First-Order Logic & Inference

Ch. 8.1–8.3, 9

Material from Dr. Marie desJardin, Some material adopted from notes by Andreas Geyer-Schulz and Chuck Dyer

Today's Class

- The last little bit of PL and FOL
  - Axioms and Theorems
  - Sufficient and Necessary
- Logical Agents
  - Reflex
  - Model-Based
  - Goal-Based
- Inference!
  - How do we use any of this?

Axioms, Definitions and Theorems

- **Axioms**: facts and rules that attempt to capture all of the (important) facts and concepts about a domain
- Axioms can be used to prove **theorems**
  - Mathematicians don't want any unnecessary (dependent) axioms –ones that can be derived from other axioms
  - Dependent axioms can make reasoning faster, however
  - Choosing a good set of axioms for a domain is a design problem!
- A **definition** of a predicate is of the form "p(X) ↔ …" and can be decomposed into two parts
  - **Necessary** description: "p(x) → …"
  - **Sufficient** description "p(x) ← …"
  - Some concepts don't have complete definitions (e.g., person(x))

More on Definitions

- Examples: define father(x, y) by parent(x, y) and male(x)
  - parent(x, y) is a necessary (**but not sufficient**) description of father(x, y)
    - father(x, y) → parent(x, y)
  - parent(x, y) ^ male(x) ^ age(x, 35) is a **sufficient** (**but not necessary**) description of father(x, y):

    father(x, y) ← parent(x, y) ^ male(x) ^ age(x, 35)
  - parent(x, y) ^ male(x) is a **necessary and sufficient** description of father(x, y)

    parent(x, y) ^ male(x) ↔ father(x, y)

Higher-Order Logics

- FOL only allows to quantify over variables, and variables can only range over objects.
- HOL allows us to quantify over relations
- Example: (quantify over functions)
  - "two functions are equal iff they produce the same value for all arguments"
  - ∀f ∀g (f = g) ↔ (∀x f(x) = g(x))
- Example: (quantify over predicates)
  - ∀r transitive(r) → (∀xyz) r(x,y) ∧ r(y,z) → r(x,z))
- More expressive, but undecidable.

Expressing Uniqueness

- Sometimes we want to say that there is a single, unique object that satisfies a certain condition
- "There exists a unique x such that king(x) is true"
  - ∃x king(x) ∧ ∀y (king(y) → x=y)
  - ∃x king(x) ∧ ¬∃y (king(y) ∧ x≠y)
  - ∃! x king(x)
- "Every country has exactly one ruler"
  - ∀c country(c) → ∃! r ruler(c,r)
- Iota operator: "ι x P(x)" means "the unique x such that p(x) is true"
  - "The unique ruler of Freedonia is dead"
  - dead(ι x ruler(freedonia,x))

Logical Agents

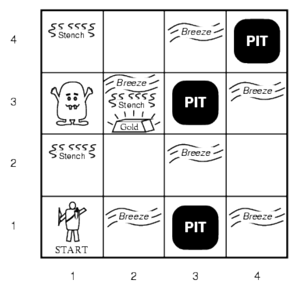
Logical Agents for Wumpus World

Three (non-exclusive) agent architectures:

- **Reflex** agents
  - Have rules that classify situations, specifying how to react to each possible situation
- **Model-based** agents
  - Construct an internal model of their world
- **Goal-based** agents
  - Form goals and try to achieve them

A Typical Wumpus World

- The agent always starts in the field [1,1].
- The task of the agent is to find the gold, return to the field [1,1] and climb out of the cave.



A Simple Reflex Agent

- Rules to map percepts into observations:

  ∀b,g,u,c,t Percept([Stench, b, g, u, c], t) → Stench(t)

  ∀s,g,u,c,t Percept([s, Breeze, g, u, c], t) → Breeze(t)

  ∀s,b,u,c,t Percept([s, b, Glitter, u, c], t) → AtGold(t)

- Rules to select an action given observations:

  ∀t AtGold(t) → Action(Grab, t)

A Simple Reflex Agent

- Some difficulties:
- Climb?
  - There is no percept that indicates the agent should climb out – position and holding gold are not part of the percept sequence
- Loops?
  - The percept will be repeated when you return to a square, which should cause the same response (unless we maintain some internal model of the world)

KB-Agents Summary

Wumpus percepts:
[Stench, Breeze, Glitter, Bump, Scream]

- Logical agents
  - Reflex: rules map directly from percepts → beliefs or percepts → actions

    ∀b,g,u,c,t Percept([Stench, b, g, u, c], t) → Stench(t)

    ∀t AtGold(t) → Action(Grab, t)
  - Model-based: construct a *model* (set of t/f beliefs about sentences) as they learn; map from models → actions

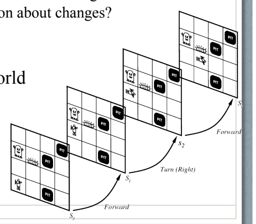
    Action(Grab, t) → HaveGold(t)

    HaveGold(t) → Action(RetraceSteps, t)
  - Goal-based: form goals, then try to accomplish them
  - Encoded as a rule:

    (∀s) Holding(Gold,s) → GoalLocation([1,1],s)

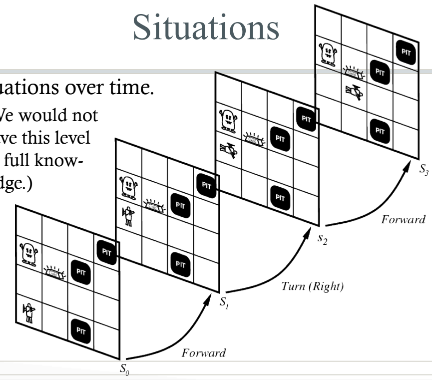
Representing Change

- Representing change in the world in logic can be tricky.
- One way is just to change the KB
  - Add and delete sentences from the KB to reflect changes
  - How do we remember the past, or reason about changes?
- **Situation calculus** is another way
- A **situation** is a snapshot of the world at some instant in time
- When the agent performs an action A in situation S1, the result is a new situation S2.



Situations

- Situations over time.
  - (We would not have this level of full knowledge.)



Situation Calculus

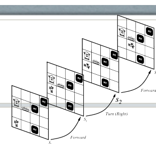
- A **situation** is:
  - A snapshot of the world
  - At an interval of time
  - During which nothing changes
- Every true or false statement is made wrt. a situation
  - Add **situation variables** to every predicate.
  - *at(Agent,1,1)* becomes *at(Agent,1,1,s0)*: *at(Agent,1,1)* **is true in** situation (i.e., state) s0.

Situation Calculus

- Alternatively, add a special 2nd-order predicate, **holds(f,s),** that means "f is true in situation s." E.g., holds(at(Agent,1,1),s0)
- Or: add a new function, **result(a,s),** that maps a situation s into a new situation as a result of performing action a. For example, result(forward, s) is a function that returns the successor state (situation) to s
- Example: The action *agent-walks-to-location-y* could be represented by

(∀x)(∀y)(∀s) (at(Agent,x,s) ∧ ¬onbox(s)) → at(Agent,y,result(walk(y),s))

Situations Summary



- Representing a dynamic world
  - Situations ($s_0 \ldots s_n$): the world in situation 0-n
    Teaching(DrM,s_0) — today,10:10,whenNotSick, ...
  - Add 'situation' argument to statements
    AtGold(t,s_0)
  - Or, add a 'holds' predicate that says 'sentence is true in this situation'
    holds(At[2,1], s_1)
  - Or, add a result(action, situation) function that takes an action and situation, and returns a new situation
    results(Action(goNorth), s_0) → s_1

Deducing Hidden Properties

- From the perceptual information we obtain in situations, we can **infer properties of locations**
  l = location, s = situation
  ∀l,s at(Agent,l,s) ∧ Breeze(s) → Breezy(l)
  ∀l,s at(Agent,l,s) ∧ Stench(s) → Smelly(l)
- Neither Breezy nor Smelly need situation arguments because pits and Wumpuses do not move around

Deducing Hidden Properties II

- We need to write some rules that relate various aspects of a single world state (as opposed to across states)
- There are two main kinds of such rules:
  - **Causal rules** reflect assumed direction of causality:
    (∀l1,l2,s) At(Wumpus,l1,s) ∧ Adjacent(l1,l2) → Smelly(l2)
    (∀ l1,l2,s) At(Pit,l1,s) ∧ Adjacent(l1,l2) → Breezy(l2)
- Systems that reason with causal rules are called **model-based reasoning** systems

Deducing Hidden Properties II

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Deducing Hidden Properties II

- We need to write some rules that relate various aspects of a single world state (as opposed to across states)
- There are two main kinds of such rules:
  - **Diagnostic rules** infer the presence of **hidden properties** directly from the percept-derived information. We have already seen two:
    (∀ l,s) At(Agent,l,s) ∧ Breeze(s) → Breezy(l)
    (∀ l,s) At(Agent,l,s) ∧ Stench(s) → Smelly(l)

Frames: A Data Structure

- A **frame** divides knowledge into **substructures** by representing "stereotypical situations."
- Situations can be visual scenes, structures of physical objects,
- Useful for representing commonsense knowledge.

Slots	Fillers
publisher	Thomson
title	Expert Systems
author	Giarratano
edition	Third
year	1998
pages	600

Slot	Fillers
name	computer
specialization_of	a_kind_of machine
types	(desktop, laptop,mainframe,super) if-added: Procedure ADD_COMPUTER
speed	default: faster if-needed: Procedure FIND_SPEED
location	(home,office,mobile)
under_warranty	(yes, no)

intelligence.worldofcomputing.net/knowledge-representation/frames.html#.WCHhCNxBo8A

Representing Change: The Frame Problem

- **Frame axioms**: If property x doesn't change as a result of applying action a in state s, then it stays the same.
  - On (x, z, s) ∧ Clear (x, s) →
    On (x, table, Result(Move(x, table), s)) ∧
    ¬On(x, z, Result (Move (x, table), s))
  - On (y, z, s) ∧ y≠ x → On (y, z, Result (Move (x, table), s))
  - The proliferation of frame axioms becomes very cumbersome in complex domains

The Frame Problem II

- **Successor-state axiom**: General statement that characterizes **every way** in which a particular predicate can become true:
  - Either it can be **made true**, or it can **already be true and not be changed**:
  - On (x, table, Result(a,s)) ↔
    [On (x, z, s) ∧ Clear (x, s) ∧ a = Move(x, table)] v
    [On (x, table, s) ∧ a ≠ Move (x, z)]
- In complex worlds with longer chains of action, even these are too cumbersome
  - Planning systems use special-purpose inference to reason about the expected state of the world at any point in time during a multi-step plan

Qualification Problem

- Qualification problem:
  - How can you possibly characterize every single effect of an action, or every single exception that might occur?
  - When I put my bread into the toaster, and push the button, it will become toasted after two minutes, unless…
    - The toaster is broken, or…
    - The power is out, or…
    - I blow a fuse, or…
    - A neutron bomb explodes nearby and fries all electrical components, or…
    - A meteor strikes the earth, and the world we know it ceases to exist, or…

Ramification Problem

- How do you describe every effect of every action?
  - When I put my bread into the toaster, and push the button, the bread will become toasted after two minutes, and…
    - The crumbs that fall off the bread onto the bottom of the toaster over tray will also become toasted, and…
    - Some of the aforementioned crumbs will become burnt, and…
    - The outside molecules of the bread will become "toasted," and…
    - The inside molecules of the bread will remain more "breadlike," and…
    - The toasting process will release a small amount of humidity into the air because of evaporation, and…
    - The heating elements will become a tiny fraction more likely to burn out the next time I use the toaster, and…
    - The electricity meter in the house will move up slightly, and…

Knowledge Engineering!

- Modeling the "right" conditions and the "right" effects at the "right" level of abstraction is very difficult
- Knowledge engineering (creating and maintaining knowledge bases for intelligent reasoning) is a **field**
- Many researchers hope that automated knowledge acquisition and machine learning tools can fill the gap:
  - Our intelligent systems should be able to **learn** about the conditions and effects, just like we do.
  - Our intelligent systems should be able to learn when to pay attention to, or reason about, certain aspects of processes, depending on the context.

Preferences Among Actions

- A problem with the Wumpus world knowledge base: It's hard to decide which action is best!
  - Ex: to decide between a *forward* and a *grab*, axioms describing when it is okay to move would have to mention glitter.
- This is not modular!
- We can solve this problem by **separating facts about actions from facts about goals**.
- This way our **agent can be reprogrammed just by asking it to achieve different goals**.

Preferences Among Actions

- The first step is to describe the desirability of actions independent of each other.
- In doing this we will use a simple scale: actions can be Great, Good, Medium, Risky, or Deadly.
- Obviously, the agent should always do the best action it can find:

$(\forall a,s)\ Great(a,s) \rightarrow Action(a,s)$
$(\forall a,s)\ Good(a,s) \wedge \neg(\exists b)\ Great(b,s) \rightarrow Action(a,s)$
$(\forall a,s)\ Medium(a,s) \wedge (\neg(\exists b)\ Great(b,s) \vee Good(b,s)) \rightarrow Action(a,s)$
…

Preferences Among Actions

- We use this action quality scale in the following way.
- Until it finds the gold, the basic strategy for our agent is:
  - Great actions include picking up the gold when found and climbing out of the cave with the gold.
  - Good actions include moving to a square that's OK and hasn't been visited yet.
  - Medium actions include moving to a square that is OK and has already been visited.
  - Risky actions include moving to a square that is not known to be deadly or OK.
  - Deadly actions are moving into a square that is known to have a pit or a Wumpus.

Goal-Based Agents

- Once the gold is found, it is necessary to change strategies. So now we need a new set of action values.
- We could encode this as a rule:
  - $(\forall s)$ Holding(Gold,s) → GoalLocation([1,1]),s)
- We must now decide how the agent will work out a sequence of actions to accomplish the goal.
- Three possible approaches are:
  - **Inference**: good versus wasteful solutions
  - **Search**: make a problem with operators and set of states
  - **Planning**: coming soon!

Logical Inference

Chapter 9

Model Checking

- Given KB, does sentence S hold?

  Quick review: What's a KB? What's a sentence?
- Basically **generate and test**:
  - Generate all the possible models
  - Consider the models M in which KB is TRUE
  - If ∀M S , then S is **provably true** What does model mean?
  - If ∀M ¬S, then S is **provably false**
  - Otherwise (∃M1 S ∧ ∃M2 ¬S): S is **satisfiable** but neither provably true or provably false

Efficient Model Checking

- Davis-Putnam algorithm (DPLL): Generate-and-test model checking with:
  - Early termination (short-circuiting of disjunction and conjunction)
  - Pure symbol heuristic: Any symbol that only appears negated or unnegated must be FALSE/TRUE respectively.
    - Can "conditionalize" based on instantiations already produced
  - Unit clause heuristic: Any symbol that appears in a clause by itself can immediately be set to TRUE or FALSE
- WALKSAT: Local search for satisfiability:
  - Pick a symbol to flip (toggle TRUE/FALSE), either using min-conflicts *or* choosing randomly
- …or you can use *any* local or global search algorithm!

Reminder: Inference Rules for FOL

- Inference rules for **propositional logic** apply to **FOL**
  - Modus Ponens, And-Introduction, And-Elimination, …
- New (sound) inference rules for use with quantifiers:
  - Universal elimination
  - Existential introduction
  - Existential elimination
  - Generalized Modus Ponens (GMP)

Automating FOL Inference with Generalized Modus Ponens

Automated Inference for FOL

- Automated inference using FOL is harder than PL
  - Variables can take on an infinite number of possible values
    - From their domains, anyway
    - This is a reason to do careful KR!
  - So, potentially infinite ways to apply Universal Elimination
- *Godel's Completeness Theorem* says that FOL entailment is only semidecidable*
  - If a sentence is **true** given a set of axioms, can prove it
  - If the sentence is **false**, then there is no guarantee that a procedure will ever determine this
  - **Inference may never halt**

*The "halting problem"

Generalized Modus Ponens (GMP)

- Apply modus ponens reasoning to generalized rules
- Combines And-Introduction, Universal-Elimination, and Modus Ponens
  - From $P(c)$ and $Q(c)$ and $(\forall x)(P(x) \wedge Q(x)) \rightarrow R(x)$ derive $R(c)$
- General case: **Given**
  - **atomic sentences** $P_1, P_2, \ldots, P_N$
  - **implication sentence** $(Q_1 \wedge Q_2 \wedge \ldots \wedge Q_N) \rightarrow R$
    - $Q_1, \ldots, Q_N$ and R are atomic sentences
  - **substitution** $\text{subst}(\theta, P_i) = \text{subst}(\theta, Q_i)$ for i=1,...,N
  - **Derive new sentence: subst(θ , R)**

Generalized Modus Ponens (GMP)

- **Derive new sentence: subst(θ , R)**
- Substitutions
  - subst(θ , α) denotes the result of applying a **set of substitutions,** defined by θ , to the sentence α
  - A substitution list $\theta = \{v_1/t_1, v_2/t_2, \ldots, v_n/t_n\}$ means to replace all occurrences of variable symbol v_i by term t_i
  - Substitutions are made in left-to-right order in the list
  - subst({x/IceCream, y/Ziggy}, eats(y,x)) = eats(Ziggy, IceCream)

Horn Clauses

- A Horn clause is a sentence of the form:
  $$(\forall x)\ P_1(x) \wedge P_2(x) \wedge \ldots \wedge P_n(x) \rightarrow Q(x)$$
  where:
  - there are 0 or more P_is and 0 or 1 Qs
  - the P_is and Q are positive (non-negated) literals
- Equivalently: $P_1(x) \vee P_2(x) \ldots \vee P_n(x)$ where the P_i are all atomic and **at most one** of them is positive
- Horn clauses represent a **subset** of the set of sentences representable in FOL

Horn Clauses II

- Special cases
  - $P_1 \wedge P_2 \wedge \ldots P_n \rightarrow Q$
  - $P_1 \wedge P_2 \wedge \ldots P_n \rightarrow \text{false}$
  - true $\rightarrow$ Q
- These are not Horn clauses:
  - $p(a) \vee q(a)$
  - $(P \wedge Q) \rightarrow (R \vee S)$

Forward Chaining

- Proofs start with the given axioms/premises in KB, deriving new sentences using GMP until the goal/query sentence is derived
- This defines a **forward-chaining** inference procedure because it moves "forward" from the KB to the goal [eventually]
- Inference using GMP is **complete** for KBs containing **only Horn clauses**

Forward Chaining Example

- KB:
  - allergies(X) → sneeze(X)
  - cat(Y) ∧ allergic-to-cats(X) → allergies(X)
  - cat(Felix)
  - allergic-to-cats(Lise)
- Goal:
  - sneeze(Lise)

Inference

Knowledge Base
1. Allergies lead to sneezing.
   allergies(X) → sneeze(X)
2. Cats cause allergies if allergic to cats.
   cat(Y) ∧ allergic-cats(X) → allergies(X)
3. Felix is a cat.
   cat(Felix)
4. Lise is allergic to cats.
   allergic-cats(Lise)

sneeze(Lise) ← infer truth of (query)

- Forward Chaining: apply rules

variable binding

cat(Y) ∧ allergic-cats(X) → allergies(X) ∧ cat(Felix)
→
cat(Felix) ∧ allergic-cats(X) → allergies(X) ∧ allergic-cats(Lise)
→
allergies(Lise) ∧ allergies(X) → sneeze(X)
→
sneeze(Lise) ✓

add new sentence to KB

Backward Chaining

- **Backward-chaining** deduction using GMP
  - **Complete** for KBs containing **only Horn clauses**.
- Proofs:
  - Start with the goal query
  - Find rules with that conclusion
  - Prove each of the antecedents in the implication
- Keep going until you reach premises!

Avoid loops
  Is new subgoal already on goal stack?
Avoid repeated work: has subgoal already been proved true already failed?

Backward Chaining Example

- KB:
  - allergies(X) → sneeze(X)
  - cat(Y) ∧ allergic-to-cats(X) → allergies(X)
  - cat(Felix)
  - allergic-to-cats(Lise)
- Goal:
  - sneeze(Lise)

Inference

Knowledge Base
1. Allergies lead to sneezing.
   allergies(X) → sneeze(X)
2. Cats cause allergies if allergic to cats.
   cat(Y) ∧ allergic-cats(X) → allergies(X)
3. Felix is a cat.
   cat(Felix)
4. Lise is allergic to cats.
   allergic-cats(Lise)

sneeze(Lise) ← query

- Backward Chaining: apply rules that end with the goal

variable binding

allergies(X) → sneeze(X) + sneeze(Lise)
  new query: allergies(Lise)?

cat(Y) ∧ allergic-cats(X) → allergies(X) + allergies(Lise)
  new query: cat(Y) ∧ allergic-cats(Lise)?

cat(Felix) + cat(Y) ∧ allergic-cats(Lise)
  new sentence: cat(Felix) ∧ allergic-cats(Lise) ✓

Backward Chaining Algorithm

```
function BACK-CHAIN(KB, q) returns a set of substitutions
  BACK-CHAIN-LIST(KB, [q], {})

function BACK-CHAIN-LIST(KB, qlist, θ) returns a set of substitutions
  inputs: KB, a knowledge base
          qlist, a list of conjuncts forming a query (θ already applied)
          θ, the current substitution
  static: answers, a set of substitutions, initially empty

  if qlist is empty then return {θ}
  q ← FIRST(qlist)
     for each q'_i in KB such that θ_i ← UNIFY(q, q'_i) succeeds do
        Add COMPOSE(θ, θ_i) to answers
     end
     for each sentence (p_1 ∧ ... ∧ p_n ⇒ q'_i) in KB such that θ_i ← UNIFY(q, q'_i) succeeds do
        answers ← BACK-CHAIN-LIST(KB, SUBST(θ_i, [p_1 ... p_n]), COMPOSE(θ, θ_i)) ∪ answers
     end
  return the union of BACK-CHAIN-LIST(KB, REST(qlist), θ) for each θ ∈ answers
```

Forward vs. Backward Chaining

- FC is data-driven
  - Automatic, unconscious processing
  - E.g., object recognition, routine decisions
  - May do lots of work that is irrelevant to the goal
- BC is goal-driven, appropriate for problem-solving
  - Where are my keys? How do I get to my next class?
  - Complexity of BC can be much less than linear in the size of the KB

Completeness of GMP

- GMP (using forward or backward chaining) is complete for KBs that contain only Horn clauses
- It is ***not* complete** for simple KBs that contain **non-Horn clauses**
- The following entail that S(A) is true:
  $(\forall x)\ P(x) \rightarrow Q(x)$
  $(\forall x)\ \neg P(x) \rightarrow R(x)$
  $(\forall x)\ Q(x) \rightarrow S(x)$
  $(\forall x)\ R(x) \rightarrow S(x)$
- If we want to conclude S(A), with GMP we cannot, since the second one is not a Horn clause
- It is equivalent to $P(x) \vee R(x)$