Subdivision

Materials from Denis Zorin, Peter Schroder et al. siggraph presentations
Motivation

• Still a hot topic today in computer graphics
• Advantages: simple (only need subdivision rule); local (only look at nearby vertices); arbitrary topology (since only local); no seams (unlike joining spline patches)
• Detailed study quite sophisticated
  – See online materials

In this class, we will briefly survey literature and discuss some ideas
Motivation

- E.g., Gari’s Game created using subdivision: [http://www.youtube.com/watch?v=1m7dcbIKvlw](http://www.youtube.com/watch?v=1m7dcbIKvlw)
Key Questions

- How to refine mesh?
- Where to place new vertices?
  - Provable properties about limit surface
Loop Subdivision Scheme

• How to refine mesh?
  – Refine each triangle into 4 triangles by splitting each edge and connecting new vertices

• Where to place new vertices?
  – Choose locations for new vertices as weighted average of original vertices in local neighborhood
Loop Subdivision Scheme

- Where to place new vertices?
  - Rules for extraordinary vertices and boundaries

Choose $\beta$ by analyzing continuity of limit surface

Original Loop:

$$\beta = \frac{1}{n} \left( \frac{5}{8} - \left( \frac{3}{8} + \frac{1}{4} \cos \frac{2\pi}{n} \right)^2 \right)$$

Warren:

$$\beta = \begin{cases} 
\frac{3}{8n} & n > 3 \\
\frac{3}{16} & n = 3 
\end{cases}$$
Butterfly Subdivision

- Interpolating subdivision: Larger neighborhood
Modified Butterfly Subdivision

- Need special weights near extraordinary vertices
  - For $n=3$, weights are $5/12$, $-1/12$, $-1/12$
  - For $n=4$, weights are $3/8$, $0$, $-1/8$, $0$
  - For $n \leq 5$, weights are,

\[
\frac{1}{n} \left( \frac{1}{4} + \cos \frac{2\pi j}{n} + \frac{1}{2} \cos \frac{4\pi j}{n} \right), \quad j = 0 \ldots n-1
\]

- Weight of extraordinary vertex = 1 = the sum of other weights
A Variety of Subdivision Schemes

- Triangles vs. quads
- Interpolating vs. approximating

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Results
Analysis of Continuity

- Analyzing subdivision schemes
  - Smoothness properties

- Start with curves: 4-point interpolating scheme (old points left where they are)
Calculate New Points

**Step i:**

\[ V_{-2} \quad V_{-1} \quad V_0 \quad V_1 \quad V_2 \]

**Step i+1:**

\[ V_{-2} \quad V_{-1} \quad V_0 \quad V_1 \quad V_2 \]

\[
\begin{pmatrix}
V_{-2}^{(i+1)} \\
V_{-1}^{(i+1)} \\
V_0^{(i+1)} \\
V_1^{(i+1)} \\
V_2^{(i+1)}
\end{pmatrix} =
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 \\
-\frac{1}{16} & \frac{9}{16} & -\frac{9}{16} & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & -\frac{1}{16} & \frac{9}{16} & \frac{9}{16} & -\frac{1}{16} \\
0 & 0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
V_{-2}^{(i)} \\
V_{-1}^{(i)} \\
V_0^{(i)} \\
V_1^{(i)} \\
V_2^{(i)}
\end{pmatrix}
\]

After \( n \) rounds:

\[
\mathbf{V}^{(n)} = S^n \mathbf{V}^{(0)}
\]
Fun with Subdivision Methods

• Behavior of surfaces depends on eigenvalues of the matrix

Real  Complex  Degenerate
Practical Evaluation

- Problems with Uniform Subdivision
  - Exponential growth of control mesh
  - Need several subdivisions before error is small
  - OK if you are “drawing and forgetting”, otherwise...

- Exact Evaluation at arbitrary points
- Tangent and other derivative evaluation needed

- Jos Stam SIGGRAPH 98 efficient method
  - Exact evaluation (essentially take out “subdivision”)
  - Smoothness analysis methods used to evaluate