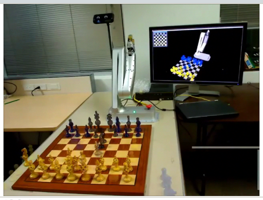
# Game Playing

AI Class 8 — Ch. 5.1-5.3, 5.4.1, 5.5





# Today's Class

- Game playing
  - State of the art and resources
  - Framework
- Game trees
  - Minimax
  - Alpha-beta pruning
  - Adding randomness

We've seen multi-agent systems, and search problems where another agent's moved need to be taken into account – but what if they are actively moving *against* us?

Adversarial search = Games!



# Why Games?

- Clear criteria for success
- Offer an opportunity to study problems involving {hostile / adversarial / competing} agents.
- Interesting, hard problems which require minimal setup
- Often define very large search spaces
  - chess $35^{100}$ nodes in search tree, $10^{40}$ legal states
- Historical reasons
- Fun! (Mostly.)



# State-of-the-art

- How good are computer game players?
  - **Chess**:
    - Deep Blue beat Gary Kasparov in 1997
    - Garry Kasparav vs. Deep Junior (Feb 2003): tie!
    - Kasparov vs. X3D Fritz (November 2003): tie! http://www.thechessdrum.net/tournaments/Kasparov-X3DFritz/index.html
    - Deep Fritz beat world champion Vladimir Kramnik (2006)
  - **Checkers**: Chinook (an AI program with a *very large* endgame database) is the world champion and can provably never be beaten. Retired in 1995
  - **Go**: Computer players have finally reached tournament-level play
    - AlphaGo beat Ke Jie (No.1 world player) in 2017
  - **Bridge**: "Expert-level" computer players exist (but no world champions yet!)
- Good places to learn more:
  - http://www.cs.ualberta.ca/~games/
  - http://www.cs.unimass.nl/icga

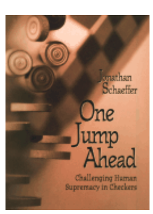


# Chinook

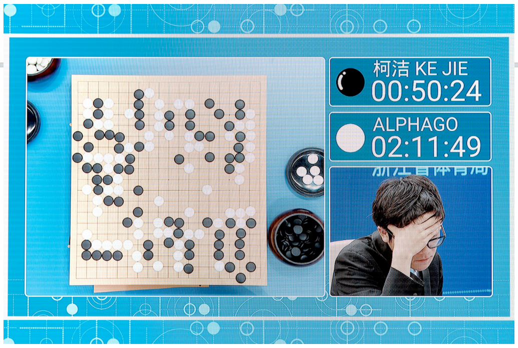
The board set for play

Red to play

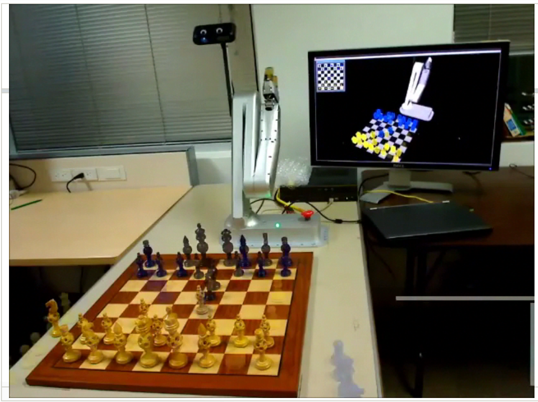
- World Man-Machine Checkers Champion, developed by researchers at the University of Alberta.
- Earned this title by competing in human tournaments, winning the right to play for the world championship, eventually defeating the best players in the world.
- Play it! http://www.cs.ualberta.ca/~chinook
- Developers have fully analyzed the game of checkers, and can provably *never* be beaten
  - (http://www.sciencemag.org/cgi/content/abstract/1144079v1)






www.wired.com/2017/05/googles-alphago-levels-board-games-power-grids



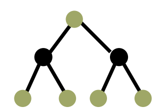
# Typical Games

- 2-person game
- Players alternate moves
- **Zero-sum**: one player's loss is the other's gain
- **Perfect information**: both players have access to complete information about the state of the game. No information is hidden from either player.
- **Deterministic**: No chance (e.g., dice) involved
- Examples: Tic-Tac-Toe, Checkers, Chess, Go, Nim, Othello
- Not: Bridge, Solitaire, Backgammon, ...



# How to Play (How to Search)

- Obvious approach:
  - From **current game state:**
  - Consider all the legal moves you can make
  - Compute new position resulting from each move
  - Evaluate each resulting position
  - Decide which is best
  - Make that move
  - Wait for your opponent to move and repeat
- Key problems are:
  - Representing the "board"
  - Generating all legal next boards
  - **Evaluating a position**






# Evaluation function

- **Evaluation function** or **static evaluator** is used to evaluate the "goodness" of a game position
  - Unlike heuristic search, where evaluation function is a positive estimate of cost from start node to a goal, passing through $n$
- Zero-sum assumption allows *one* evaluation function to describe goodness of a board for *both* players (how?)
  - $f(n) >> 0$: position $n$ good for me and bad for you
  - $f(n) << 0$: position $n$ bad for me and good for you
  - $f(n) = 0 \pm \varepsilon$ : position $n$ is a neutral position
  - $f(n) = +\infty$: win for me
  - $f(n) = -\infty$: win for you



# Evaluation function examples

- Example of an evaluation function for Tic-Tac-Toe:
  - $f(n)$ = [#3-lengths open for ×] - [#3-lengths open for O]
  - A 3-length is a complete row, column, or diagonal
- Alan Turing's function for chess
  - $f(n) = w(n)/b(n)$
  - $w(n)$ = sum of the **point value** of white's pieces
  - $b(n)$ = sum of black's

## Evaluation function examples

- Most evaluation functions are specified as a **weighted sum** of position features:
  - $f(n) = w_1 * feat_1(n) + w_2 * feat_2(n) + \ldots + w_n * feat_k(n)$
- Example features for chess: piece count, piece placement, squares controlled, …
- Deep Blue had over **8000** features in its nonlinear evaluation function!

square control, rook-in-file, x-rays, king safety, pawn structure, passed pawns, ray control, outposts, pawn majority, rook on the 7th blockade, restraint, trapped pieces, color complex, …

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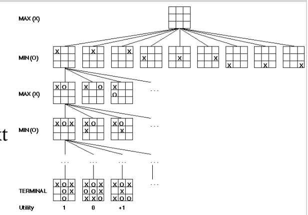
## Game trees

- Problem spaces for typical games are represented as trees
- Player must decide best single **move** to make next
- Root node = the current board configuration
- Arcs = possible legal moves for a player



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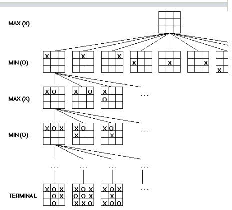
## Game trees

- **Static evaluator function**
  - Rates a board position
  - f(board) = R with f>0 "white" (me), f<0 for black (you)
- If it is **my turn** to move:
  - Root is labeled "**MAX**" node
  - Otherwise it is a "**MIN**" node (**opponent's turn**)
- Each level's nodes are all MAX or all MIN
- Nodes at level *i* are opposite those at level *i* +1



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## Minimax Procedure

- Create start node: MAX node, current board state
- Expand nodes down to a **depth** of *lookahead*
- Apply evaluation function at each leaf node
- "Back up" values for each non-leaf node until a value is computed for the root node
  - MIN: backed-up value is **lowest** of children's values
  - MAX: backed-up value is **highest** of children's values
- Pick operator associated with the child node whose backed-up value set the value at the root

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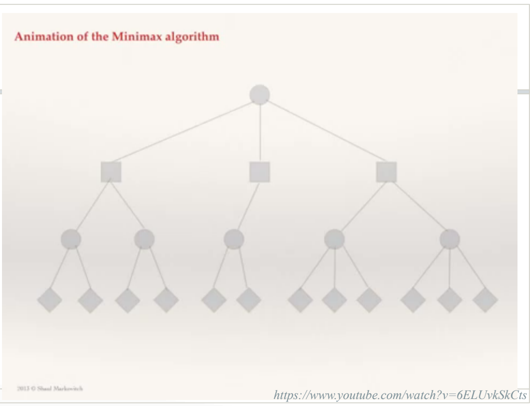
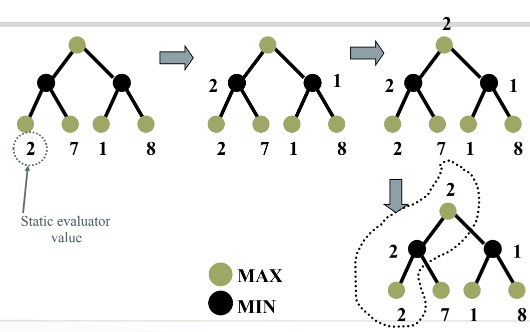


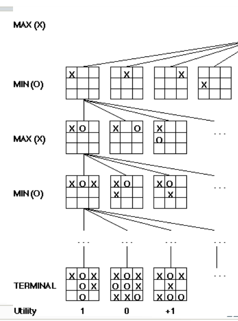

## Minimax Algorithm



# Example: Nim

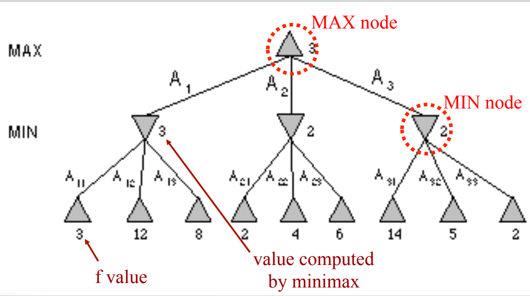
- In Nim, there are a certain number of objects (coins, sticks, etc.) on the table – we'll play 7-coin Nim
- Each player in turn has to pick up either one or two objects
- Whoever picks up the last object loses

# Partial Game Tree for Tic-Tac-Toe




- f(n) = +1 if position is a win for X.
- f(n) = -1 if position is a win for O.
- f(n) = 0 if position is a draw.

# Minimax Tree



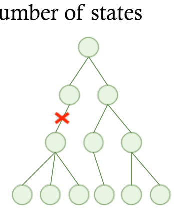

# Nim Game Tree

- **In-class exercise:**
- Draw minimax search tree for 4-coin Nim
- Things to consider:
  - What's your start state?
  - What's the maximum depth of the tree? Minimum?
- Pick up either one or two objects
- Whoever picks up the last object loses



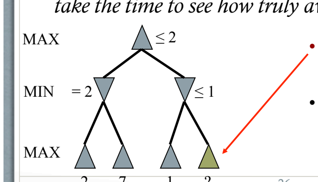
# Improving Minimax

- Basic problem: must examine a number of states that is exponential in $d$ !
- Solution: judicious **pruning** of the search tree
- "Cut off" whole sections that **can't** be part of the best solution
  - Or, sometimes, **probably won't**
  - Can be a completeness vs. efficiency tradeoff, esp. in stochastic problem spaces



# Alpha-Beta Pruning

- We can improve on the performance of the minimax algorithm through **alpha-beta pruning**
- Basic idea: *"If you have an idea that is surely bad, don't take the time to see how truly awful it is."* – Pat Winston




- We don't need to compute the value at this node.
- No matter what it is, it can't affect the value of the root node.

# Alpha-Beta Pruning

- Traverse search tree in *depth-first order*
- At each **MAX** node n, $\alpha(n)$ = maximum value found so far
- At each **MIN** node n, $\beta(n)$ = minimum value found so far
  - $\alpha$ starts at $-\infty$ and increases, $\beta$ starts at $+\infty$ and decreases
- **$\beta$–cutoff**: Given a MAX node *n*,
  - Cut off search below *n* (i.e., don't look at any more of *n*'s children) if:
  - $\alpha(n) \geq \beta(i)$ for some MIN node ancestor *i* of *n*
- **$\alpha$–cutoff**:
  - Stop searching below MIN node *n* if:
  - $\beta(n) \leq \alpha(i)$ for some MAX node ancestor *i* of *n*

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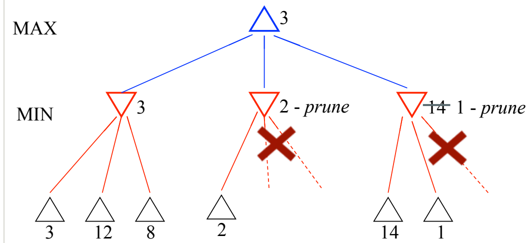
# Alpha-beta Example ($b$=3)

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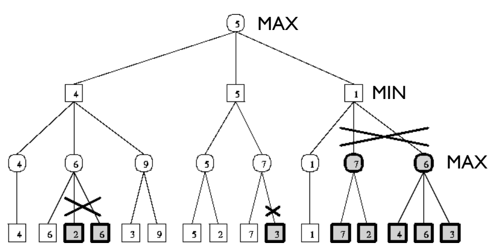
# Alpha-Beta Pruning

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# Effectiveness of Alpha-Beta

- Alpha-beta is guaranteed to:
  - Compute the same value for the root node as minimax
  - With $\leq$ computation
- **Worst case:** nothing pruned, examine $b^d$ leaf nodes, where each node has $b$ children and a $d$-ply search is performed
- **Best case:** examine only $(2b)^{d/2}$ leaf nodes.
  - Result is you can search twice as deep as minimax!
  - When each player's best move is the first alternative generated
- In Deep Blue, empirically, alpha-beta pruning took average branching factor from ~35 to ~6!

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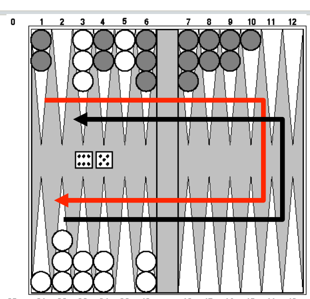
# Games of Chance

- Backgammon: a two-player game with uncertainty
- Players roll dice to determine what moves to make
- White has just rolled 5 and 6 and has four legal moves:
  - 5-10, 5-11
  - 5-11, 19-24
  - 5-10, 10-16
  - 5-11, 11-16
- Good for decision making in adversarial problems with skill *and* luck



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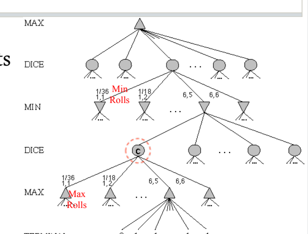
# Game Trees with Chance

- **Chance nodes** (circles) represent random events
- For a random event with N outcomes:
  - Chance node has N distinct children
  - Each has a probability
- Example:
  - Rolling 2 dice → 21 distinct outcomes
  - Not all equally likely!

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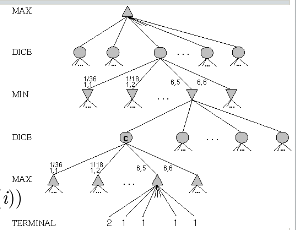
# Game Trees with Chance

- Use minimax to compute values for MAX and MIN nodes
- Use **expected values** for chance nodes
- Over a max node, as in C:

  expectimax(C) = $\Sigma_i(P(d_i) * \text{maxvalue}(i))$

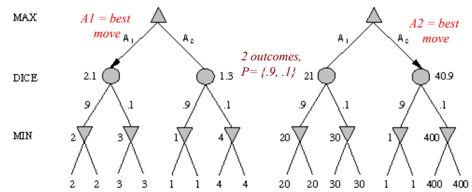
- Over a min node:

  expectimin(C) = Σi(P(di) * minvalue(i))






# Meaning of the Evaluation Function




- Dealing with probabilities and expected values means we have to be careful about the "meaning" of values returned by the static evaluator.
- A "relative-order preserving" change of values would not change decision of minimax, but could change the decision with chance nodes.



# Example: Oopsy-Nim

- Starts out like Nim
  - Each player in turn has to pick up either one or two objects
  - Sometimes (probability = 0.25), when you try to pick up two objects, you drop them both
  - Picking up a single object always works

- Question: Why can't we draw the entire game tree?
- **Exercise: Draw the 2-ply game tree (2 moves per player)**

# Nim Game Tree

- **In-class exercise:**
- Draw minimax search tree for 4-coin Nim
- Things to consider:
  - What's your start state?
  - What's the maximum depth of the tree? Minimum?