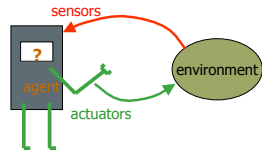


Decision Making Under Uncertainty

AI CLASS 10 (CH. 15.1-15.2.1, 16.1-16.3)



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Material from Marie desJardins, Lie Geisse, Jean-Claude Latombe, Daphne Koller, and Pinda Matuszek

Bookkeeping

- HW 3 out
 - Group work for non-programming parts!
 - Heavy on CSPs and probability
 - Forms groups today or in Piazza
- **Note corrected error in turnin instructions:** Parts I-III as PDF (not I-IV)
- HW1 and HW2
 - If you haven't gotten comments on HW1, you will soon
 - HW2 should be graded by Friday
- Soon: form project teams!

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Today's Class

- Making Decisions Under Uncertainty
 - Tracking Uncertainty over Time
 - Decision Making under Uncertainty
 - Decision Theory
 - Utility

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Introduction

- The world is not a well-defined place.
- Sources of uncertainty
 - Uncertain **inputs**: What's the temperature?
 - Uncertain (imprecise) **definitions**: Is Trump a good president?
 - Uncertain (unobserved) **states**: What's the top card?
- There is uncertainty in **inferences**
 - If I have a blistery, itchy rash and was gardening all weekend I **probably** have poison ivy

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Sources of Uncertainty

- | | |
|---|--|
| <ul style="list-style-type: none"> • Uncertain inputs <ul style="list-style-type: none"> • Missing data • Noisy data • Uncertain knowledge <ul style="list-style-type: none"> • >1 cause → >1 effect • Incomplete knowledge of causality • Probabilistic effects | <ul style="list-style-type: none"> • Uncertain outputs <ul style="list-style-type: none"> • All uncertain: <ul style="list-style-type: none"> • Reasoning-by-default • Abduction & induction • Incomplete deductive inference • Result is derived correctly but wrong in real world |
|---|--|

Probabilistic reasoning only gives **probabilistic results** (summarizes uncertainty from various sources)

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Reasoning Under Uncertainty

- People constantly make decisions anyhow.
 - Very successfully!
 - How?
 - More formally: how do we **reason under uncertainty** with **inexact knowledge**?
- Step one: **understanding what we know**

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PART I: MODELING UNCERTAINTY OVER TIME

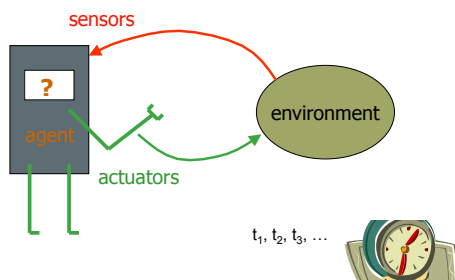
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States and Observations

- Agents don't have a continuous view of world
 - People don't either!
- We see things as a series of snapshots:
- Observations**, associated with **time slices**
 - t_1, t_2, t_3, \dots
- Each snapshot contains all variables, observed or not
 - X_t = (unobserved) state variables at time t ; observation at t is E_t
- This is **world state at time t**

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Temporal Probabilistic Agent



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Uncertainty and Time

- The world changes
 - Examples: diabetes management, traffic monitoring
- Tasks: **track** changes; **predict** changes
- Basic idea:
 - For each time step, copy state and evidence variables
 - Model uncertainty in **change over time** (the Δ)
 - Incorporate new observations as they arrive

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Uncertainty and Time

- Basic idea:
 - Copy state and evidence variables for each time step
 - Model uncertainty in change over time
 - Incorporate new observations as they arrive
- X_t = unobserved/unobservable state variables at time t :
 $BloodSugar_t, StomachContents_t$
- E_t = evidence variables at time t :
 $MeasuredBloodSugar_t, PulseRate_t, FoodEaten_t$
- Assuming discrete time steps

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States (more formally)

- Change is viewed as series of snapshots
 - Time slices/timesteps
 - Each describing the **state** of the world at a particular time
 - So we also refer to these as *states*
- Each time slice/timestep/state is represented as a set of random variables indexed by t :
 - the set of unobservable **state variables** X_t
 - the set of observable **evidence variables** E_t

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Observations (more formally)

- Time slice (a set of random variables indexed by t):
 - the set of unobservable state variables X_t
 - the set of observable evidence variables E_t
- An **observation** is a set of observed variable instantiations at some timestep
- Observation at time t : $E_t = e_t$
 - (for some values e_t)
- $X_{a:b}$ denotes the set of variables from X_a to X_b

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Transition and Sensor Models

- So how do we model change over time?**

- Transition model
 - Models how the world changes over time
 - Specifies a probability distribution...
 - Over state variables at time t
 - Given values at previous times
- Sensor model
 - Models how evidence (sensor data) gets its values
 - E.g.: $\text{BloodSugar}_t \rightarrow \text{MeasuredBloodSugar}_t$

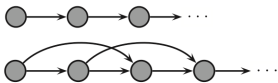
This can get exponentially large...

$$P(X_t | X_{0:t-1})$$

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Markov Assumption(s)

- Markov Assumption:**
 - X_t depends on some finite (usually fixed) number of previous X_i 's
- First-order Markov process:** $P(X_t | X_{0:t-1}) = P(X_t | X_{t-1})$
 - k^{th} order: depends on previous k time steps



- Sensor Markov assumption:** $P(E_t | X_{0:t}, E_{0:t-1}) = P(E_t | X_t)$
 - Agent's observations depend *only* on actual current state of the world

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Stationary Process

- Infinitely many possible values of t
 - Does each timestep need a distribution?
 - That is, do we need a distribution of what the world looks like at t_3 , given t_2 AND a distribution for t_{16} given t_{15} AND ...
- Assume **stationary process:**
 - Changes in the world state are governed by laws that do not themselves change over time
 - Transition model $P(X_t | X_{t-1})$ and sensor model $P(E_t | X_t)$ are time-invariant, i.e., they are the same for all t

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Complete Joint Distribution

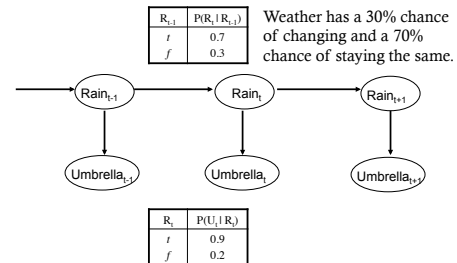
- Given:
 - Transition model: $P(X_t | X_{t-1})$
 - Sensor model: $P(E_t | X_t)$
 - Prior probability: $P(X_0)$
- Then we can specify a **complete joint distribution** of a sequence of states:

$$P(X_0, X_1, \dots, X_t, E_1, \dots, E_t) = P(X_0) \prod_{i=1}^t P(X_i | X_{i-1}) P(E_i | X_i)$$

- What's the joint probability of instantiations?

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Example



Fully worked out HMM for rain: www2.isye.gatech.edu/~yinye77/isy66416_17/Lecture6.pdf

Inference Tasks

- **Filtering** or monitoring: $P(\mathbf{X}_t | e_1, \dots, e_t)$:
 - Compute the current belief state, given all evidence to date
- **Prediction**: $P(\mathbf{X}_{t+k} | e_1, \dots, e_t)$:
 - Compute the probability of a future state
- **Smoothing**: $P(\mathbf{X}_k | e_1, \dots, e_t)$:
 - Compute the probability of a past state (hindsight)
- **Most likely explanation**: $\arg \max_{x_1, \dots, x_t} P(x_1, \dots, x_t | e_1, \dots, e_t)$
 - Given a sequence of observations, find the sequence of states that is most likely to have generated those observations

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Examples

- **Filtering**: What is the probability that it is raining today, given all of the umbrella observations up through today?
- **Prediction**: What is the probability that it will rain the day after tomorrow, given all of the umbrella observations up through today?
- **Smoothing**: What is the probability that it rained yesterday, given all of the umbrella observations through today?
- **Most likely explanation**: If the umbrella appeared the first three days but not on the fourth, what is the most likely weather sequence to produce these umbrella sightings?

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Filtering

- Maintain a current state estimate and update it
 - Instead of looking at all observed values in history
 - Also called **state estimation**
- Given result of filtering up to time t , agent must compute result at $t+1$ from new evidence e_{t+1} :

$$P(\mathbf{X}_{t+1} | e_{1:t+1}) = f(e_{t+1}, P(\mathbf{X}_t | e_{1:t}))$$

... for some function f .

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Recursive Estimation

1. Project current state forward ($t \rightarrow t+1$)
2. Update state using new evidence e_{t+1}

$P(\mathbf{X}_{t+1} | e_{1:t+1})$ as function of e_{t+1} and $P(\mathbf{X}_t | e_{1:t})$:

$$P(\mathbf{X}_{t+1} | e_{1:t+1}) = P(\mathbf{X}_{t+1} | e_{1:t}, e_{t+1})$$

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Recursive Estimation

- $P(\mathbf{X}_{t+1} | e_{1:t+1})$ as a function of e_{t+1} and $P(\mathbf{X}_t | e_{1:t})$:

$$\begin{aligned} P(X_{t+1} | e_{1:t+1}) &= P(X_{t+1} | e_{1:t}, e_{t+1}) && \text{dividing up evidence} \\ &= \alpha P(e_{t+1} | X_{t+1}, e_{1:t}) P(X_{t+1} | e_{1:t}) && \text{Bayes rule} \\ &= \alpha P(e_{t+1} | X_{t+1}) P(X_{t+1} | e_{1:t}) && \text{sensor Markov assumption} \end{aligned}$$

- $P(e_{t+1} | \mathbf{X}_{1:t+1})$ updates with new evidence (from sensor)
- One-step prediction by conditioning on current state X :

$$= \alpha P(e_{t+1} | X_{t+1}) \sum_{x_t} P(X_{t+1} | x_t) P(x_t | e_{1:t})$$

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Recursive Estimation

- One-step prediction by conditioning on current state X :

$$= \alpha P(e_{t+1} | X_{t+1}) \sum_{x_t} \underbrace{P(X_{t+1} | x_t)}_{\text{transition model}} \underbrace{P(x_t | e_{1:t})}_{\text{current state}}$$

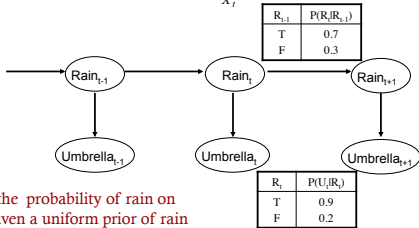
- ...which is what we wanted!
- So, think of $P(\mathbf{X}_t | e_{1:t})$ as a "message" $f_{1:t}$
 - Carried forward along the time steps
 - Modified at every transition, updated at every new observation
- This leads to a recursive definition:

$$f_{1:t+1} = \alpha \text{FORWARD}(f_{1:t}, e_{t+1})$$

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Group Exercise: Filtering

$$P(X_{t+1} | e_{1:t+1}) = \alpha P(e_{t+1} | X_{t+1}) \sum_{X_t} P(X_{t+1} | X_t) P(X_t | e_{1:t})$$



What is the probability of rain on Day 2, given a uniform prior of rain on Day 0, $U_1 = \text{true}$, and $U_2 = \text{true}$?

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PART II: DECISION MAKING UNDER UNCERTAINTY

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Decision Making Under Uncertainty

- Many environments have multiple possible outcomes
- Some outcomes may be good; others may be bad
- Some may be very likely; others unlikely
- **What's a poor agent to do?**

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Reasoning Under Uncertainty

- How do we **reason** under uncertainty and with inexact knowledge?
 - Heuristics
 - Mimic heuristic knowledge processing methods used by experts
 - Empirical associations
 - Experiential reasoning based on limited observations
 - **Probabilities**
 - Objective (frequency counting)
 - Subjective (human experience)

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Decision-Making Tools

- Decision Theory
 - Normative: how *should* agents make decisions?
 - Descriptive: how *do* agents make decisions?
- **Utility** and utility functions
 - Something's **perceived ability to satisfy needs or wants**
 - A mathematical function that ranks alternatives by utility



What is Decision Theory?

- Mathematical study of strategies for optimal decision-making
 - Options involve different risks
 - **Expectations** of gain or loss
- The study of identifying:
 - The values, uncertainties and other issues relevant to a decision
 - The resulting optimal decision for a rational agent

Decision Theory

- Combines **probability** and **utility** → Agent that makes **rational** decisions (takes rational actions)
 - On average, lead to desired outcome
- First-pass simplifications:
 - Want most desirable *immediate* outcome (episodic)
 - nondeterministic, partially observable world
- Definition of **action**:
- An action a in state s leads to outcome s' , RESULT:
 - RESULT(a) is a random variable; domain is possible outcomes
 - $P(\text{RESULT}(a) = s' \mid a, e)$

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Expected Value

- Expected Value
 - The **predicted future value** of a variable, calculated as:
 - The sum of all possible values
 - Each multiplied by the probability of its occurrence

$$\text{A } \$1000 \text{ bet for a } 20\% \text{ chance to win } \$10,000 \\ [20\%(\$10,000) + 80\%(\$0)] = \$2000$$

Satisficing

- Satisficing: achieving a goal **sufficiently**
 - Achieving the goal “more” does not increase utility of resulting state
 - Portmanteau of “satisfy” and “suffice”



Win a baseball game by 1 point now, or 2 points in another inning?
 Full credit for a search is $\leq 3K$ nodes visited. You're at 2K. Spend an hour making it 1K?
 Do you stop the coin flipping game at 1-0, or continue playing, hoping for 2-0?
 At the end of semester, you can stop with a B. Do you take the exam?
 You're thirsty. Water is good. Is more water better?

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Value Function

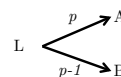
- Provides a ranking of alternatives, but not a meaningful metric scale
- Also known as an “ordinal utility function”
- Sometimes, only relative judgments (value functions) are necessary
- At other times, absolute judgments (utility functions) are required

Rational Agents

- Rationality (an overloaded word).
- A rational agent...
 - Behaves according to a **ranking over possible outcomes**
 - Which is:
 - Complete (covers all situations)
 - Consistent
 - Optimizes over strategies to best serve a desired interest
- Humans are none of these.

Preferences

- An agent chooses among:
 - Prizes (A, B, etc.)
 - Lotteries (situations with uncertain prizes and probabilities)



- Notation:
 - $A > B$ A preferred to B
 - $A \sim B$ Indifference between A and B
 - $A \succsim B$ B not preferred to A

Rational Preferences

- Preferences of a rational agent must obey constraints
 - Transitivity $(A > B) \wedge (B > C) \Rightarrow (A > C)$
 - Monotonicity $(A > B) \Rightarrow [p > q \Leftrightarrow [p, A; 1-p, B] > [q, A; 1-q, B])$
 - Orderability $(A > B) \vee (B > A) \vee (A \sim B)$
 - Substitutability $(A \sim B) \Rightarrow [p, A; 1-p, C] \sim [p, B; 1-p, C]$
 - Continuity $(A > B > C \Rightarrow \exists p [p, A; 1-p, C] \sim B)$
- Rational preferences give behavior that **maximizes expected utility**
- Violating these constraints leads to irrationality
 - For example: an agent with intransitive preferences can be induced to give away all its money.

Expected Utility

- Goal: find best of expected outcomes
- Random variable X with:
 - n values x_1, \dots, x_n
 - Distribution (p_1, \dots, p_n)
- X is the state reached after doing an action A under uncertainty
 - state = some state of the world at some timestep
- Utility function $U(s)$ is the utility of a state, i.e., **desirability**

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Expected Utility

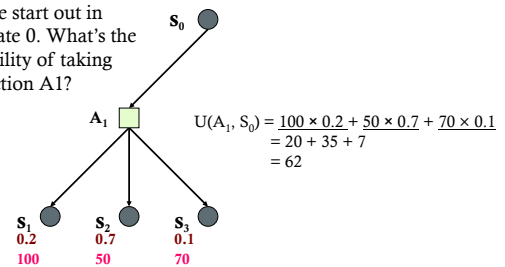
- X is state reached after doing an action A under uncertainty
- $U(s)$ is the utility of a state \leftarrow **desirability**
- $EU(a|e)$: The **expected utility** of action A, given evidence, is the **average utility of outcomes** (states in S), weighted by probability an action occurs:

$$EU[A] = \sum_{i=1, \dots, n} p(x_i|A)U(x_i)$$

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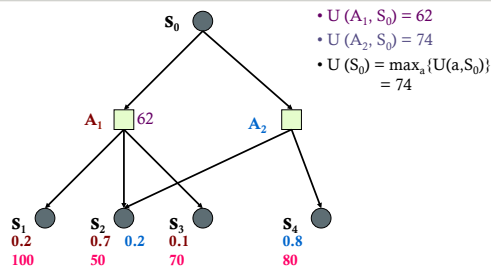
One State/One Action Example

- We start out in state 0. What's the utility of taking action A1?



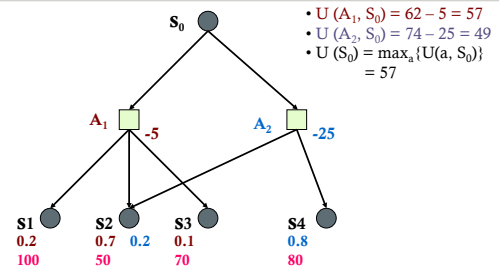
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One State/Two Actions Example



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Introducing Action Costs



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MEU Principle

- A **rational agent** should choose the action that maximizes agent's expected utility
- This is the basis of the field of **decision theory**
- The MEU principle provides a **normative criterion** for rational choice of action
- ...AI is solved!

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Not Quite...

- Must have a **complete** model of:
 - Actions
 - Utilities
 - States
- Even if you have a complete model, decision making is computationally **intractable**
- In fact, a truly rational agent takes into account the utility of reasoning as well (**bounded rationality**)
- Nevertheless, great progress has been made in this area
 - We are able to solve much more complex decision-theoretic problems than ever before

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Money

- Money does not behave as a utility function
 - That is, people don't maximize expected value of *dollar assets*.
- People are risk-averse:
 - Given a lottery L with expected monetary value EMV(L), usually $U(L) < U(\text{EMV}(L))$
 - **Want to bet \$1000 for a 20% chance to win \$10,000?**
 $[20\%(\$10,000) + 80\%(\$0)] = \$2000 > [100\%(\$1000)]$
- Expected Utility Hypothesis
 - rational behavior maximizes the expectation of some function $u \dots$ which in need not be monetary

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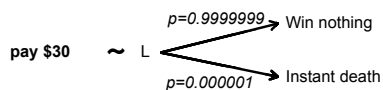
Money Versus Utility

- Money \pm Utility
 - More money is better, but not always in a linear relationship to the amount of money
- Expected Monetary Value
- Risk-averse: $U(L) < U(\text{EMV}(L))$
- Risk-seeking: $U(L) > U(\text{EMV}(L))$
- Risk-neutral: $U(L) = U(\text{EMV}(L))$

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Maximizing Expected Utility

- Utilities map **states** to **real numbers**. Which numbers?
 - People are very bad at mapping their own preferences
- Standard approach to assessment of human utilities:
 - Compare a state A to a standard lottery L_p that has "best possible prize" u^+ with probability p
 - "worst possible catastrophe" u^- with probability $(1-p)$
 - adjust lottery probability p until $A \sim L_p$



Actual Utility Scales

- Micromorts: one-millionth chance of death
 - Useful for:
 - Russian roulette
 - Paying to reduce product risks, etc.
- QALYs: quality-adjusted life years
 - Useful for:
 - Medical decisions involving substantial risk