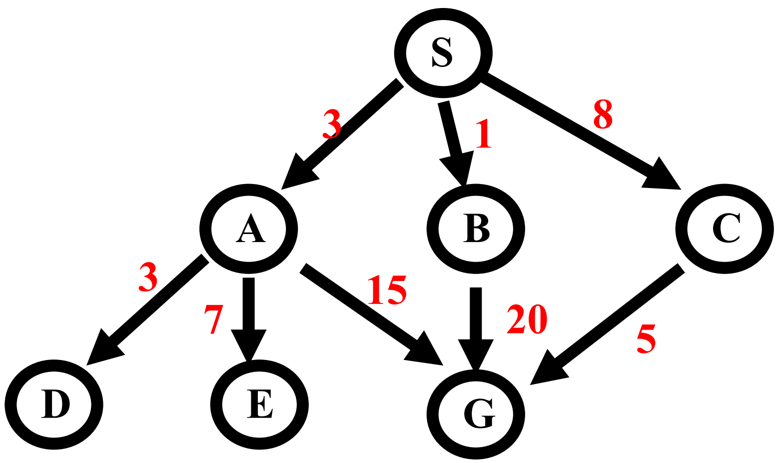
Artificial Intelligence

Class 4: Uninformed Search (Ch. 3.4)



Some material adapted from slides by Gang Hua of Stevens Institute of Technology
Some material adapted from slides by Charles R. Dyer, University of Wisconsin-Madison

Bookkeeping

- HW 1 due 9/19, 11:59pm – **Monday night**
- Piazza
  - Thank you all for using Piazza!
  - “General questions (i.e., anything that another student may also be wondering about) should be posted here.”
- Reminder, from syllabus, about Piazza posts:
  - [posts] on Piazza must follow the academic integrity guidelines
  - So post about clarifications, resources, or debugging help, but
  - **Not (for example) hints about specific answers**
  - **Or examples!**

Today's Class

- Specific algorithms
  - Breadth-first search
  - Depth-first search
  - Uniform cost search
  - Depth-first iterative deepening
- Example problems revisited

> "This is the essence of search—following up one option now and putting the others aside for later, in case the first choice does not lead to a solution."
>
> – *R&N pg. 75*

Key Procedures to Define

- EXPAND
  - Generate all successor nodes of a given node
- GOAL-TEST
  - Test if state satisfies all goal conditions
- QUEUEING-FUNCTION
  - Used to maintain a ranked list of nodes that are candidates for expansion

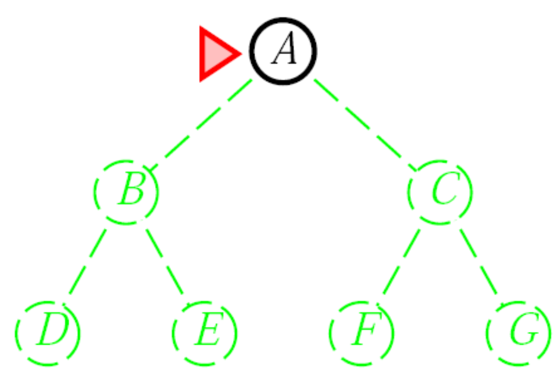
Review: Characteristics

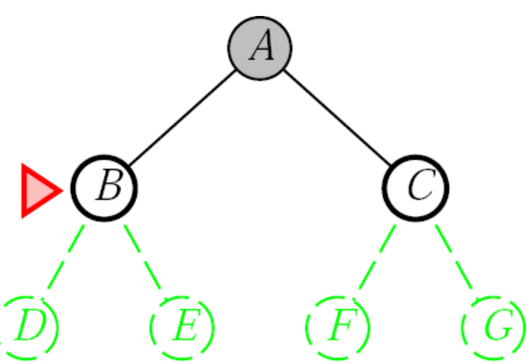
- **Completeness:** Is the algorithm guaranteed to find a solution?
- **Optimality:** Does it find the optimal solution?
  - (The solution with the lowest path cost of all possible solutions)
- **Time complexity:** How long does it take to find a solution?
- **Space complexity:** How much memory is needed to perform the search?

R&N pg. 68, 80

Generation vs. Expansion

- **Selecting** a state means making that node current
- **Expanding** the current state means applying every legal action to the current state
- Which **generates** a new set of nodes





R&N pg. 68, 80

Pre-Reading Quiz

- How does **breadth-first search** instantiate the EXPAND, GOAL-TEST, and QUEUING-FUNCTION components of state space search?
  - What does breadth-first search remind you of? (A simple abstract data type)
- How does **uniform-cost search** instantiate these search components?
  - Uniform-cost search may be less familiar.
  - Do you know another name for this type of search?
  - Can you give a real-world equivalent/example?
- How does **depth-first search** instantiate these search components?
  - What does depth-first search remind you of?
- **Why does it matter when the goal test is applied (expansion time vs. generation time)?**

Pre-Reading Quiz

- How does **breadth-first search** instantiate the EXPAND, GOAL-TEST, and QUEUING-FUNCTION components of state space search?

  - EXPAND: always expand *shallowest* unexpanded node
  - GOAL-TEST: test a node when it is **expanded**
  - QUEUEING-FUNCTION: FIFO
    - What does breadth-first search remind you of? (A simple abstract data type)

Pre-Reading Quiz

- How does **uniform-cost search** instantiate these search components?
  - Uniform-cost search may be less familiar.
  - Do you know another name for this type of search?
  - Can you give a real-world equivalent/example?

  - EXPAND: always expand *lowest path cost* unexpanded node
    - Store frontier as priority queue
  - GOAL-TEST: test a node when it is **selected for expansion**
    - First generated node may not be on optimal path
  - QUEUEING-FUNCTION: priority queue

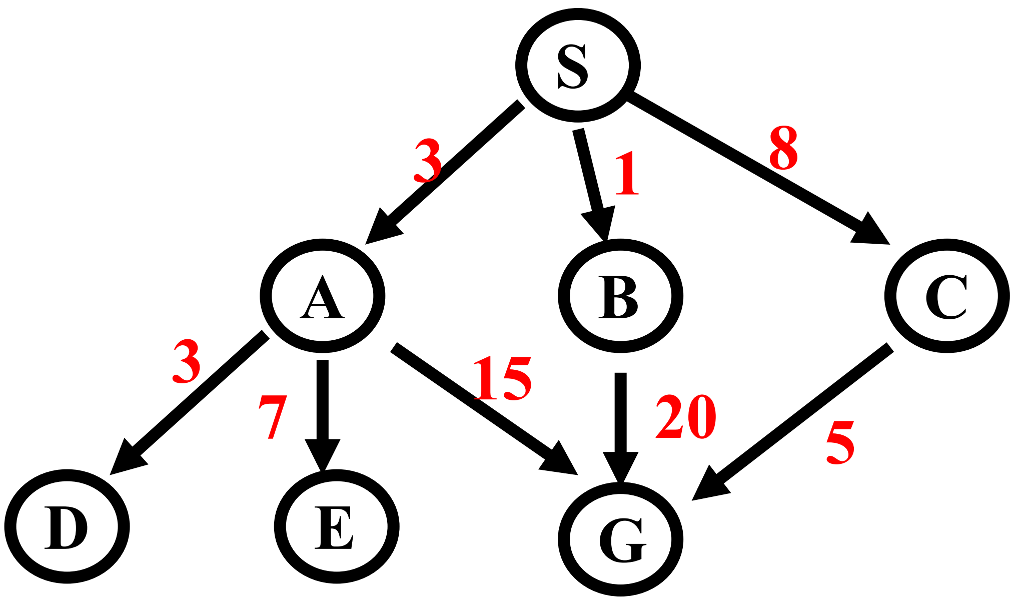
Pre-Reading Quiz

- How does **depth-first search** instantiate these search components?
  - What does depth-first search remind you of?

  - EXPAND: always expand *deepest* unexpanded node
  - GOAL-TEST: test a node when it is **expanded**
  - QUEUEING-FUNCTION: LIFO

Pre-Reading Quiz

- **Why does it matter when the goal test is applied (expansion time vs. generation time)?**
- Optimality and complexity of the algorithms are strongly affected!



Uninformed vs. Informed Search

- Uninformed search strategies
  - Use no information about the "direction" of the goal node(s)
  - Also known as "blind search"
  - Methods: Breadth-first, depth-first, depth-limited, uniform-cost, depth-first iterative deepening, bidirectional
- Informed search strategies (**next** class...)
  - Use information about the domain to (try to) (usually) head in the general direction of the goal node(s)
  - Also known as "heuristic search"
  - Methods: Hill climbing, best-first, greedy search, beam search, A, A*

Breadth-First

- Enqueue nodes in **FIFO** (first-in, first-out) order
- Characteristics:
  - **Complete** (meaning?)
  - **Optimal** (i.e., admissible) if all operators have the same cost
  - Otherwise, not optimal but finds solution with shortest path length
  - **Exponential time and space complexity**, $O(b^d)$, where:
    - *d* is the depth of the solution
    - *b* is the branching factor (number of children) at each node
- Takes a **long time to find long-path solutions**

Breadth-First: Analysis

- Takes a **long time to find long-path solutions**
  - Must look at all shorter length possibilities first
  - A complete search tree of depth d where each non-leaf node has b children:

$$1 + b + b2 + \ldots + bd = (b(d+1) - 1)/(b-1) \text{ nodes}$$

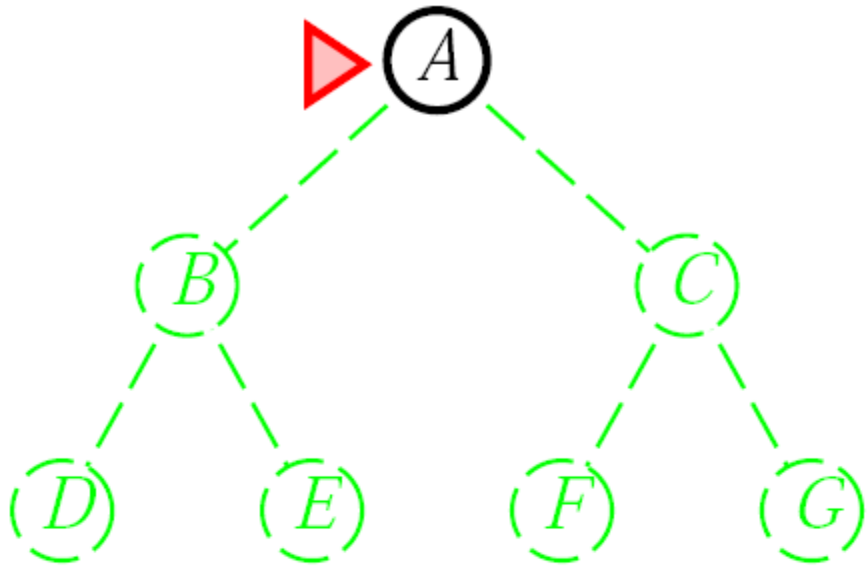
  - What if we expand nodes when they are selected?

Breadth-First: O(Example)

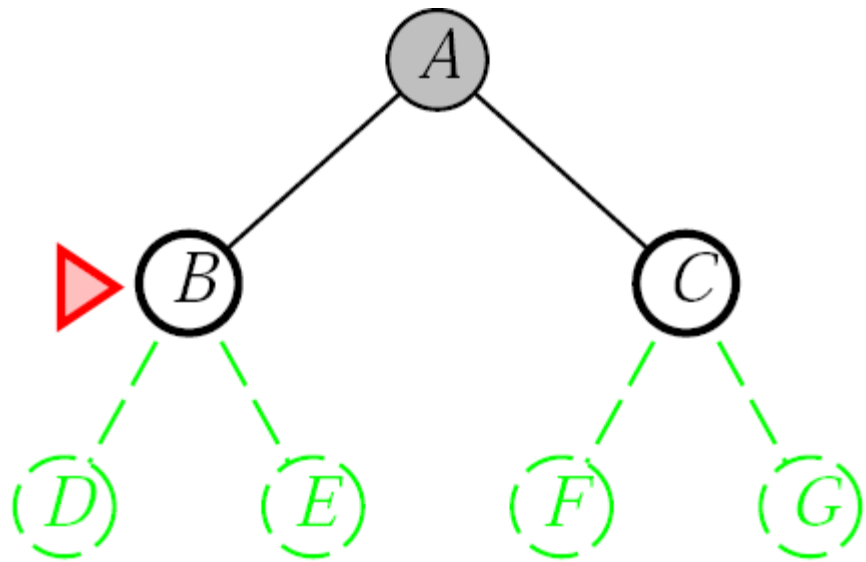
1 + b + b2 + … + bd = (b(d+1) - 1)/(b-1) nodes

- Tree where: d=12
- Every node at depths 0, ..., 11 has 10 children (b=10)
- Every node at depth 12 has 0 children
- 1 + 10 + 100 + 1000 + ... + 1012 = (1013 - 1)/9 = O(1012) nodes in the complete search tree
- If BFS expands 1000 nodes/sec and each node uses 100 bytes of storage
- Will take 35 years to run in the worst case
- Will use 111 terabytes of memory

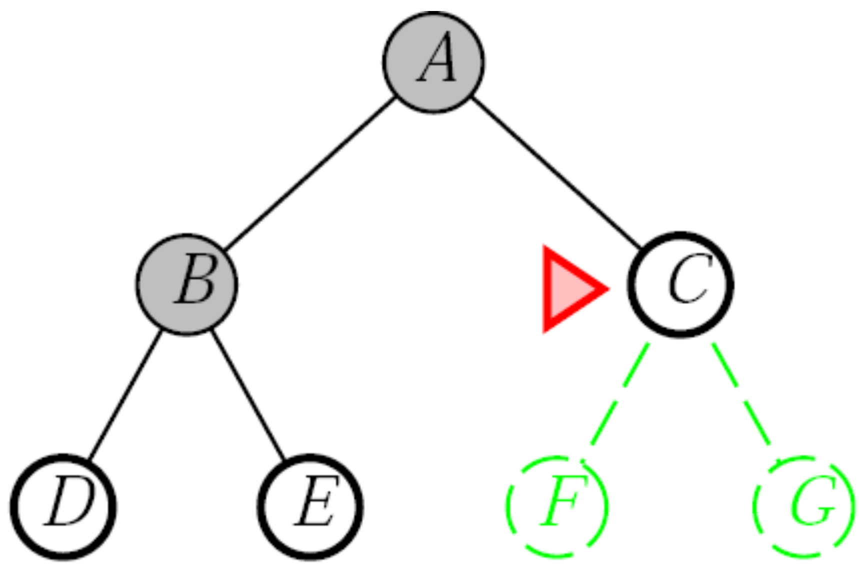
BFS



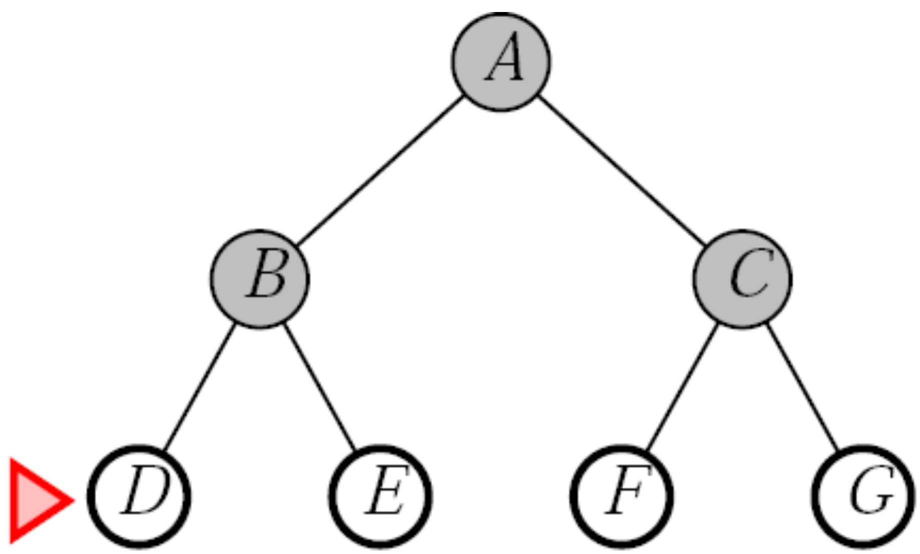
BFS



BFS



BFS



Depth-First (DFS)

- Enqueue nodes on nodes in **LIFO** (last-in, first-out) order
  - That is, nodes used as a stack data structure to order nodes
- Characteristics:
  - **Might not terminate** without a "depth bound"
    - I.e., cutting off search below a fixed depth D ("depth-limited search")
  - **Not complete**
    - With or without cycle detection, and with or without a cutoff depth
  - **Exponential time**, $O(b^d)$, but only **linear space**, O(bd)
    - Can find **long solutions quickly** if lucky
    - And **short solutions slowly** if unlucky

Loops?

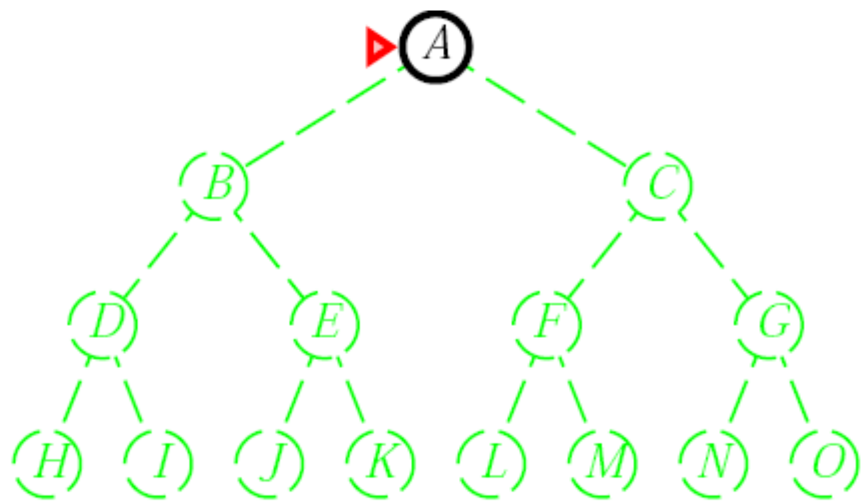
Infinite search spaces?

Depth-First (DFS): Analysis

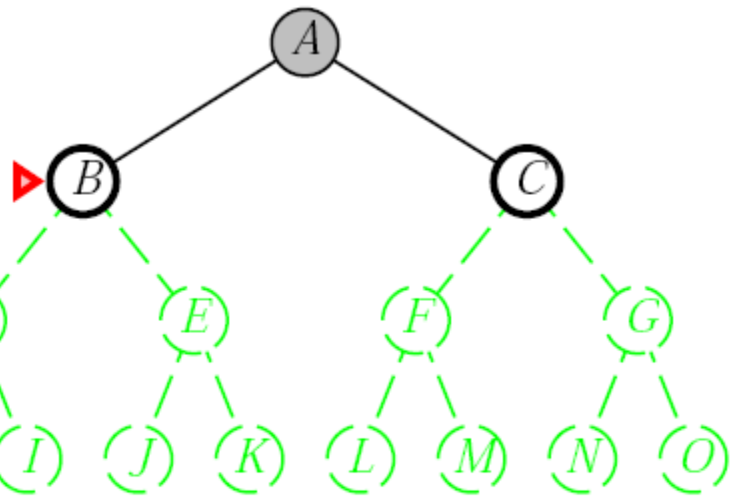
- DFS:
  - Can find **long solutions quickly** if lucky
  - And **short solutions slowly** if unlucky
- When search hits a dead end
  - Can only back up one level at a time*
  - Even if the “problem” occurs because of a bad operator choice near the top of the tree
  - Hence, only does “chronological backtracking”

* Why?

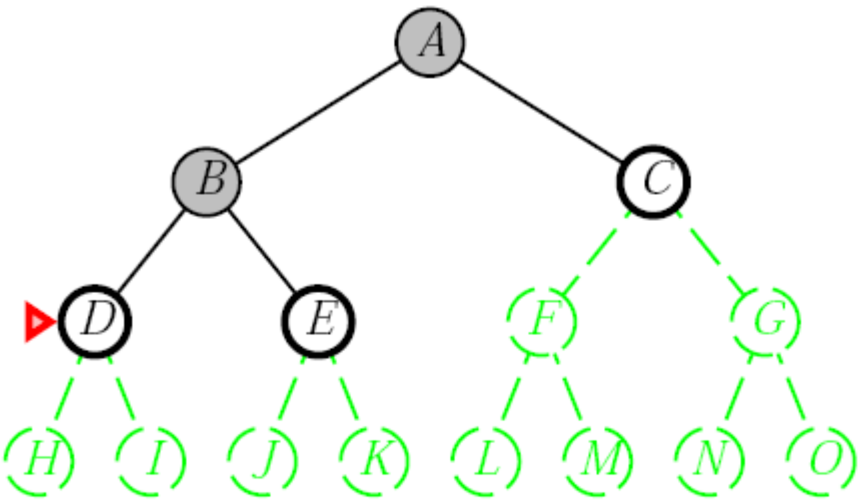
DFS



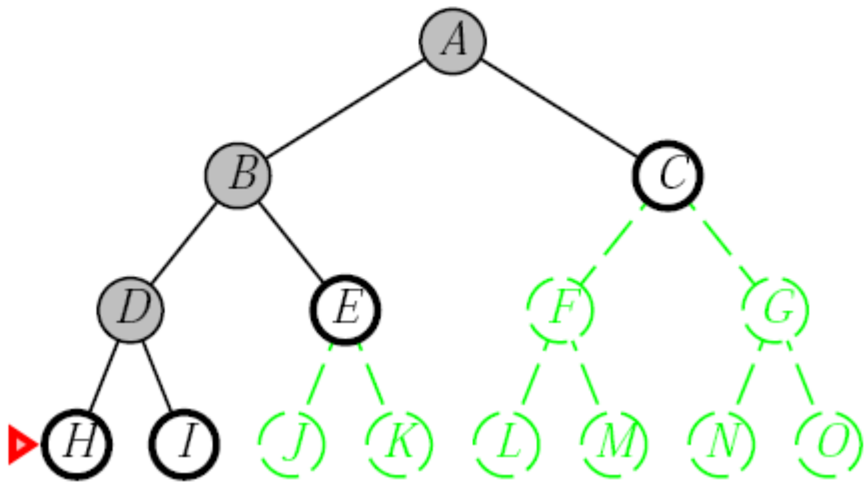
DFS



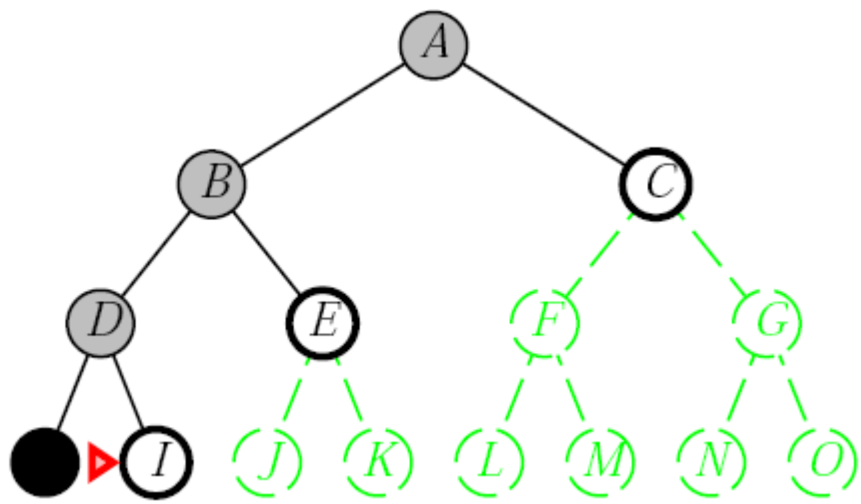
DFS



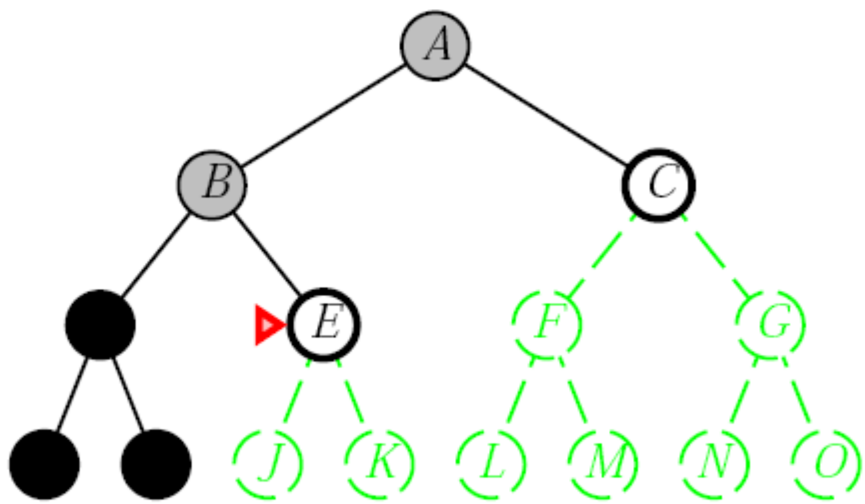
DFS



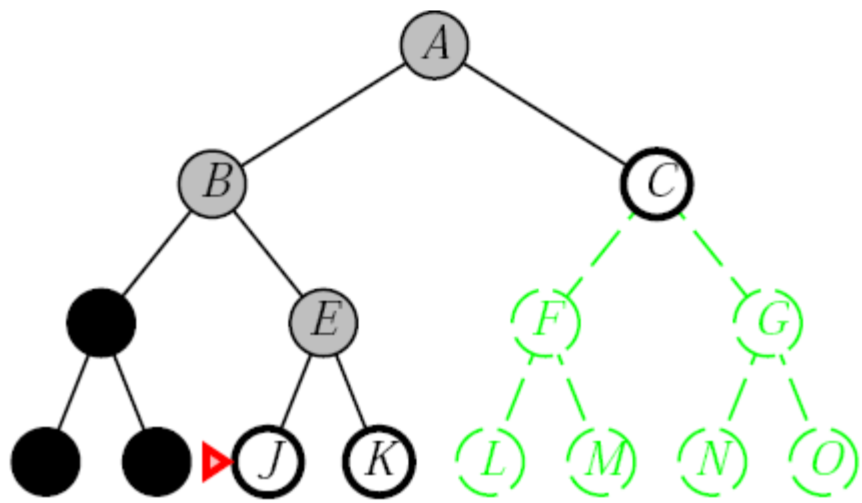
DFS



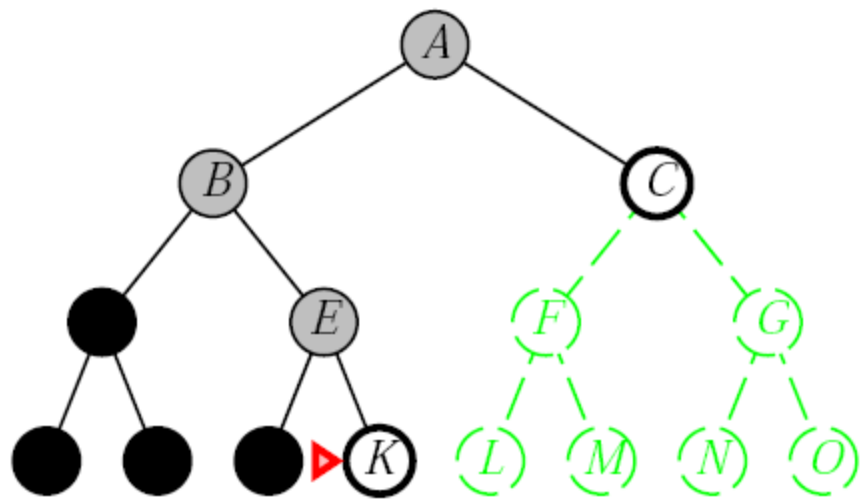
DFS



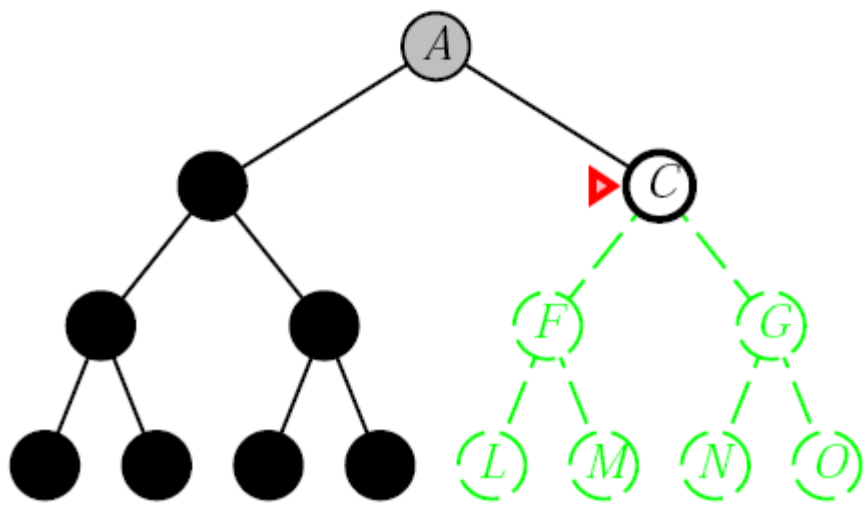
DFS



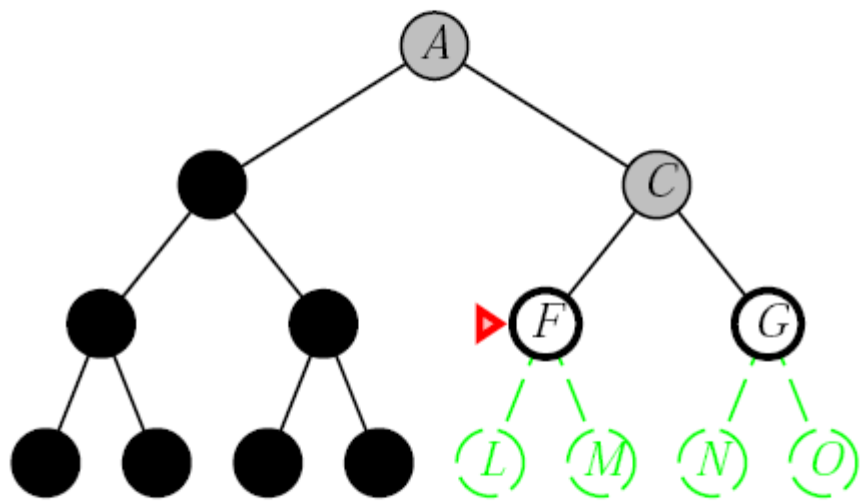
DFS



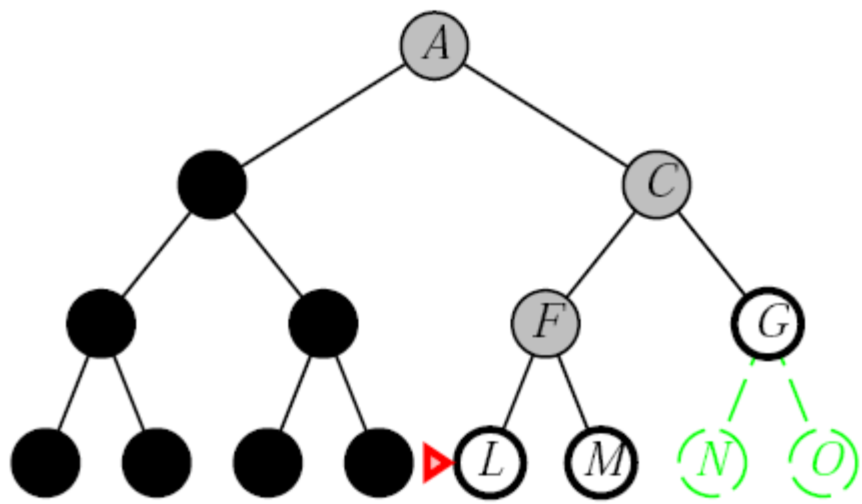
DFS



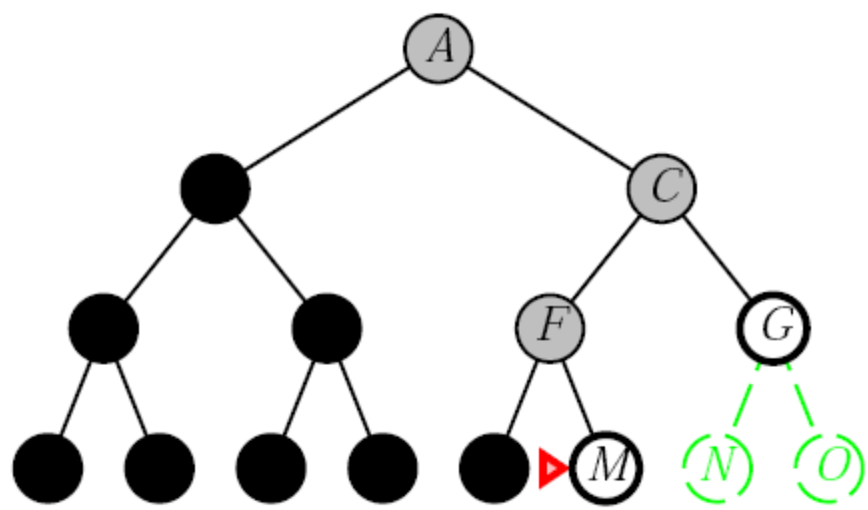
DFS



DFS



DFS



UCS Implementation

- For each frontier node, save the total cost of the path from the initial state to that node
- Expand the frontier node with the lowest path cost
- Equivalent to breadth-first if step costs all equal
- Equivalent to Dijkstra's algorithm in general

Uniform-Cost (UCS)

- Enqueue nodes by **path cost**:
  - Let g(n) = cost of path from start node to current node n
  - Sort nodes by increasing value of g
  - Identical to breadth-first search **if** all operators have equal cost
- *"Dijkstra's Algorithm"* in algorithms literature
- *"Branch and Bound Algorithm"* in operations research literature
- **Complete** (*)
- **Optimal/Admissible** (*)
  - Admissibility depends on the goal test being applied *when a node is removed from the nodes list*, not when its parent node is expanded and the node is first generated
- **Exponential time and space complexity**, $O(b^d)$

Depth-First Iterative Deepening (DFID)

1. DFS to depth 0 (i.e., treat start node as having no successors)
2. Iff no solution found, do DFS to depth 1

```
until solution found do
    DFS with depth cutoff c
    c = c+1
```

- **Complete**
- **Optimal/Admissible** if all operators have the same cost
  - Otherwise, not optimal, guarantees finding solution of shortest length
- Time complexity is a little worse than BFS or DFS because nodes near the top of the search tree are generated multiple times
- Because most nodes are near the bottom of a tree, worst case time complexity is still exponential, $O(b^d)$

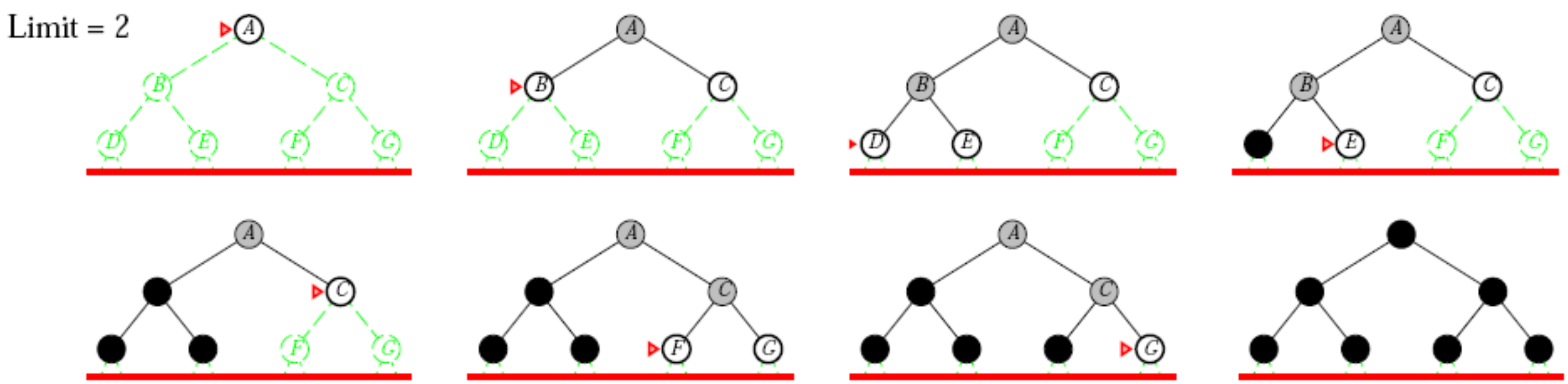
Depth-First Iterative Deepening

- If branching factor is *b* and solution is at depth *d*, then nodes at depth *d* are generated once, nodes at depth *d*-1 are generated twice, etc.
  - Hence $b^d + 2b^{(d-1)} + \ldots + db <= b^d / (1 - 1/b)^2 = O(b^d)$.
  - If b=4, then worst case is $1.78 * 4^d$, i.e., 78% more nodes searched than exist at depth d (in the worst case).
- **Linear space complexity**, O(bd), like DFS
- Has advantage of both BFS (completeness) and DFS (limited space, finds longer paths more quickly)
- Generally preferred for **large state spaces** where **solution depth is unknown**

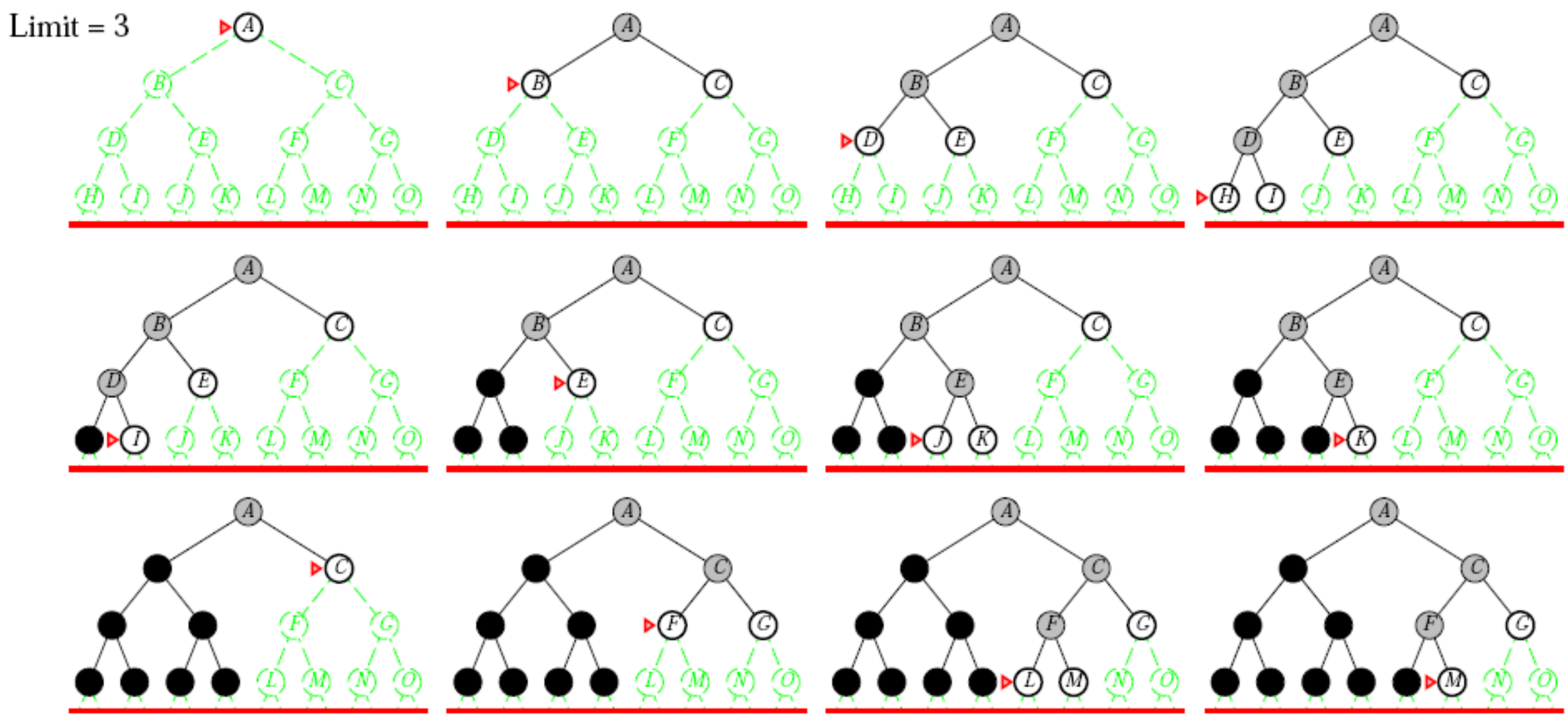
Iterative deepening search (l=1)

Limit = 1

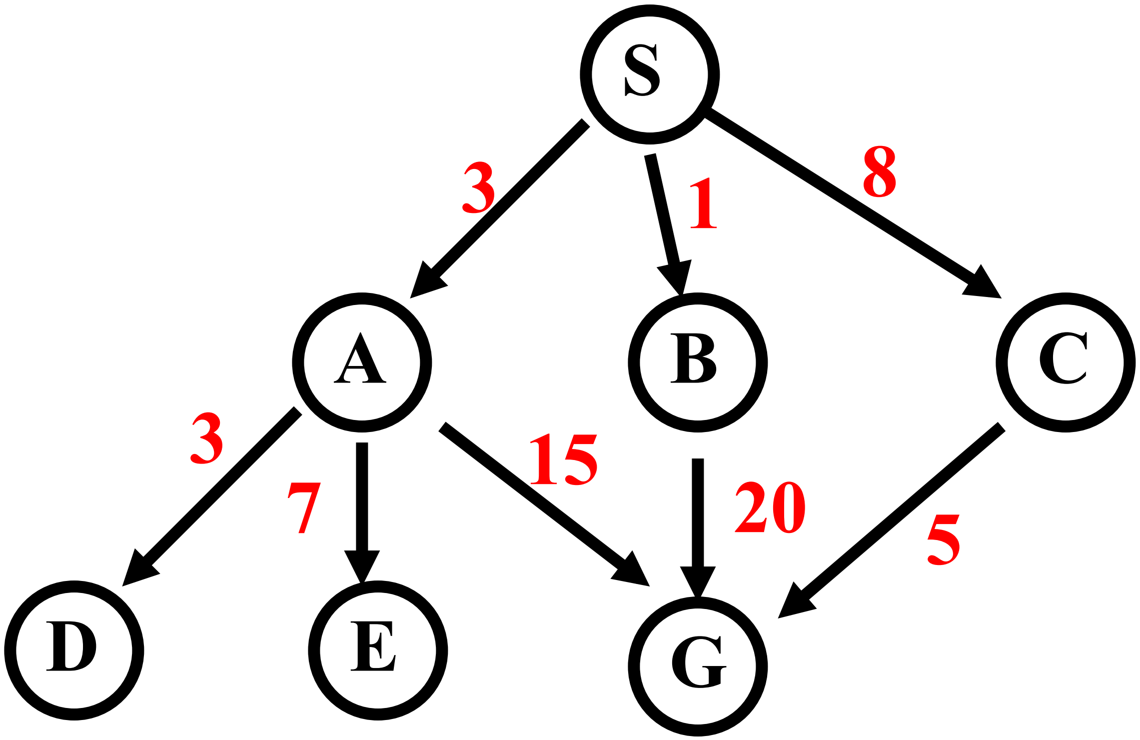
Iterative deepening search (l=2)



Iterative deepening search (l=3)



Example for Illustrating Search Strategies



Depth-First Search

Expanded node	**Nodes list**
	$\{ S^0 \}$
S^0	$\{ A^3\ B^1\ C^8 \}$
A^3	$\{ D^6\ E^{10}\ G^{18}\ B^1\ C^8 \}$
D^6	$\{ E^{10}\ G^{18}\ B^1\ C^8 \}$
E^{10}	$\{ G^{18}\ B^1\ C^8 \}$
G^{18}	$\{ B^1\ C^8 \}$

Solution path found is S A G, cost 18

Number of nodes expanded (including goal node) = 5

Breadth-First Search

Expanded node	Nodes list
	{ S^0 }
S^0	{ A^3 B^1 C^8 }
A^3	{ B^1 C^8 D^6 E^{10} G^{18} }
B^1	{ C^8 D^6 E^{10} G^{18} G^{21} }
C^8	{ D^6 E^{10} G^{18} G^{21} G^{13} }
D^6	{ E^{10} G^{18} G^{21} G^{13} }
E^{10}	{ G^{18} G^{21} G^{13} }
G^{18}	{ G^{21} G^{13} }

Solution path found is S A G , cost 18

Number of nodes expanded (including goal node) = 7

Uniform-Cost Search

Expanded node	**Nodes list**
	{ S^0 }
S^0	{ B^1 A^3 C^8 }
B^1	{ A^3 C^8 G^{21} }
A^3	{ D^6 C^8 E^{10} G^{18} G^{21} }
D^6	{ C^8 E^{10} G^{18} G^1 }
C^8	{ E^{10} G^{13} G^{18} G^{21} }
E^{10}	{ G^{13} G^{18} G^{21} }
G^{13}	{ G^{18} G^{21} }

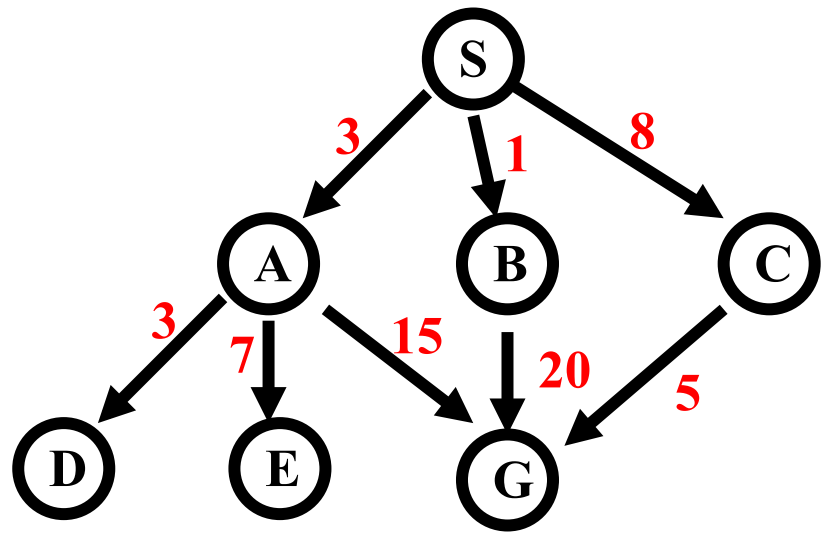
Solution path found is S A G, cost 13

Number of nodes expanded (including goal node) = 7

How they Perform

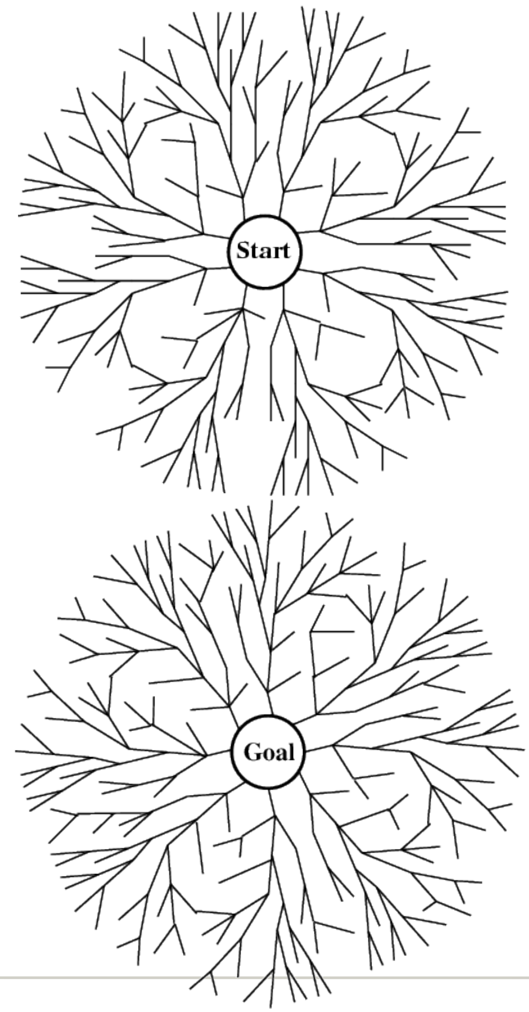
- **Depth-First Search:**
  - Expanded nodes: S A D E G
  - Solution found: S A G (cost 18)
- **Breadth-First Search**:
  - Expanded nodes: S A B C D E G
  - Solution found: S A G (cost 18)
- **Uniform-Cost Search**:
  - Expanded nodes: S A D B C E G
  - Solution found: S B G (cost 13)

  This is the only uninformed search that worries about costs.
- **Iterative-Deepening Search**:
  - nodes expanded: S S A B C S A D E G
  - Solution found: S A G (cost 18)



Bi-directional Search

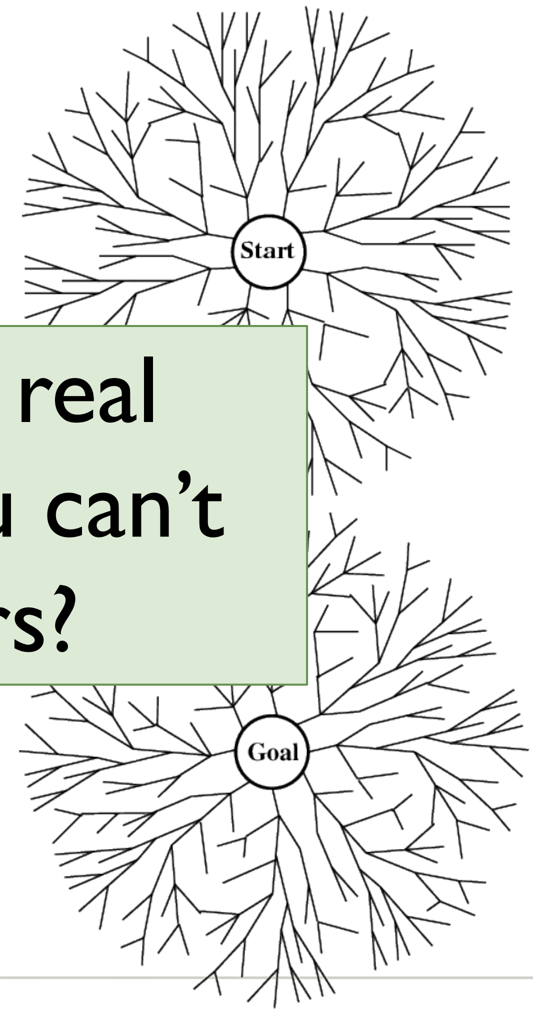
- Alternate searching from
  - start state → goal
  - goal state → start
- Stop when the frontiers intersect.
- Works well only when there are unique start and goal states
- Requires ability to generate "predecessor" states.
- Can (sometimes) find a solution fast



Bi-directional Search

- Alternate searching from
  - start state → goal
  - goal state → start
- Stop when the frontiers intersect
- W
  u
- R
  "predecessor" states.
- Can (sometimes) find a solution fast

For next time: What's a real world problem where you can't generate predecessors?



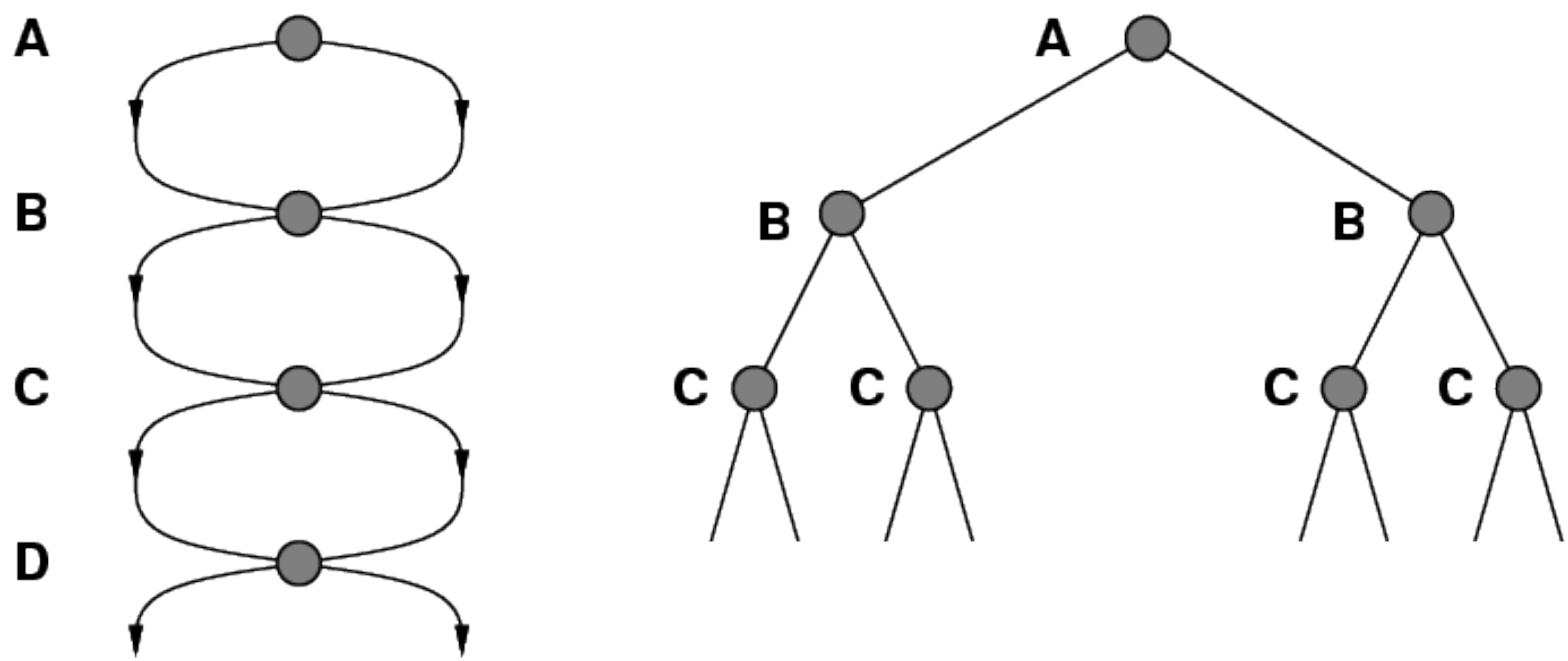
Comparing Search Strategies

Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Time	b^d	b^d	b^m	b^l	b^d	$b^{d/2}$
Space	b^d	b^d	bm	bl	bd	$b^{d/2}$
Optimal?	Yes	Yes	No	No	Yes	Yes
Complete?	Yes	Yes	No	Yes, if $l \geq d$	Yes	Yes

Avoiding Repeated States

- In increasing order of effectiveness in reducing size of state space and with increasing computational costs:
    1. Do not return to the state you just came from.
    2. Do not create paths with cycles in them.
    3. Do not generate any state that was ever created before.
- Effect depends on frequency of loops in state space.

A State Space that Generates an Exponentially Growing Search Space



Holy Grail Search

Expanded node	**Nodes list**
	{ S^0 }
S^0	{ C^8 A^3 B^1 }
C^8	{ G^{13} A^3 B^1 }
G^{13}	{ A^3 B^1 }

Solution path found is S C G, cost 13 (optimal)

Number of nodes expanded (including goal node) = 3

(minimum possible!)

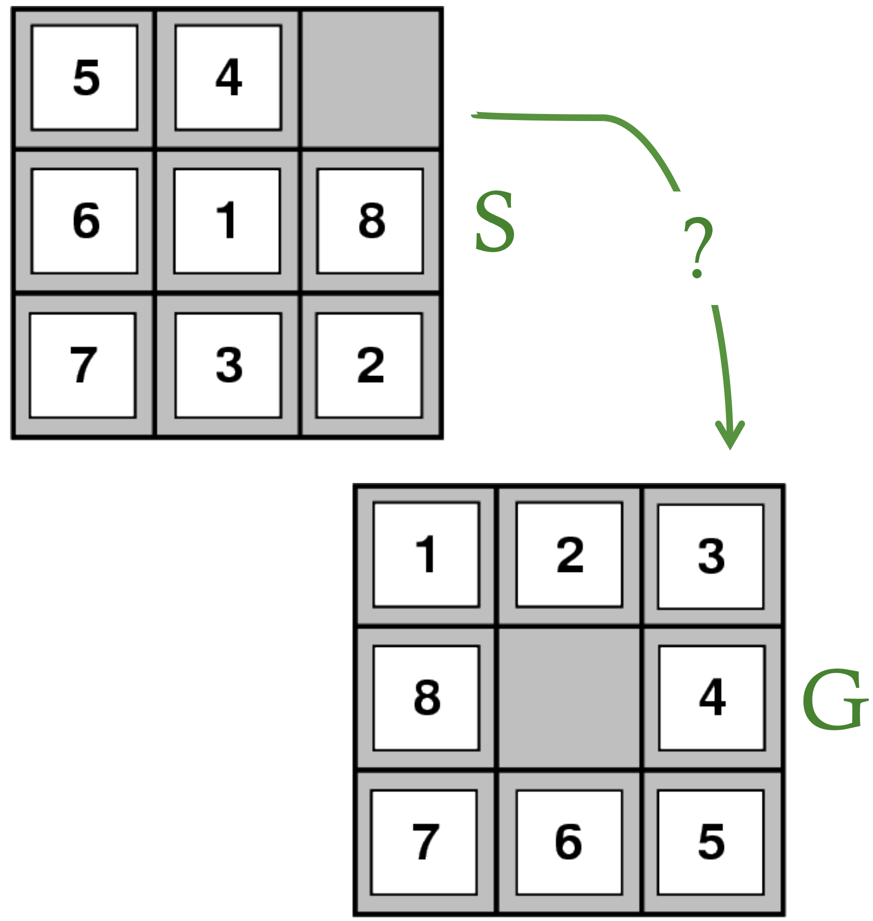
Holy Grail Search

Why not go straight to the solution, without any wasted detours off to the side?

<foreshadowing> **If only we knew where we were headed…** </foreshadowing>

8-Puzzle Revisited

- What's a good algorithm?
  - Depth-first search?
  - Breadth-first search?
  - Uniform-cost?
  - Iterative deepening?



"Satisficing"

- Wikipedia:

  "**Satisficing** is … searching until
  an **acceptability threshold** is met"

- Contrast with **optimality**
- A combination of *satisfy* and *suffice*
- Introduced by Herbert A. Simon in 1956

Another piece of
problem
definition

“Satisficing”

- Wikipedia:

  “**Satisficing** is … searching until
  an **acceptability threshold** is met”

- Contrast with **optimality**
  - Satisficeable problems *do not benefit from* finding the optimal solution

Another piece of **problem definition**

- A combination of *satisfy* and *suffice*

- Introduced by Herbert A. Simon in 1956