- Complete
- Optimal/Admissible if all operators have the same cost
  - Otherwise, not optimal, but guarantees finding solution of shortest length
- Time complexity is a little worse than BFS or DFS because nodes near the top of the search tree are generated multiple times
- Because most nodes are near the bottom of a tree, worst case time complexity is still exponential, O(bd)

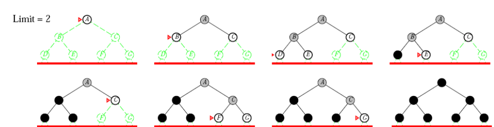
Depth-First Iterative Deepening

- If branching factor is *b* and solution is at depth *d*, then nodes at depth *d* are generated once, nodes at depth *d*-1 are generated twice, etc.
  - Hence $b^d + 2b^{(d-1)} + ... + db \leq b^d / (1 - 1/b)^2 = O(b^d)$.
  - If b=4, then worst case is $1.78 * 4^d$, i.e., 78% more nodes searched than exist at depth d (in the worst case).
- **Linear space complexity**, O(bd), like DFS
- Has advantage of both BFS (completeness) and DFS (limited space, finds longer paths more quickly)
- Generally preferred for **large state spaces** where **solution depth is unknown**

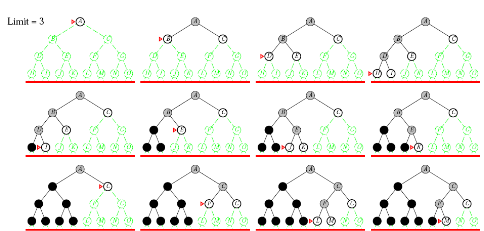
Iterative deepening search (c=1)



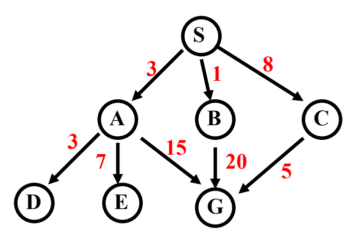
Iterative deepening search (c=2)



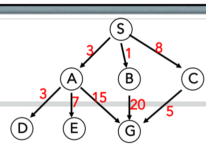
Iterative deepening search (c=3)



Example for Illustrating Search Strategies



Depth-First Search

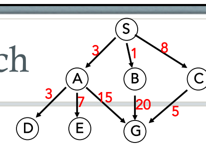


Expanded node	Nodes list
	{ S^0 }
S^0	{ A^3 B^1 C^8 }
A^3	{ D^6 E^{10} G^{18} B^1 C^8 }
D^6	{ E^{10} G^{18} B^1 C^8 }
E^{10}	{ G^{18} B^1 C^8 }
G^{18}	{ B^1 C^8 }

Solution path found is S A G, cost 18

Number of nodes expanded (including goal node) = 5

Breadth-First Search

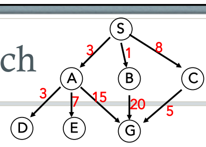


Expanded node	Nodes list
	{ S^0 }
S^0	{ A^3 B^1 C^8 }
A^3	{ B^1 C^8 D^6 E^{10} G^{18} }
B^1	{ C^8 D^6 E^{10} G^{18} G^{21} }
C^8	{ D^6 E^{10} G^{18} G^{21} G^{13} }
D^6	{ E^{10} G^{18} G^{21} G^{13} }
E^{10}	{ G^{18} G^{21} G^{13} }
G^{18}	{ G^{21} G^{13} }

Solution path found is S A G , cost 18

Number of nodes expanded (including goal node) = 7

Uniform-Cost Search



Expanded node	Nodes list
	{ S^0 }
S^0	{ B^1 A^3 C^8 }
B^1	{ A^3 C^8 G^{21} }
A^3	{ D^6 C^8 E^{10} G^{18} G^{21} }
D^6	{ C^8 E^{10} G^{18} G^1 }
C^8	{ E^{10} G^{13} G^{18} G^{21} }
E^{10}	{ G^{13} G^{18} G^{21} }
G^{13}	{ G^{18} G^{21} }

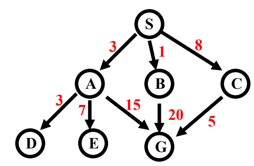
Solution path found is S C G, cost 13

Number of nodes expanded (including goal node) = 7

How they Perform

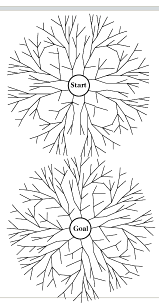
- **Depth-First Search:**
  - Expanded nodes: S A D E G
  - Solution found: S A G (cost 18)
- **Breadth-First Search**:
  - Expanded nodes: S A B C D E G
  - Solution found: S A G (cost 18)
- **Uniform-Cost Search**:
  - Expanded nodes: S A D B C E G
  - Solution found: S C G (cost 13)

  This is the only uninformed search that worries about costs.
- **Iterative-Deepening Search**:
  - nodes expanded: S S A B C S A D E G



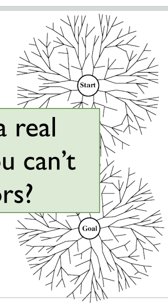
Bi-directional Search

- Alternate searching from
  - start state → goal
  - goal state → start
- Stop when the frontiers intersect.
- Works well only when there are unique start and goal states
- Requires ability to generate “predecessor” states.
- Can (sometimes) find a solution fast



Bi-directional Search

- Alternate searching from
  - start state → goal
  - goal state → start
- St…
- W… un…
- R… “predecessor” states.
- Can (sometimes) find a solution fast



For next time: What’s a real world problem where you can’t generate predecessors?

Comparing Search Strategies

	Complete	Optimal	Time complexity	Space complexity
Breadth first search:	yes	yes	$O(b^d)$	$O(b^d)$
Depth first search	no	no	$O(b^m)$	$O(bm)$
Depth limited search	if l >= d	no	$O(b^l)$	$O(bl)$
depth first iterative deepening search	yes	yes	$O(b^d)$	$O(bd)$
bi-directional search	yes	yes	$O(b^{d/2})$	$O(b^{d/2})$

b is branching factor, d is depth of the shallowest solution,
m is the maximum depth of the search tree, l is the depth limit

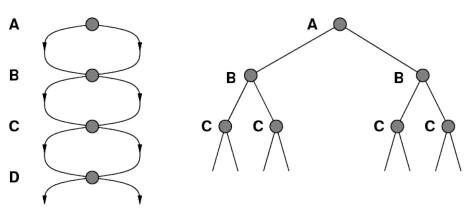
Avoiding Repeated States

- Ways to reduce size of state space (with increasing computational costs)
- In increasing order of effectiveness:

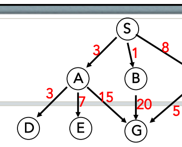
1. Do not return to the state you just came from.
2. Do not create paths with cycles in them.
3. Do not generate any state that was ever created before.

- Effect depends on frequency of loops in state space.

A State Space that Generates an Exponentially Growing Search Space



Holy Grail Search



Expanded node	Nodes list
	$\{ S^0 \}$
S^0	$\{C^8\ A^3\ B^1 \}$
C^8	$\{ G^{13}\ A^3\ B^1 \}$
G^{13}	$\{ A^3\ B^1 \}$

Solution path found is S C G, cost 13 (optimal)
Number of nodes expanded (including goal node) = 3
(minimum possible!)

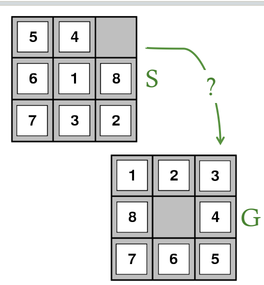
Holy Grail Search

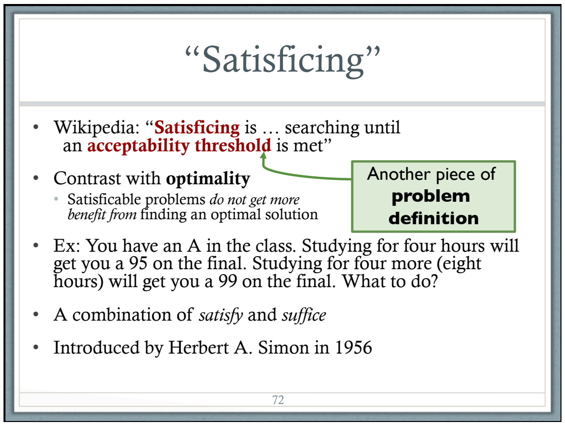
Why not go straight to the solution, without any wasted detours off to the side?

<foreshadowing> **If only we knew where we were headed…** </foreshadowing>

8-Puzzle Revisited

- What's a good algorithm?
  - Depth-first search?
  - Breadth-first search?
  - Uniform-cost?
  - Iterative deepening?




“Satisficing”
Wikipedia: “Satisficing is … searching until an acceptability threshold is met”
Contrast with optimality
Satisficable problems do not get more benefit from finding an optimal solution
Another piece of problem definition
Ex: You have an A in the class. Studying for four hours will get you a 95 on the final. Studying for four more (eight hours) will get you a 99 on the final. What to do?
A combination of satisfy and suffice
Introduced by Herbert A. Simon in 1956
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