Uninformed Search
Chapter 3

Some material adopted from notes by Charles R. Dyer, University of Wisconsin-Madison

Big Idea

Allen Newell and Herb Simon developed the problem space principle as an AI approach in the late 60s/early 70s.

"The rational activity in which people engage to solve a problem can be described in terms of (1) a set of states of knowledge, (2) operators for changing one state into another, (3) constraints on applying operators and (4) control knowledge for deciding which operator to apply next."


Today’s topics

• Goal-based agents
• Representing states and operators
• Example problems
• Generic state-space search algorithm
• Specific algorithms
  — Breadth-first search
  — Depth-first search
  — Uniform cost search
  — Depth-first iterative deepening
• Example problems revisited

Example: 8-Puzzle

Given an initial configuration of 8 numbered tiles on a 3x3 board, move the tiles in such a way so as to produce a desired goal configuration of the tiles.

Start State

<table>
<thead>
<tr>
<th>5</th>
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<th>6</th>
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<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>3</td>
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</table>

Goal State

<table>
<thead>
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<th>3</th>
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</thead>
<tbody>
<tr>
<td>8</td>
<td>4</td>
<td>5</td>
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<table>
<thead>
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<tbody>
<tr>
<td>7</td>
<td>6</td>
<td>5</td>
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</tbody>
</table>
Building goal-based agents
To build a goal-based agent we need to answer the following questions:

– How do we represent the state of the ‘world’?
– What is the goal to be achieved?
– What are the actions?
– What relevant information should be encoded to describe the state of the world and the available transitions, and solve the problem?

What is the goal to be achieved?

• Could describe a situation we want to achieve, a set of properties that we want to hold, etc.
• Requires defining a “goal test” so we know what it means to have achieved/satisfied our goal.
• A hard question, rarely tackled in AI; usually assume the system designer or user specifies the goal to be achieved.
• Psychologists and motivational speakers stress the importance of establishing clear goals as a first step towards solving a problem.
• What are your goals???

What are the actions?

• Characterize the primitive actions or events that are available for making changes in the world in order to achieve a goal.
• Deterministic world: no uncertainty in an action’s effects. Given an action (a.k.a. operator or move) and a description of the current world state, the action completely specifies
  – whether that action can be applied to the current world (i.e., is it applicable and legal?), and
  – what the exact state of the world will be after the action is performed in the current world (i.e., no need for “history” information to compute the next state)

Representing actions

• Actions in this framework can all be considered as discrete events that occur at an instant of time, e.g.:
  if “Mary is in class” and performs action “go home,” then next state is “at home.” There is no representation of a time where she’s neither in class nor at home (i.e., in the state of “going home”).
• Number of actions/operators depends on the representation used in describing a state
  – 8-puzzle: we could specify 4 possible moves for each of the 8 tiles, resulting in a total of $4^8=32$ operators.
  – On the other hand, we could specify four moves for the “blank” square and we would only need 4 operators.
• Representational shift can simplify a problem!
Representing states

- What information is necessary to describe all relevant aspects to solving the goal?
- The size of a problem is usually described in terms of the number of states that are possible.
  - Tic-Tac-Toe has about $3^9$ states.
  - Checkers has about $10^{40}$ states.
  - Rubik’s Cube has about $10^{18}$ states.
  - Chess has about $10^{120}$ states in a typical game.
  - Theorem provers may deal with an infinite space

- state space size $\approx$ solution difficulty

Closed World Assumption

- We will generally use the Closed World Assumption
- All necessary information about a problem domain is available in each percept so that each state is a complete description of the world
- There is no incomplete information at any point in time

Some example problems

- Toy problems and micro-worlds
  - 8-Puzzle
  - Missionaries and Cannibals
  - Cryptarithmic
  - Remove 5 Sticks
  - Water Jug Problem

- Real-world problems

8-Puzzle

Given an initial configuration of 8 numbered tiles on a 3x3 board, move the tiles in such a way so as to produce a desired goal configuration of the tiles.

What are the states, goal test, actions?
8 puzzle

- **State:** 3 x 3 array configuration of the tiles on the board
- **Operators:** Move Blank Square Left, Right, Up or Down
  - A more efficient operator encoding than one with four possible moves for each of eight distinct tiles
- **Initial State:** A particular configuration of the board
- **Goal:** A particular configuration of the board

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15 puzzle

- Popularized, but not invented by, **Sam Loyd**
- In the late 1800s he offered $1000 to all who could find a solution
- He sold many puzzles
- The states form two disjoint spaces
- From the initial configuration, there is no path to the solution!

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Missionaries and Cannibals

There are 3 missionaries, 3 cannibals, and 1 boat that can carry up to two people on one side of a river.

- **Goal:** Move all the missionaries and cannibals across the river.
- **Constraint:** Missionaries can’t be outnumbered by cannibals on either side of the river, or else the missionaries are killed.
- **State:** configuration of missionaries and cannibals and boat on each side of the river.
- **Operators:** Move boat containing some set of occupants across the river (in either direction) to the other side.

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HW2: What are the states, goal test, actions?
Missionaries and Cannibals Solution

<table>
<thead>
<tr>
<th>Steps</th>
<th>Near side</th>
<th>Far side</th>
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<tbody>
<tr>
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<td>MMMCCC</td>
<td>B  -</td>
</tr>
<tr>
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<td>B  CC</td>
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<tr>
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</tr>
<tr>
<td>11</td>
<td>-</td>
<td>B  MMMCCC</td>
</tr>
</tbody>
</table>

Cryptarithmic

- Find an assignment of digits (0..9) to letters so that a given arithmetic expression is true. Examples:
  SEND + MORE = MONEY and
  \[
  \begin{array}{ccc}
  \text{SEND} & \text{+} & \text{MORE} \\
  \text{FORTY} & \text{+} & \text{TEN} \\
  \text{SIXTY} & \text{=} & \text{F=2, O=9, R=7, etc.} \\
  \end{array}
  \]
- The solution is NOT a sequence of actions that transforms the initial state into the goal state
- The solution is a goal node that includes an assignment of digits to each of the distinct letters in the given problem.

What are the states, goal test, actions?

Remove 5 Sticks

Given the following configuration of sticks, remove exactly five sticks so that the remaining ones form exactly three squares.

Other tasks:
- Remove 4 sticks and leave 4 squares
- Remove 3 sticks and leave 4 squares
- Remove 4 sticks and leave 3 squares

Water Jug Problem

<table>
<thead>
<tr>
<th>Operators</th>
<th>Cond.</th>
<th>Transition</th>
<th>Effect</th>
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</thead>
<tbody>
<tr>
<td>Empty5</td>
<td>-</td>
<td>(x,y)→(0,y)</td>
<td>Empty 5G jug</td>
</tr>
<tr>
<td>Empty2</td>
<td>-</td>
<td>(x,y)→(x,0)</td>
<td>Empty 2G jug</td>
</tr>
<tr>
<td>2to5</td>
<td>x ≤ 3</td>
<td>(x,2)→(x+2,0)</td>
<td>Pour 2G into 5G</td>
</tr>
<tr>
<td>5to2</td>
<td>x ≥ 2</td>
<td>(x,0)→(x-2,2)</td>
<td>Pour 5G into 2G</td>
</tr>
<tr>
<td>5to2part</td>
<td>y &lt; 2</td>
<td>(1,y)→(0,y+1)</td>
<td>Pour partial 5G into 2G</td>
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</table>
Some more real-world problems

• Route finding
• Touring (traveling salesman)
• Logistics
• VLSI layout
• Robot navigation
• Theorem proving
• Learning

Knowledge representation issues

• What’s in a state?
  – Is boat color relevant to solving the M&C problem? Is sunspot activity relevant to predicting the stock market? What to represent is a very hard problem that is usually left to the system designer to specify.

• The right level of abstraction to describe the world
  – Too fine-grained and we’ll “miss the forest for the trees.” Too coarse-grained and we’ll miss critical details for solving the problem.

• Number of states depends on the representation and abstraction level. E.g., for Remove-5-Sticks
  – if we represent individual sticks, there are 17-choose-5 possible ways of removing 5 sticks
  – If we represent the “squares” defined by 4 sticks, there are 6 squares initially and we must remove 3 squares, so only 6-choose-3 ways of removing 3 squares

Formalizing search in a state space

• A state space is a graph \((V, E)\) where \(V\) is a set of nodes and \(E\) is a set of arcs, and each arc is directed from a node to another node

• Each node is a data structure with a state description and other info, such as the node’s parent, the name of the operator that generated it from that parent, and other bookkeeping data

• Each arc corresponds to an instance of one of the operators. When the operator is applied to the state associated with the arc’s source node, then the resulting state is the state associated with the arc’s destination node

Formalizing search (2)

• Each arc has a fixed, positive cost associated with it corresponding to the cost of the operator.

• Each node has a set of successor nodes corresponding to all of the legal operators that can be applied at the source node’s state.
  – Expanding a node means generating its successor nodes and adding them and their associated arcs to the state-space graph

• One or more nodes are designated as start nodes.

• A goal test predicate is applied to a state to determine if its associated node is a goal node.
Class Exercise

- Representing a 2x2 Sudoku puzzle as a search space
- Fill in the grid so that every row, every column, and every 2x2 box contains the digits 1 through 4.
  - What are the states?
  - What are the operators?
  - What are the constraints (on operator application)?
  - What is the description of the goal state?

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Formalizing search (3)

- A solution is a sequence of operators that is associated with a path in a state space from a start node to a goal node.
- The cost of a solution is the sum of the arc costs on the solution path.
  - If all arcs have the same (unit) cost, then the solution cost is just the length of the solution (number of steps / state transitions)
Formalizing search (4)

• **State-space search** is the process of searching through a state space for a solution by making explicit a sufficient portion of an implicit state-space graph to find a goal node.
  
  – For large state spaces, it’s impractical to represent the whole space.
  – Initially $V = \{S\}$, where $S$ is the start node; when $S$ is expanded, its successors are generated and those nodes are added to $V$ and the associated arcs are added to $E$. This process continues until a goal node is found.

• A node implicitly or explicitly represents a partial solution path (+ cost of partial solution path) from $S$ to the given node.
  – In general, from this node there are many possible paths (and therefore solutions) that have this partial path as a prefix.

State-space search algorithm

```plaintext
function general-search (problem, QUEUEING-FUNCTION)
;; problem describes the start state, operators, goal test, and operator costs
;; queueing-function is a comparator function that ranks two states
;; general-search returns either a goal node or failure

nodes = MAKE-QUEUE(MAKE-NODE(problem.INITIAL-STATE))

loop
  if EMPTY(nodes) then return "failure"
  node = REMOVE-FRONT(nodes)
  if problem.GOAL-TEST(node.STATE) succeeds
    then return node
  nodes = QUEUEING-FUNCTION(nodes, EXPAND(node, problem.OPERATORS))
end

;; Note: The goal test is NOT done when nodes are generated
;; Note: This algorithm does not detect loops
```

Key procedures to be defined

• EXPAND
  – Generate all successor nodes of a given node

• GOAL-TEST
  – Test if state satisfies all goal conditions

• QUEUEING-FUNCTION
  – Used to maintain a ranked list of nodes that are candidates for expansion

Bookkeeping

• Typical node data structure includes:
  – State at this node
  – Parent node
  – Operator applied to get to this node
  – Depth of this node (number of operator applications since initial state)
  – Cost of the path (sum of each operator application so far)
Some issues

• Search process constructs a search tree, where
  – root is the initial state and
  – leaf nodes are nodes
    • not yet expanded (i.e., they are in the list “nodes”) or
    • having no successors (i.e., they’re “deadends” because
      no operators were applicable and yet they’re not goals)
  • Search tree may be infinite because of loops even if
    state space is small
  • Return a path or a node depending on problem.
    – E.g., in cryptarithmetic return a node; in 8-puzzle, a path
  • Changing definition of the QUEUEING-FUNCTION
    leads to different search strategies

Evaluating search strategies

• Completeness
  – Guarantees finding a solution whenever one exists
• Time complexity
  – How long (worst or average case) does it take to find a
    solution? Usually measured in terms of the number of
    nodes expanded
• Space complexity
  – How much space is used by the algorithm? Usually
    measured in terms of the maximum size of the “nodes”
    list during the search
• Optimality/Admissibility
  – If a solution is found, is it guaranteed to be an optimal
    one? That is, is it the one with minimum cost

Uninformed vs. informed search

• Uninformed search strategies
  – Aka “blind search,”
  – use no information about the likely “direction” of the goal
    node(s)
  – Example methods: breadth-first, depth-first, depth-limited,
    uniform-cost, depth-first iterative deepening, bidirectional
• Informed search strategies
  – aka “heuristic search”
  – use information about the domain to (try to) (usually) head
    in the general direction of the goal node(s)
  – Example methods: hill climbing, best-first, greedy search,
    beam search, A, A*

Example for illustrating uninformed search strategies
Classic uninformed search methods

- The four classic uninformed search methods
  - Breadth first search
  - Depth first search
  - Uniform cost search (a generalization of BFS)
  - Iterative deepening (a blend of DFS and BFS)
- To which we can add another technique
  - Bi-directional search (a hack on BFS)

Breadth-First

- Enqueue nodes on nodes in FIFO (first-in, first-out) order.
- Complete
- Optimal (i.e., admissible) if all operators have the same cost. Otherwise, not optimal but finds solution with shortest path length.
- Exponential time and space complexity, \(O(b^d)\), where \(d\) is the depth of the solution and \(b\) is the branching factor (i.e., number of children) at each node
- Will take a long time to find solutions with a large number of steps because must look at all shorter length possibilities first
  - A complete search tree of depth \(d\) where each non-leaf node has \(b\) children, has a total of \(1 + b + b^2 + \ldots + b^d = (b^{d+1} - 1)/(b-1)\) nodes
  - For a complete search tree of depth 12, where nodes at depths 0..11 have 10 children and nodes at depth 12 have 0, there are \(1+10+100+1000\ldots+10^{12} = (10^{13}-1)/9 = O(10^{12})\) nodes in the complete search tree
  - If BFS expands 1000 nodes/sec and each node uses 100 bytes of storage, then BFS will take 35 years to run in the worst case, and it will use 111 terabytes of memory!

Depth-First (DFS)

- Enqueue nodes on nodes in LIFO (last-in, first-out) order, i.e., nodes used as a stack data structure to order nodes.
- May not terminate without a “depth bound,” i.e., cutting off search below a fixed depth \(D\) (“depth-limited search”)
- Not complete (with or without cycle detection, and with or without a cutoff depth)
- Exponential time, \(O(b^d)\), but only linear space, \(O(bd)\)
- Can find long solutions quickly if lucky (and short solutions slowly if unlucky!)
- When search hits a deadend, can only back up one level at a time even if the “problem” occurs because of a bad operator choice near the top of the tree. Hence, only does “chronological backtracking”
**Depth-First Search**

**Expanded node** | **Nodes list**
---|---
$S^0$ | \{ A$^3$ B$^1$ C$^8$ \}
A$^3$ | \{ D$^6$ E$^{10}$ G$^{18}$ B$^1$ C$^8$ \}
D$^6$ | \{ E$^{10}$ G$^{18}$ B$^1$ C$^8$ \}
E$^{10}$ | \{ G$^{18}$ B$^1$ C$^8$ \}
G$^{18}$ | \{ B$^3$ C$^8$ \}

Solution path found is S A G, cost 18
Number of nodes expanded (including goal node) = 5

**Uniform-Cost (UCS)**
- Enqueue nodes by path cost. I.e., let $g(n) =$ cost of path from start to current node $n$. Sort nodes by increasing value of $g$.
- Called “Dijkstra’s Algorithm” in the algorithms literature and similar to “Branch and Bound Algorithm” in operations research literature
- **Complete (\*)**
- **Optimal/Admissible (\*)**
  - Admissibility depends on the goal test being applied *when a node is removed from the nodes list*, not when its parent node is expanded and the node is first generated
- **Exponential time and space complexity**, $O(b^d)$

**Uniform-Cost Search**

**Expanded node** | **Nodes list**
---|---
$S^0$ | \{ S$^0$ \}
S$^0$ | \{ B$^1$ A$^3$ C$^8$ \}
B$^1$ | \{ A$^3$ C$^8$ G$^{21}$ \}
A$^3$ | \{ D$^6$ C$^8$ E$^{10}$ G$^{18}$ G$^{21}$ \}
D$^6$ | \{ C$^8$ E$^{10}$ G$^{18}$ G$^1$ \}
C$^8$ | \{ E$^{10}$ G$^{13}$ G$^{18}$ G$^{21}$ \}
E$^{10}$ | \{ G$^{13}$ G$^{18}$ G$^{21}$ \}
G$^{13}$ | \{ G$^{18}$ G$^{21}$ \}

Solution path found is S C G, cost 13
Number of nodes expanded (including goal node) = 7

**Depth-First Iterative Deepening (DFID)**
- First do DFS to depth 0 (i.e., treat start node as having no successors), then, if no solution found, do DFS to depth 1, etc.
- **Complete**
- **Optimal/Admissible** if all operators have the same cost. Otherwise, not optimal but guarantees finding solution of shortest length (like BFS).
- **Time complexity** a bit worse than BFS or DFS. Nodes near top of the search tree are generated many times, but since almost all nodes are near the tree bottom, worst case time complexity is still exponential, $O(b^d)$
Depth-First Iterative Deepening

• If branching factor is $b$ and solution is at depth $d$, then nodes at depth $d$ are generated once, nodes at depth $d-1$ are generated twice, etc.
  - Hence $b^d + 2b^{(d-1)} + ... + db \leq b^d / (1 - 1/b) = O(b^d)$.
  - If $b=4$, then worst case is $1.78 * 4^d$, i.e., 78% more nodes searched than exist at depth $d$ (in the worst case).
• Linear space complexity, $O(bd)$, like DFS
• Has advantage of BFS (i.e., completeness) and also advantages of DFS (i.e., limited space and finds longer paths more quickly)
• Generally preferred for large state spaces where solution depth is unknown

Searching Backward from Goal

• Usually a successor function is reversible
  - i.e., we can generate a node’s predecessors in the graph
• If we know a single goal (rather than a goal’s properties), we could search backward to the initial state
• It might be more efficient
  - Depends on whether the graph fans in or out

How they perform

• Depth-First Search:
  - Expanded nodes: S A D E G
  - Solution found: S A G (cost 18)
• Breadth-First Search:
  - Expanded nodes: S A B C D E G
  - Solution found: S A G (cost 18)
• Uniform-Cost Search:
  - Expanded nodes: S A D B C E G
  - Solution found: S C G (cost 13)
  *This is the only uninformed search that worries about costs.*
• Iterative-Deepening Search:
  - nodes expanded: S S A B C S A D E G
  - Solution found: S A G (cost 18)

Bi-directional search

• Alternate searching from the start state toward the goal and from the goal state toward the start.
• Stop when the frontiers intersect.
• Works well only when there are unique start and goal states.
• Requires the ability to generate “predecessor” states.
• Can (sometimes) lead to finding a solution more quickly.
Comparing Search Strategies

<table>
<thead>
<tr>
<th>Search Strategy</th>
<th>Breadth First</th>
<th>Uniform Cost</th>
<th>Depth First</th>
<th>Depth Limited</th>
<th>Iterative Deepening</th>
<th>Bidiirectional</th>
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<tbody>
<tr>
<td>Time Space</td>
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<td>( b^1 )</td>
<td>( b^1 )</td>
<td>( b^1 )</td>
<td>( b^1 )</td>
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<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td>Complete</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes, if ( f \geq c )</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Avoiding Repeated States

- In increasing order of effectiveness in reducing size of state space and with increasing computational costs:
  1. Never return to the state you just came from
  2. Never create paths with cycles in them
  3. Never generate any state that was ever created before

- Net effect depends on frequency of “loops” in state space

A State Space that Generates an Exponentially Growing Search Space

Holy Grail Search

Expanded node | Nodes list
---|---
\( S^0 \) | \{ \( S^0 \) \}
\( C^8 \) | \{ \( C^8, A^3, B^1 \) \}
\( G^{13} \) | \{ \( A^3, B^1 \) \}

Solution path found is \( S \rightarrow C \rightarrow G \), cost 13 (optimal)

Number of nodes expanded (including goal node) = 3 (as few as possible!)

If only we knew where we were headed...