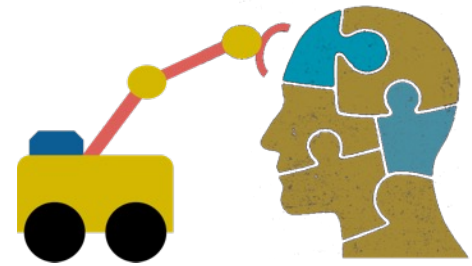
9.3.2



First-Order Logic (FOL) part 2

Overview

- We’ll first give some examples of how to translate between FOL and English
- Then look at modelling family relations in FOL
- And finally touch on a few other topics

Translating English to FOL

Every gardener likes the sun

$\forall x\ gardener(x) \rightarrow likes(x, Sun)$

All purple mushrooms are poisonous

$\forall x\ mushroom(x) \land purple(x) \rightarrow poisonous(x)$

No purple mushroom is poisonous (two ways)

$\neg\exists x\ purple(x) \land mushroom(x) \land poisonous(x)$

$\forall x\ mushroom(x) \land purple(x) \rightarrow \neg poisonous(x)$

English to FOL: Counting

- Using with numbers & simple math can seem awkward
- Use **= predicate** to identify different individuals

There are <u>at least</u> two purple mushrooms

$\exists x \, \exists y$ mushroom(x) $\wedge$ purple(x) $\wedge$ mushroom(y) $\wedge$ purple(y) $\wedge$ $\neg$**(x=y)**

This says that there exisit an x and a y such that

- "x is a purple mushroom" and
- "y is a purple mushroom" and
- "x and y are not the same objects"

English to FOL: Counting

There are exactly two purple mushrooms

$\exists x \, \exists y \, mushroom(x) \wedge purple(x) \wedge mushroom(y) \wedge purple(y) \wedge \neg\mathbf{(x=y)} \wedge \forall z \, (mushroom(z) \wedge purple(z)) \rightarrow \mathbf{((x=z) \vee (y=z))}$

This says that

- "x is a purple mushroom" and
- "y is a purple mushroom" and
- "x and y are not the same objects"
- If there's a purple mushroom z, then either z=x or z=y

Saying there are 802 different Pokemon is hard!

Direct use of FOL is not for everything!

Translating English to FOL

What do these mean?

- **You can fool *some of* the people *all of* the time**

- **You can fool *all of* the people *some of* the time**

Translating English to FOL

What do these mean?

Both English statements are ambiguous

- **You can fool *some of* the people *all of* the time**

  #1 There is a nonempty subset of people so easily fooled that you can fool that subset every time*

  #2 For any given time, there is a non-empty subset at that time that you can fool

- **You can fool *all of* the people *some of* the time**

  #1 There are one or more times when it's possible to fool everyone*

  #2 Each individual can be fooled at some point in time

* Most common interpretation, I think

To represent these in logic we need some terms

- **person(x):** True iff x is a person
- **time(t):** True iff t is a point in time
- **canFool(x, t):** True iff x can be fooled at time t

Note: *iff* = *if and only if* = $\leftrightarrow$

Translating English to FOL

You can fool *some of* the people *all of* the time

#1 There is a nonempty group of people so easily fooled that you can fool that group every time*

≡ There's (at least) one person you can fool every time

$\exists \mathbf{x}\ \forall \mathbf{t}\ \ person(x) \land time(t) \rightarrow canFool(x, t)$

#2 For any given time, there is a non-empty group at that time you can fool

≡ For every time, there's a person at that time you can fool

$\forall \mathbf{t}\ \exists \mathbf{x}\ \ person(x) \land time(t) \rightarrow canFool(x, t)$

* Most common interpretation, I think

Translating English to FOL

You can fool *all of* the people *some of* the time

#1 There's at least one time when you can fool everyone*

$\exists \mathbf{t}\ \forall \mathbf{x}$ time(t) $\wedge$ person(x) $\rightarrow$ canFool(x, t)

#2 Everybody can be fooled at some point in time

$\forall \mathbf{x}\ \exists \mathbf{t}$ person(x) $\wedge$ time(t) $\rightarrow$ canFool(x, t)

* Most common interpretation, I think

Limits of classical logic

- Note that there's no easy, natural way to talk about a few, many, most, almost all …
- This is natural in human languages
  - There are **many** people you can fool **most** of the time
  - There are a **few** people you can fool **almost every** time
- We also can't have exceptions
  - All birds can fly, **except for** penguins, ostriches and a few other species
- There are non-classical logic systems that can handles these problems

Representation Design

- Many options for representing even a simple fact, e.g., something's color as red, green or blue, e.g.:
  - green(kermit)
  - color(kermit, green)
  - hasProperty(kermit, color, green)
- Choice can influence how easy it is to use
- Last option of representing properties & relations as triples used by modern knowledge graphs
  - **Let's us ask:** What color is Kermit? What are Kermit's properties?, What green things are there? What colors are there? What properties are there? Tell me everything you know about Kermit, …

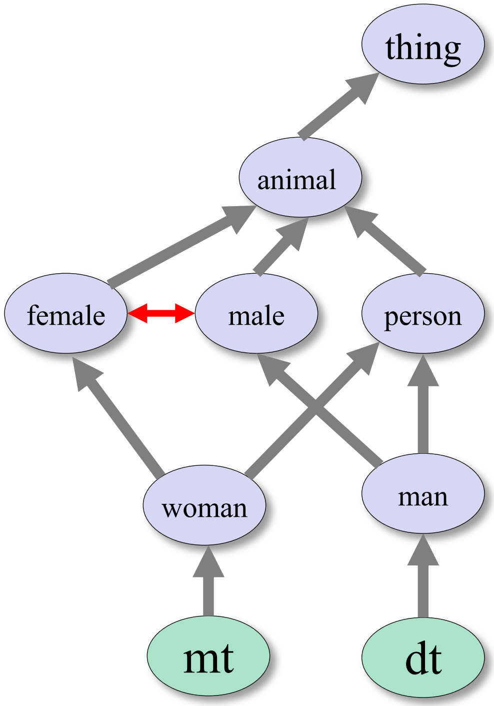
Simple genealogy KB in FOL

Design a knowledge base using FOL that

- Has facts of immediate family relations, e.g., spouses, parents, etc.
- Defines more complex relations (ancestors, relatives)
- Detect inconsistencies, e.g., a person is her own parent
- Infers relations, e.g., grandparent from parent
- Answers queries about relationships between people

How do we approach this?

- Design an initial ontology of types, e.g.
  - person, animal, man, woman, …
- Types form a taxonomy or lattice*, e.g.
  - person(X) <=> man(X) ∨ woman(Y)
  - man(X) <=> person(X) ∧ male(X)
  - woman(X) <=> person(X) ∧ female(X)
  - female(X) <=> ~ male(X)
- Make assertions about individuals, e.g.
  - man(donaldTrump)
  - woman(melaniaTrump)



* In a lattice, objects can have multiple immediate types

Extend with relations and constraints

- Simple two argument relations, e.g.
  - spouse, has_child, has_parent
- Add general constraints to relations, e.g.
  - spouse(X,Y) => ~ (X = Y)
  - spouse(X,Y) => person(X) ∧ person(Y)
  - spouse(X,Y) => (man(X) ∧ woman(Y)) ∨ (woman(X) ∧ man(Y))*
- Add FOL sentences for inference, e.g.
  - spouse(X,Y) ⬄ spouse(Y,X)
- Add instance data
  - e.g., spouse(djt, mt)

** Note this constraint is a traditional one than no longer holds*

Example: A simple genealogy KB in FOL

Predicates:

- parent(X, Y), child(X, Y), father(X, Y), daughter(X, Y), etc.
- spouse(X, Y), husband(X, Y), wife(X,Y)
- ancestor(X, Y), descendant(X, Y)
- male(X), female(Y)
- relative(X, Y)

Facts:

- husband(joe, mary), son(fred, joe)
- spouse(john, nancy), male(john), son(mark, nancy)
- father(jack, nancy), daughter(linda, jack)
- daughter(liz, linda)
- etc.

Example Axioms

$(\forall X,Y)$ parent(X, Y) $\leftrightarrow$ child (Y, X)

$(\forall X,Y)$ father(X, Y) $\leftrightarrow$ parent(X, Y) $\wedge$ male(X)

$(\forall X,Y)$ mother(X, Y) $\leftrightarrow$ parent(X, Y) $\wedge$ female(X)

$(\forall X,Y)$ daughter(X, Y) $\leftrightarrow$ child(X, Y) $\wedge$ female(X)

$(\forall X,Y)$ son(X, Y) $\leftrightarrow$ child(X, Y) $\wedge$ male(X)

$(\forall X,Y)$ husband(X, Y) $\leftrightarrow$ spouse(X, Y) $\wedge$ male(X)

$(\forall X,Y)$ spouse(X, Y) $\leftrightarrow$ spouse(Y, X)

…

Axioms, definitions and theorems

- **Axioms**: facts and rules that capture (important) facts & concepts in a domain; used to prove **theorems**
  - Mathematicians dislike unnecessary (dependent) axioms, i.e., ones that can be derived from others
  - Including dependent axioms can make the result easier for people to understand and reasoning faster, however
  - Choosing a good set of axioms is a design problem
- A **definition** of a predicate is of the form "p(X) ↔ …" and can be decomposed into two parts
  - **Necessary** description: "p(X) → …"
  - **Sufficient** description "p(X) ← …"
  - Some concepts have definitions (e.g., triangle) and some don't (e.g., person)

More on definitions

Example: define father(X, Y) by parent(X, Y) & male(X)

- **parent(X, Y)** is a **necessary** (but not sufficient) description of father(X, Y)

  father(X, Y) $\rightarrow$ parent(X, Y)

- **parent(X, Y) ^ male(X) ^ age(X, 35)** is a **sufficient** (but not necessary) description of father(X, Y):

  father(X, Y) $\leftarrow$ parent(X, Y) ^ male(X) ^ age(X, 35)

- **parent(X, Y) ^ male(X)** is a **necessary and sufficient** description of father(X, Y)

  parent(X, Y) ^ male(X) $\leftrightarrow$ father(X, Y)

Necessary and sufficient descriptions are definitions

Higher-order logic

- FOL only lets us quantify over variables, and **variables can only range over objects**
- HOL allows us to quantify over relations, e.g.

  "two functions are equal iff they produce the same value for all arguments"

  $\forall f\ \forall g\ (f = g) \leftrightarrow (\forall x\ f(x) = g(x))$

- E.g.: (quantify over predicates)

  $\forall r\ \text{transitive}(\ r\ ) \rightarrow (\forall xyz)\ r(x,y) \wedge r(y,z) \rightarrow r(x,z))$

- More expressive, but reasoning is undecide-able, in general

FOL used to specify semantics of KR languages

- FOL is typically not used directly
- Often used to specify semantics of knowledge representation and reasoning systems
- Examples:
  - Datalog: relational data base + rules
  - Prolog: logic oriented programming language
  - OWL: a family of semantic knowledge graph languages
  - CYC: A KB of common-sense knowledge

Examples of FOL in use

- Semantics of W3C's Semantic Web stack (RDF, RDFS, OWL) is defined in FOL
- OWL Full is equivalent to FOL
- Other OWL profiles support a subset of FOL and are more efficient
- FOL oriented knowledge representation systems have many user-friendly tools
- E.g.: Protégé system for creating, editing and exploring knowledge-based systems

Examples of FOL in use

Many practical approaches embrace the approach that "some data is better than none"

- The semantics of schema.org is only defined in natural language text
- Wikidata's knowledge graph has a rich schema
  - Many constraint/logical violations are flagged with warnings
  - However, not all, see this Wikidata query that finds people who are their own grandfather

Wikidata knowledge graph

- Community knowledge graph with ~1B statements about ~100M items
- Fine-grained **ontology** has ~2M types & ~10K properties
- Multilingual: all text values tagged with language id
- Has both a human and query interface
- Many community tools for editing, search, visualization, update

Wikidata web interface for the UMBC entity, Q735049

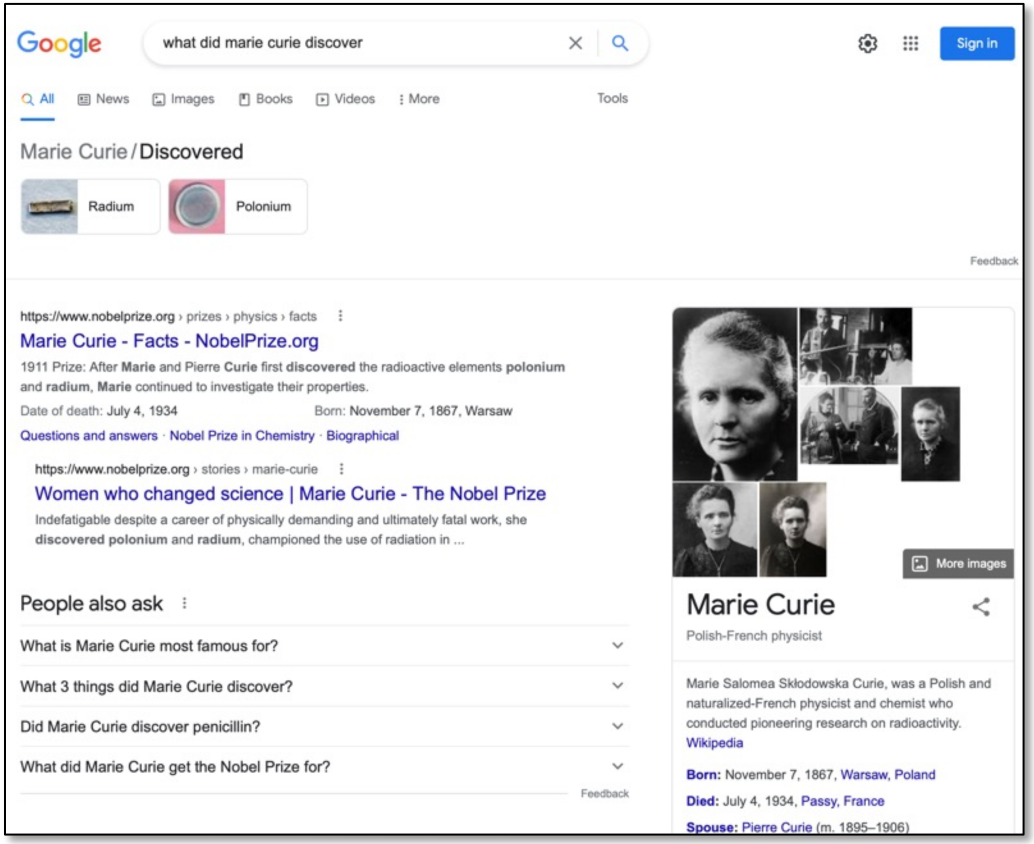
Wikidata’s huge ontology

- How can we understand an ontology with so many types?
- wdtaxonomy is a useful tool for exploring the ontology
- Given a type (e.g., Q3918, university) we can quickly see
  - Subtypes or supertypes (immediate or inferred)
  - Number of instances (immediate or inferred)
  - Direct instances
  - Number Wikimedia sites it’s in
- Implemented in javascript with a command line script

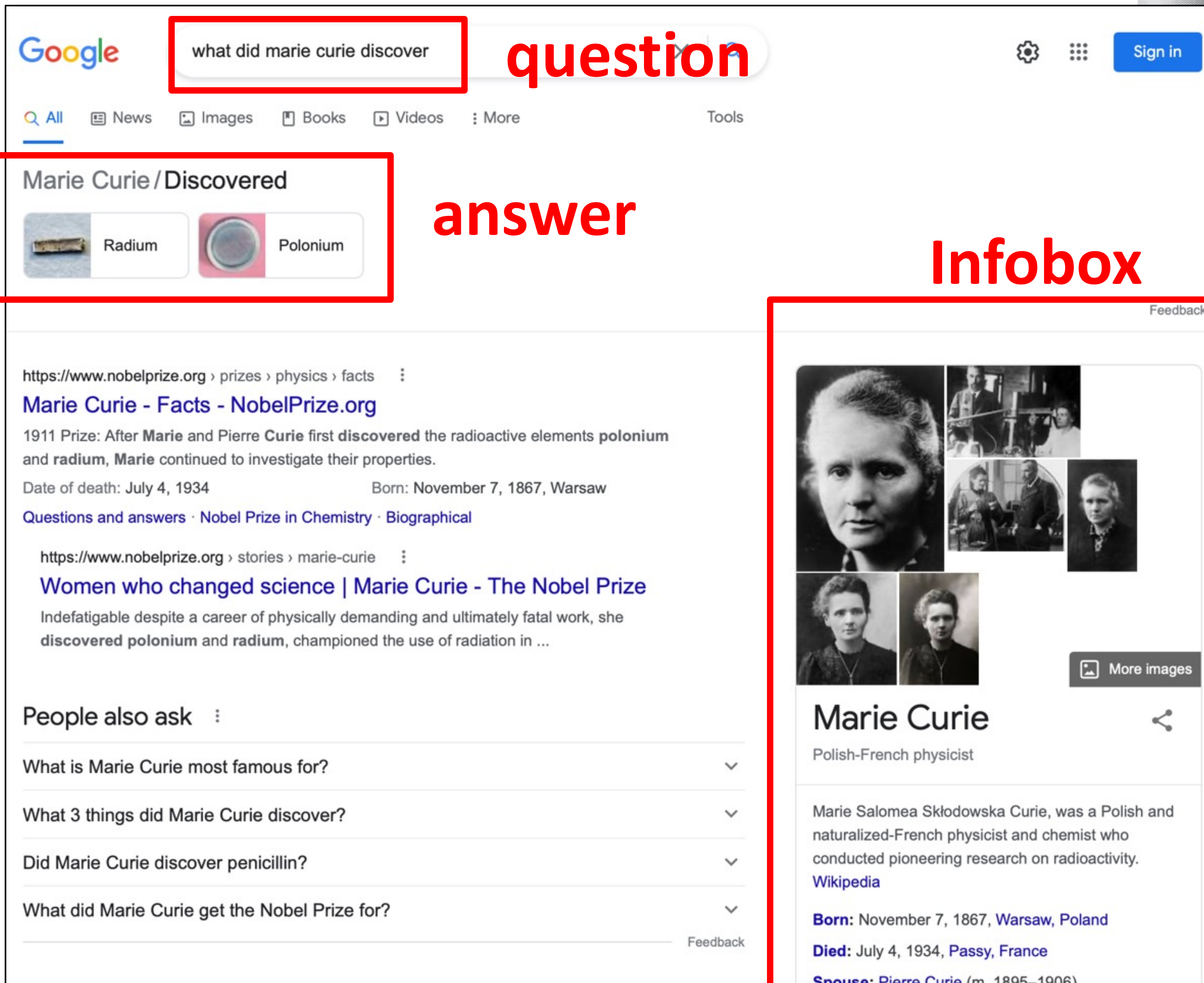
```
$$ wdtaxonomy Q3918 -c -t
university (Q3918) •163 ×15380 ↑↑
├──Universities in Germany (Q212462) •2
├──national university (Q265662) •11 ×73
├──National University (Q366354) •5
├──Imperial universities of Japan (Q562092) •12
├──Byzantine university (Q622870) •4
├──college and university rankings (Q847843) •23 ×45 ↑
├──public university (Q875538) •39 ×974 ↑
├──private university (Q902104) •32 ×846 ↑
├──new university (Q987075) •4 ×1
├──Red brick university (Q1202123) •11
├──??? (Q1305046) •2
├──institute of technology (Q1371037) •20 ×325
├──veterinary medicine school (Q1384955) •5 ×28
├──online university (Q1407393) •4 ×10 ↑
├──virtual university (Q1755248) •8 ×11
│  ╘══online university (Q1407393) •4 ×10 ↑ …
├──comprehensive university (Q1767829) •2 ×6
├──plate glass university (Q1902446) •8
├──medical university (Q1916585) •1 ×9 ↑
├──??? (Q2073922) •1
├──pontifical university (Q2120466) •18 ×37 ↑↑
├──Corporate university (Q2278672) •6
├──ancient university (Q2667285) •9 ×1
├──central university (Q3351682) •12 ×2
├──collegiate university (Q3354859) •9 ×12
├──deemed university (Q3520135) •6 ×16
├──university in France (Q3551775) •3 ×75 ↑
├──Istituto superiore per le industrie artistiche (Q3803831) •2 ×4
├──??? (Q3803846) •1 ×2
├──Smolny Institute for Noble Maidens (Q4432880) •1
├──??? (Q4475845) •2
├──federal university (Q4481793) •3 ×3
├──ecclesiastical university (Q5332280) •6 ×2
├──labor universities (Q5690751) •1 ×6
├──open university (Q6755402) •4 ×1
├──Urban university (Q7900184) •2
├──??? (Q10387922) •1
├──international university (Q10829188) •3 ×9
├──autonomous university (Q11057861) •2 ×1
├──research university (Q15936437) •9 ×224
├──Italian universities (Q20009854) •2
├──??? (Q20052016) •1 ×2
├──Canciller de Universidad (Q21547263)
├──imperial university of the Russian Empire (Q28667313) •2 ×12
├──universities in China (Q28700403) •1
├──Institute of National Importance (Q47531586) ×1 ↑
├──campusuniversity (Q59537665) ×3
├──Indiana University Bloomington Department of French and Italian (Q63441027)
├──Indiana University Department of French and Italian (Q63441251)
└──Indiana University Bloomington Department of History (Q63441447)
```

Virtual assistants and Infoboxes

- Web search engines and virtual assistants like Alexa use custom **knowledge graphs** to
  - help understand queries and content of web pages & documents
  - Answer questions
  - Show infoboxes
- Wikidata shares roots with these
- All draw on the similar knowledge, like the ~300 Wikipedia & Wikimedia sites

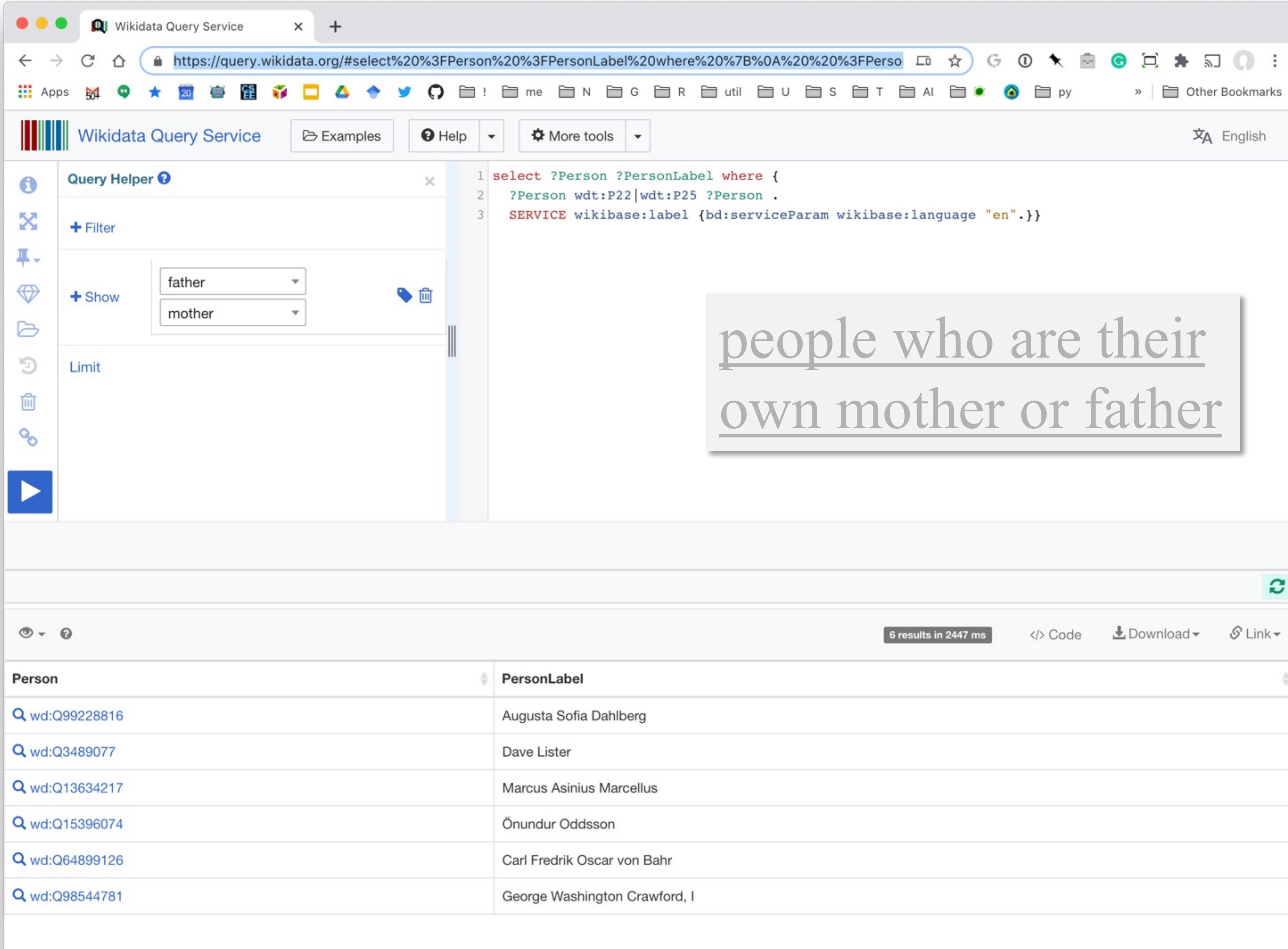


Virtual assistants & search engines

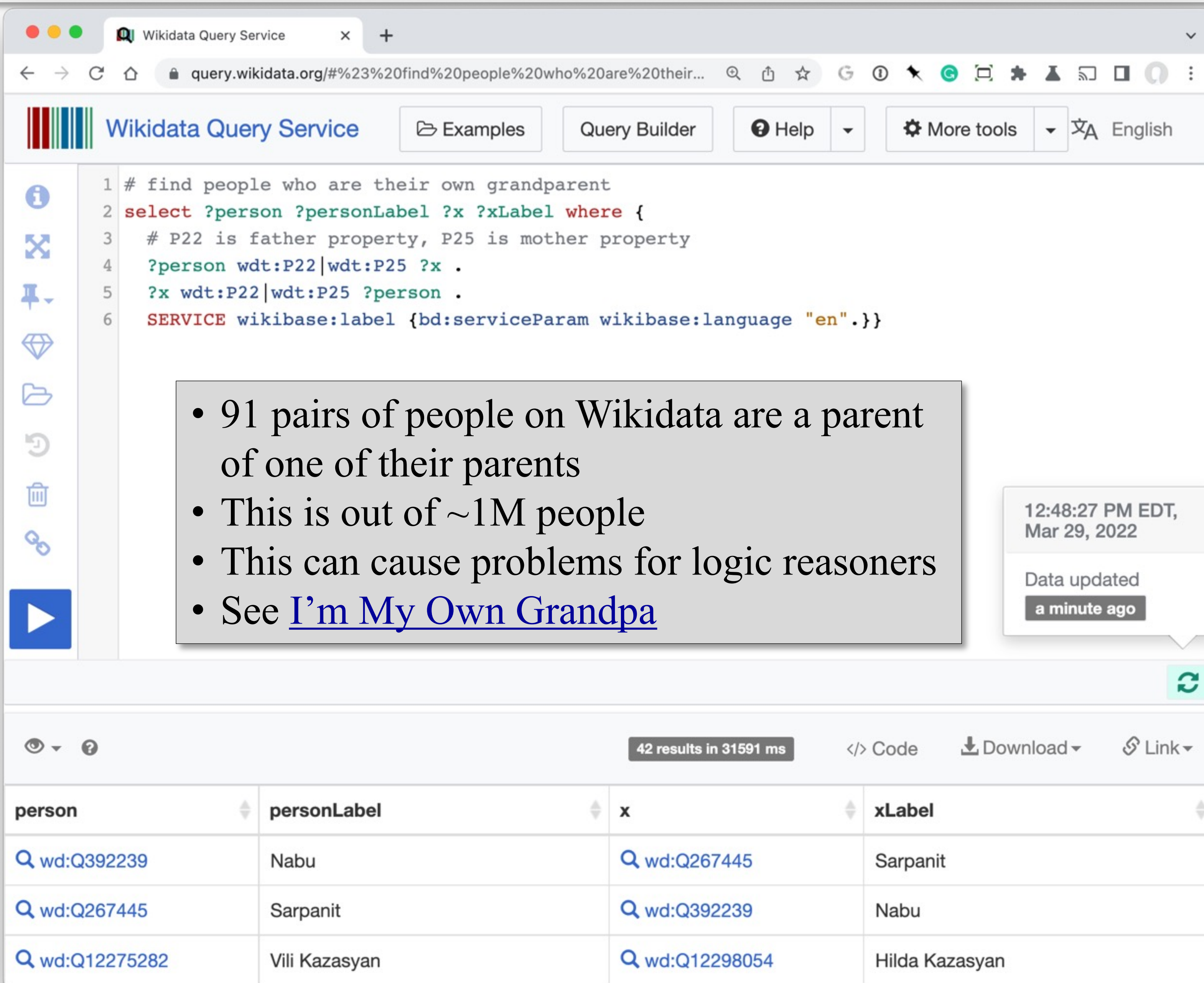


However…

- Huge knowledge bases like Wikidata made with human-provided data have some errors
- Using "too much" logic could produce logical inconsistencies
  - E.g., you can't be your own ancestor
  - This could be fatal for many classic FOL reasoning systems
- Solution?
  - Use a limited set of logical rules, e.g., for class/subclass relations like parent/child or symmetric relations like spouse
  - Use reasoners that can handle inconsistencies
  - Use variations like probabilistic logic, default logic, etc.


Wikidata Query Service
https://query.wikidata.org/#select%20%3FPerson%20%3FPersonLabel%20where%20%7B%0A%20%20%3FPerso
Examples
Help
More tools
English
Query Helper
Filter
Show
father
mother
Limit
select ?Person ?PersonLabel where {
?Person wdt:P22|wdt:P25 ?Person .
SERVICE wikibase:label {bd:serviceParam wikibase:language "en".}}
people who are their own mother or father
6 results in 2447 ms
Code
Download
Link
Person
PersonLabel
wd:Q99228816
Augusta Sofia Dahlberg
wd:Q3489077
Dave Lister
wd:Q13634217
Marcus Asinius Marcellus
wd:Q15396074
Önundur Oddsson
wd:Q64899126
Carl Fredrik Oscar von Bahr
wd:Q98544781
George Washington Crawford, I

people who are their own grandparent



- 91 pairs of people on Wikidata are a parent of one of their parents
- This is out of ~1M people
- This can cause problems for logic reasoners
- See I'm My Own Grandpa

FOL Summary

- First order logic (FOL) introduces predicates, functions and quantifiers
- More expressive, but reasoning more complex
  - Reasoning in propositional logic is NP hard, FOL is semi-decidable
- Common as an AI knowledge representation language
  - Other KR languages (e.g., OWL) often defined by mapping them to FOL
- FOL variables range over objects
  - HOL variables range over functions, predicates or sentences
- Some practical systems avoid enforcing rigid FOL constraints due to having noisy data

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