First-Order Logic & Inference

AI Class 19 (Ch. 8.1–8.3, 9)

Material from Dr. Marie desJardin, Some material adopted from notes by Andreas Geyer-Schulz and Chuck Dyer

Bookkeeping & Today

- HW5 out by 11:59pm
- Project designs Thursday
  - Not trivial to grade!
- Today:
  - A couple of midterm questions
  - Reasoning with logic-based agents
  - Logical inference
  - Model checking

Eliding +/- ...

Score	|Students|
A > 70	15
B > 60	12
C > 50	2
D ≤ 50	

These are *very very approximate.*

Midterm: State Spaces

- **Describe the <u>state space</u> for the following puzzle:**
  - You are given a grid of 5 color bars (rows). Each bar contains exactly 5 colors, with no duplications.
  - The colors on each bar can be changed by rotating the bar to the right, one step at a time.
  - ~~The goal is to rotate each bar until every column contains every color, and no column contains the same color more than once.~~
  - This is an initial state – so, it's a member of the state space.

Green	Blue	Orange	Red	Yellow
Green	Orange	Yellow	Blue	Red
Blue	Red	Green	Yellow	Orange
Green	Orange	Yellow	Blue	Red
Green	Orange	Red	Yellow	Blue

- What other states can you generate?
- How many total states is that?

Midterm: State Spaces

- **What is the difference between Nash Equilibrium and Pareto Optimality?**
  - **Nash equilibrium** is when no player in a game can increase their payoff by ***unilaterally*** changing their actions.
    - Social or individual good?
    - Can you have >1 Nash equilibrium?
  - **Pareto optimal** is when it is not possible to make ***any*** player better off in the game **without hurting another player**.
    - Social or individual good?

Logical Agents

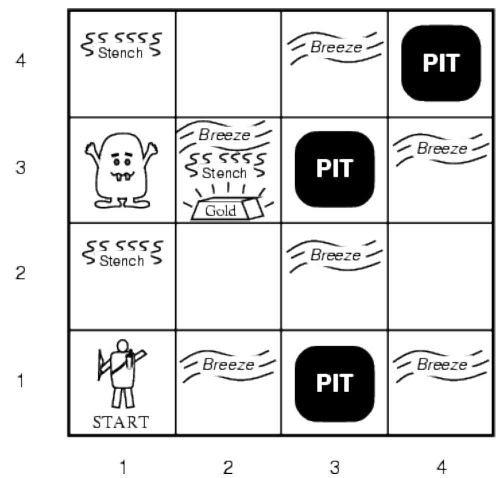
Logical Agents for Wumpus World

Three (non-exclusive) agent architectures:

- **Reflex** agents
  - Have rules that classify situations, specifying how to react to each possible situation
- **Model-based** agents
  - Construct an internal model of their world
- **Goal-based** agents
  - Form goals and try to achieve them

A Typical Wumpus World

- The agent always starts in the field [1,1].
- The task of the agent is to find the gold, return to the field [1,1] and climb out of the cave.



A Simple Reflex Agent

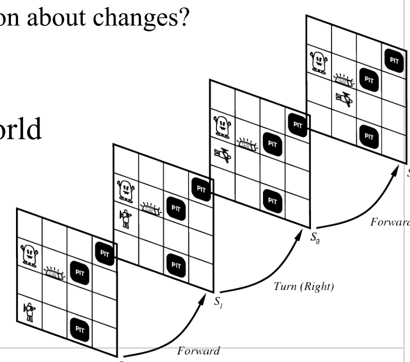
- Rules to map percepts into observations:
  ∀b,g,u,c,t Percept([Stench, b, g, u, c], t) → Stench(t)
  ∀s,g,u,c,t Percept([s, Breeze, g, u, c], t) → Breeze(t)
  ∀s,b,u,c,t Percept([s, b, Glitter, u, c], t) → AtGold(t)
- Rules to select an action given observations:
  ∀t AtGold(t) → Action(Grab, t)

A Simple Reflex Agent

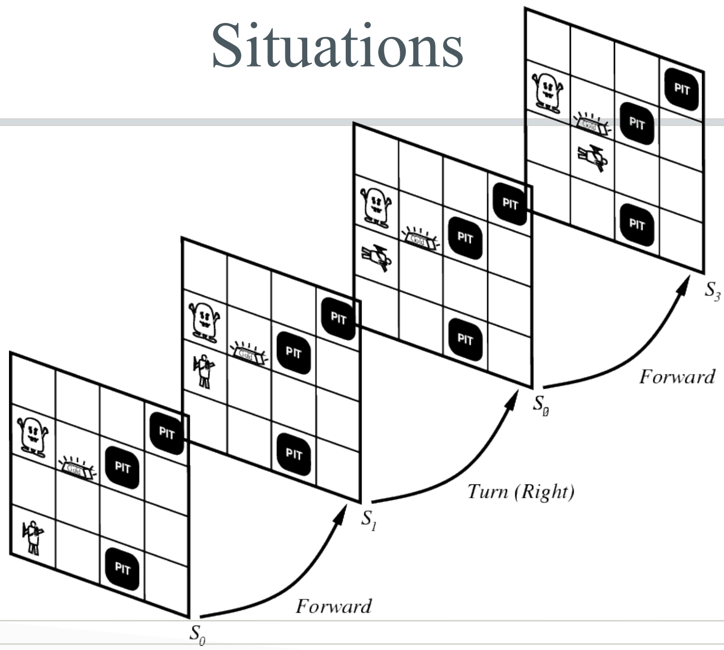
- Some difficulties:
- Climb?
  - There is no percept that indicates the agent should climb out – position and holding gold are not part of the percept sequence
- Loops?
  - The percept will be repeated when you return to a square, which should cause the same response (unless we maintain some internal model of the world)

Representing Change

- Representing change in the world in logic can be tricky.
- One way is just to change the KB
  - Add and delete sentences from the KB to reflect changes
  - How do we remember the past, or reason about changes?
- **Situation calculus** is another way
- A **situation** is a snapshot of the world at some instant in time
- When the agent performs an action A in situation S1, the result is a new situation S2.



Situations



Situation Calculus

- A **situation** is:
  - A snapshot of the world
  - At an interval of time
  - During which nothing changes
- Every true or false statement is made wrt. a situation
  - Add **situation variables** to every predicate.
  - *at(Agent,1,1)* becomes *at(Agent,1,1,s0)*:
    at(Agent,1,1) **is true in** situation (i.e., state) s0.

Situation Calculus

- Alternatively, add a special 2nd-order predicate, **holds(f,s),** that means "f is true in situation s." E.g., holds(at(Agent,1,1),s0)
- Or: add a new function, **result(a,s),** that maps a situation s into a new situation as a result of performing action a. For example, result(forward, s) is a function that returns the successor state (situation) to s
- Example: The action *agent-walks-to-location-y* could be represented by

$(\forall x)(\forall y)(\forall s)\ (at(Agent,x,s) \wedge \neg onbox(s)) \rightarrow at(Agent,y,result(walk(y),s))$

Deducing Hidden Properties

- From the perceptual information we obtain in situations, we can **infer properties of locations**

  $\forall l,s\ at(Agent,l,s) \wedge Breeze(s) \rightarrow Breezy(l)$

  $\forall l,s\ at(Agent,l,s) \wedge Stench(s) \rightarrow Smelly(l)$
- Neither Breezy nor Smelly need situation arguments because pits and Wumpuses do not move around

Deducing Hidden Properties II

- We need to write some rules that relate various aspects of a single world state (as opposed to across states)
- There are two main kinds of such rules:
  - **Causal rules** reflect assumed direction of causality:

    (∀l1,l2,s) At(Wumpus,l1,s) ∧ Adjacent(l1,l2) → Smelly(l2)

    (∀ l1,l2,s) At(Pit,l1,s) ∧ Adjacent(l1,l2) → Breezy(l2)
- Systems that reason with causal rules are called **model-based reasoning** systems

Deducing Hidden Properties II

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Deducing Hidden Properties II

- We need to write some rules that relate various aspects of a single world state (as opposed to across states)
- There are two main kinds of such rules:
  - **Diagnostic rules** infer the presence of **hidden properties** directly from the percept-derived information. We have already seen two:

    $(\forall\ l,s)\ At(Agent,l,s) \wedge Breeze(s) \rightarrow Breezy(l)$

    $(\forall\ l,s)\ At(Agent,l,s) \wedge Stench(s) \rightarrow Smelly(l)$

Frames: A Data Structure

- A **frame** divides knowledge into **substructures** by representing "stereotypical situations."
- Situations can be visual scenes, structures of physical objects,
- Useful for representing commonsense knowledge.

Slots	Fillers
publisher	Thomson
title	Expert Systems
author	Giarratano
edition	Third
year	1998
pages	600

Slot	Fillers
name	computer
specialization_of	a_kind_of machine
types	(desktop, laptop,mainframe,super) if-added: Procedure ADD_COMPUTER
speed	default: faster if-needed: Procedure FIND_SPEED
location	(home,office,mobile)
under_warranty	(yes, no)

intelligence.worldofcomputing.net/knowledge-representation/frames.html#.WCHhCNxBo8A

Representing Change: The Frame Problem

- **Frame axioms**: If property x doesn't change as a result of applying action a in state s, then it stays the same.
  - On (x, z, s) ∧ Clear (x, s) → On (x, table, Result(Move(x, table), s)) ∧ ¬On(x, z, Result (Move (x, table), s))
  - On (y, z, s) ∧ y≠ x → On (y, z, Result (Move (x, table), s))
  - The proliferation of frame axioms becomes very cumbersome in complex domains

The Frame Problem II

- **Successor-state axiom**: General statement that characterizes **every way** in which a particular predicate can become true:
  - Either it can be **made true**, or it can **already be true and not be changed**:
  - On (x, table, Result(a,s)) ↔ [On (x, z, s) ∧ Clear (x, s) ∧ a = Move(x, table)] v [On (x, table, s) ∧ a ≠ Move (x, z)]
- In complex worlds with longer chains of action, even these are too cumbersome
  - Planning systems use special-purpose inference to reason about the expected state of the world at any point in time during a multi-step plan

Qualification Problem

- Qualification problem:
  - How can you possibly characterize every single effect of an action, or every single exception that might occur?
  - When I put my bread into the toaster, and push the button, it will become toasted after two minutes, unless…
    - The toaster is broken, or…
    - The power is out, or…
    - I blow a fuse, or…
    - A neutron bomb explodes nearby and fries all electrical components, or…
    - A meteor strikes the earth, and the world we know it ceases to exist, or…

Ramification Problem

- How do you describe every effect of every action?
  - When I put my bread into the toaster, and push the button, the bread will become toasted after two minutes, and…
    - The crumbs that fall off the bread onto the bottom of the toaster over tray will also become toasted, and…
    - Some of the aforementioned crumbs will become burnt, and…
    - The outside molecules of the bread will become "toasted," and…
    - The inside molecules of the bread will remain more "breadlike," and…
    - The toasting process will release a small amount of humidity into the air because of evaporation, and…
    - The heating elements will become a tiny fraction more likely to burn out the next time I use the toaster, and…
    - The electricity meter in the house will move up slightly, and…

Knowledge Engineering!

- Modeling the "right" conditions and the "right" effects at the "right" level of abstraction is very difficult
- Knowledge engineering (creating and maintaining knowledge bases for intelligent reasoning) is a **field**
- Many researchers hope that automated knowledge acquisition and machine learning tools can fill the gap:
  - Our intelligent systems should be able to **learn** about the conditions and effects, just like we do.
  - Our intelligent systems should be able to learn when to pay attention to, or reason about, certain aspects of processes, depending on the context.

Preferences Among Actions

- A problem with the Wumpus world knowledge base that we have built so far is that it is difficult to decide which action is best among a number of possibilities.
- For example, to decide between a forward and a grab, axioms describing when it is OK to move to a square would have to mention glitter.
- This is not modular!
- We can solve this problem by **separating facts about actions from facts about goals**. This way our **agent can be reprogrammed just by asking it to achieve different goals**.

Preferences Among Actions

- The first step is to describe the desirability of actions independent of each other.
- In doing this we will use a simple scale: actions can be Great, Good, Medium, Risky, or Deadly.
- Obviously, the agent should always do the best action it can find:

  $(\forall a,s)\ Great(a,s) \rightarrow Action(a,s)$

  $(\forall a,s)\ Good(a,s) \wedge \neg(\exists b)\ Great(b,s) \rightarrow Action(a,s)$

  $(\forall a,s)\ Medium(a,s) \wedge (\neg(\exists b)\ Great(b,s) \vee Good(b,s)) \rightarrow Action(a,s)$

  ...

Preferences Among Actions

- We use this action quality scale in the following way.
- Until it finds the gold, the basic strategy for our agent is:
  - Great actions include picking up the gold when found and climbing out of the cave with the gold.
  - Good actions include moving to a square that's OK and hasn't been visited yet.
  - Medium actions include moving to a square that is OK and has already been visited.
  - Risky actions include moving to a square that is not known to be deadly or OK.
  - Deadly actions are moving into a square that is known to have a pit or a Wumpus.

Goal-Based Agents

- Once the gold is found, it is necessary to change strategies. So now we need a new set of action values.
- We could encode this as a rule:
  - $(\forall s)$ Holding(Gold,s) → GoalLocation([1,1]),s)
- We must now decide how the agent will work out a sequence of actions to accomplish the goal.
- Three possible approaches are:
  - **Inference**: good versus wasteful solutions
  - **Search**: make a problem with operators and set of states
  - **Planning**: coming soon!

Logical Inference

Chapter 9

Model Checking

- Given KB, does sentence S hold?

Quick review: What's a KB? What's a sentence?

- Basically **generate and test**:
  - Generate all the possible models
  - Consider the models M in which KB is TRUE
  - If ∀M S , then S is **provably true**
  - If ∀M ¬S, then S is **provably false**
  - Otherwise (∃M1 S ∧ ∃M2 ¬S): S is **satisfiable** but neither provably true or provably false

What does model mean?

Efficient Model Checking

- Davis-Putnam algorithm (DPLL): Generate-and-test model checking with:
  - Early termination (short-circuiting of disjunction and conjunction)
  - Pure symbol heuristic: Any symbol that only appears negated or unnegated must be FALSE/TRUE respectively.
    - Can "conditionalize" based on instantiations already produced
  - Unit clause heuristic: Any symbol that appears in a clause by itself can immediately be set to TRUE or FALSE
- WALKSAT: Local search for satisfiability:
  - Pick a symbol to flip (toggle TRUE/FALSE), either using min-conflicts *or* choosing randomly
- …or you can use *any* local or global search algorithm!

Reminder: Inference Rules for FOL

- Inference rules for **propositional logic** apply to **FOL**
  - Modus Ponens, And-Introduction, And-Elimination, …
- New (sound) inference rules for use with quantifiers:
  - Universal elimination
  - Existential introduction
  - Existential elimination
  - Generalized Modus Ponens (GMP)

Automating FOL Inference with Generalized Modus Ponens

Automated Inference for FOL

- Automated inference using FOL is harder than PL
  - Variables can take on an infinite number of possible values
    - From their domains, anyway
    - This is a reason to do careful KR!
  - So, potentially infinite ways to apply Universal Elimination
- *Godel's Completeness Theorem* says that FOL entailment is only semidecidable*
  - If a sentence is **true** given a set of axioms, can prove it
  - If the sentence is **false**, then there is no guarantee that a procedure will ever determine this
  - **Inference may never halt**

*The "halting problem"

Generalized Modus Ponens (GMP)

- Apply modus ponens reasoning to generalized rules
- Combines And-Introduction, Universal-Elimination, and Modus Ponens
  - From $P(c)$ and $Q(c)$ and $(\forall x)(P(x) \wedge Q(x)) \rightarrow R(x)$ derive $R(c)$
- General case: **Given**
  - **atomic sentences** $P_1, P_2, \ldots, P_N$
  - **implication sentence** $(Q_1 \wedge Q_2 \wedge \ldots \wedge Q_N) \rightarrow R$
    - $Q_1, \ldots, Q_N$ and R are atomic sentences
  - **substitution** subst(θ , P_i) = subst(θ , Q_i) for i=1,…,N
  - **Derive new sentence: subst(θ , R)**

Generalized Modus Ponens (GMP)

- **Derive new sentence: subst(θ , R)**
- Substitutions
  - subst(θ , α) denotes the result of applying a **set of substitutions,** defined by θ , to the sentence α
  - A substitution list $\theta = \{v_1/t_1, v_2/t_2, \ldots, v_n/t_n\}$ means to replace all occurrences of variable symbol v_i by term t_i
  - Substitutions are made in left-to-right order in the list
  - subst({x/IceCream, y/Ziggy}, eats(y,x)) = eats(Ziggy, IceCream)

Horn Clauses

- A Horn clause is a sentence of the form:

  $$(\forall x)\ P_1(x) \wedge P_2(x) \wedge \ldots \wedge P_n(x) \rightarrow Q(x)$$

  where:
  - there are 0 or more P_is and 0 or 1 Qs
  - the P_is and Q are positive (non-negated) literals
- Equivalently: $P_1(x) \vee P_2(x) \ldots \vee P_n(x)$ where the P_i are all atomic and **at most one** of them is positive
- Prolog is based on Horn clauses
- Horn clauses represent a **subset** of the set of sentences representable in FOL

Horn Clauses II

- Special cases
  - $P_1 \wedge P_2 \wedge \ldots P_n \rightarrow Q$
  - $P_1 \wedge P_2 \wedge \ldots P_n \rightarrow$ false
  - true $\rightarrow Q$
- These are not Horn clauses:
  - $p(a) \vee q(a)$
  - $(P \wedge Q) \rightarrow (R \vee S)$

Forward Chaining

- Proofs start with the given axioms/premises in KB, deriving new sentences using GMP until the goal/query sentence is derived
- This defines a **forward-chaining** inference procedure because it moves "forward" from the KB to the goal [eventually]
- Inference using GMP is **complete** for KBs containing **only Horn clauses**

Forward Chaining Example

- KB:
  - allergies(X) → sneeze(X)
  - cat(Y) ∧ allergic-to-cats(X) → allergies(X)
  - cat(Felix)
  - allergic-to-cats(Lise)
- Goal:
  - sneeze(Lise)

Forward Chaining Algorithm

procedure FORWARD-CHAIN(KB, p)

  if there is a sentence in KB that is a renaming of p **then return**
  Add p to KB
  for each $(p_1 \wedge \ldots \wedge p_n \Rightarrow q)$ **in** KB such that for some i, UNIFY$(p_i, p) = \theta$ succeeds **do**
    FIND-AND-INFER(KB, $[p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n]$, q, θ)
  end

---

procedure FIND-AND-INFER(KB, $premises$, $conclusion$, θ)

  if $premises = [\,]$ **then**
    FORWARD-CHAIN(KB, SUBST(θ, $conclusion$))
  else for each p' **in** KB such that UNIFY(p', SUBST(θ, FIRST($premises$))) $= \theta_2$ **do**
    FIND-AND-INFER(KB, REST($premises$), $conclusion$, COMPOSE(θ, θ_2))
  end

Backward Chaining

- **Backward-chaining** deduction using GMP
  - **Complete** for KBs containing **only Horn clauses**.
- Proofs:
  - Start with the goal query
  - Find rules with that conclusion
  - Prove each of the antecedents in the implication
- Keep going until you reach premises!

Avoid loops
    Is new subgoal already on goal stack?
Avoid repeated work: has subgoal
    already been proved true
    already failed?

Backward Chaining Example

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- Goal:
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Backward Chaining Algorithm

function BACK-CHAIN(KB, q) **returns** a set of substitutions

  BACK-CHAIN-LIST(KB, $[q]$, $\{\}$)

---

function BACK-CHAIN-LIST(KB, $qlist$, θ) **returns** a set of substitutions
  inputs: KB, a knowledge base
      $qlist$, a list of conjuncts forming a query (θ already applied)
      θ, the current substitution
  static: $answers$, a set of substitutions, initially empty

  if $qlist$ is empty **then return** $\{\theta\}$
  $q \leftarrow$ FIRST($qlist$)
    for each q'_i **in** KB such that $\theta_i \leftarrow$ UNIFY(q, q'_i) succeeds **do**
      Add COMPOSE(θ, θ_i) to $answers$
    end
    for each sentence ($p_1 \wedge \ldots \wedge p_n \Rightarrow q'_i$) **in** KB such that $\theta_i \leftarrow$ UNIFY(q, q'_i) succeeds **do**
      $answers \leftarrow$ BACK-CHAIN-LIST(KB, SUBST(θ_i, $[p_1 \ldots p_n]$), COMPOSE(θ, θ_i)) $\cup$ $answers$
    end
  return the union of BACK-CHAIN-LIST(KB, REST($qlist$), θ) for each $\theta \in answers$

Forward vs. Backward Chaining

- FC is data-driven
  - Automatic, unconscious processing
  - E.g., object recognition, routine decisions
  - May do lots of work that is irrelevant to the goal
- BC is goal-driven, appropriate for problem-solving
  - Where are my keys? How do I get to my next class?
  - Complexity of BC can be much less than linear in the size of the KB

Completeness of GMP

- GMP (using forward or backward chaining) is complete for KBs that contain only Horn clauses
- It is ***not* complete** for simple KBs that contain **non-Horn clauses**
- The following entail that S(A) is true:
  $(\forall x)\ P(x) \rightarrow Q(x)$
  $(\forall x)\ \neg P(x) \rightarrow R(x)$
  $(\forall x)\ Q(x) \rightarrow S(x)$
  $(\forall x)\ R(x) \rightarrow S(x)$
- If we want to conclude S(A), with GMP we cannot, since the second one is not a Horn clause
- It is equivalent to $P(x) \vee R(x)$