



# Logic Puzzles

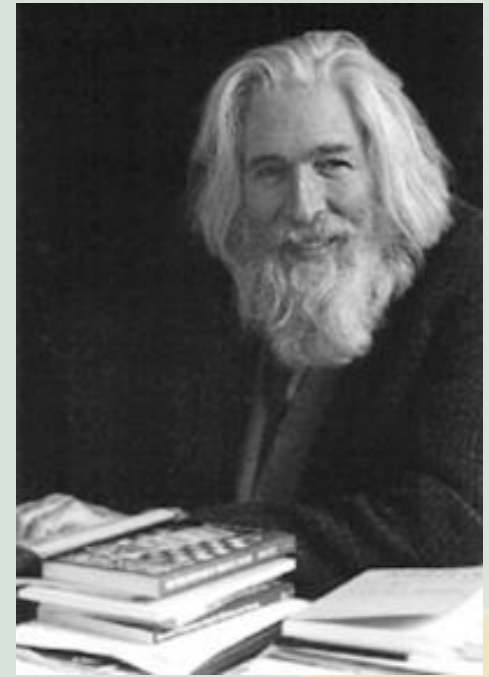
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# What are logic puzzles?

- “A puzzle deriving from the mathematics field of deduction”
- Produced by Charles Lutwidge Dodgson
- A puzzle that can be solved using logical reasoning
- It helps work with rules of logic (and, or, xor, etc.)
- Programs that carry out logical reasoning use these puzzles to illustrate capabilities

# The Master of Logic Puzzles

- High School dropout who got a Ph.D. in logic at Princeton
- Wrote many books on logic puzzles such as *Alice in Puzzle-Land* and *To Mock a Mockingbird*
- Most famous for his “Knights and Knaves Problem”



Raymond Smullyan

# Knights and Knaves



I don't lie!

Neither  
do I!



- Encounter two people
- Knights always tell the truth
- Knaves always lie
- Figure out whether each person is a knight or a knave from their statements
- Example: A says, "At least one of us is a knave" and B says nothing

# Knight and Knave Problem

A says “At least one of us is a knave” and B says nothing.

$P(x)$ :  $x$  is a knight

$\neg P(x)$ :  $x$  is a knave

Suppose A is a knave.

$\neg P(A) \Leftrightarrow T$

What A says must be false

$\neg P(A) \vee \neg P(B) \Leftrightarrow F$

Check:

$\neg P(A) \vee \neg P(B) \Leftrightarrow T \vee \neg P(B) \Leftrightarrow T$

**Impossible**

A is a knight and what A says must be true.

$P(A)$

$\neg P(A) \vee \neg P(B)$

$\therefore \neg P(B)$

**Answer:**

**A is a knight.**

**B is a knave.**

# Knight and Knave Problem

A says "The two of us are both knight" and B says "A is a knave."

$P(x)$ : x is a knight

$\neg P(x)$ : x is a knave

Suppose A is a knight.

$P(A) \Leftrightarrow T$

What A says must be true

$P(A) \wedge P(B) \Leftrightarrow T$

$P(B) \Leftrightarrow T$

**Impossible**

However, B says

$\neg P(A) \Leftrightarrow T$

$P(A) \Leftrightarrow F$

A is a knave and what A says is false.

$\neg P(A) \Leftrightarrow T$

$P(A) \wedge P(B) \Leftrightarrow F \wedge P(B) \Leftrightarrow F$

B is a knight because his statement (A is a knave) is true.

**Answer:**

**A is a knave.**

**B is a knight.**

# Knight and Knave Problem

**A says, “I am a knave or B is a knight” and B says nothing.**

- A is a knight
- B is a knight

**Both A and B say, “I am a knight.”**

- Cannot determine the answer

**A says, “We are both knaves” and B says nothing.**

- A is a knave
- B is a knight

**A says, “B is a knight” and B says, “The two of us are opposite types.”**

- A is a knave
- B is a knave

# Knight, Knave and Spy Problem

from *Alice in Puzzle-Land*

Added rule: Spy can lie or tell the truth.

There is one spy, one knight, and one knave.

A says that C is a knave. B says that A is a knight. C says "I am the spy."

Which one is the spy, which one is the knight, which one is the knave?

Knight(x): x is a knight

Knave(x): x is a knave

Spy(x): x is a spy

From C's statement, C can't be a knight because a knight never lie about his identity.

Therefore, C is either a knave or a spy.



# Knight, Knave and Spy Problem cont.

from *Alice in Puzzle-Land*

Suppose C is a spy.

$$\neg \text{Knight}(C) \wedge \neg \text{Knave}(C) \wedge \text{Spy}(C) \Leftrightarrow T$$

$$\neg \text{Knave}(C) \Leftrightarrow T \quad (\text{simplification})$$

$$\text{Knave}(C) \Leftrightarrow F$$

What A says is false, so A is knave.

$$\neg \text{Knight}(A) \wedge \text{Knave}(A) \wedge \neg \text{Spy}(A) \Leftrightarrow T$$

$$\neg \text{Knight}(A) \Leftrightarrow T \quad (\text{simplification})$$

B must be a knight, and what B says must be true.

Check:

$$\text{Knight}(A) \Leftrightarrow T$$

$$\neg \text{Knight}(A) \Leftrightarrow F$$

There is one spy, one knight, and one knave.

A says that C is a knave.  
B says that A is a knight. C says "I am the spy."

**Impossible**

$\therefore$  C isn't a spy.

**Answer:**

C is a knave.

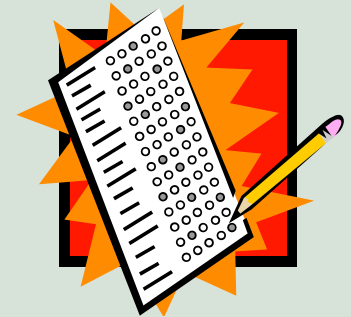
A is telling the truth, so A is a knight.

B is a spy.

# Multiple Choice Help

You encounter a problem on an exam with only answer choices, the question has been omitted. Here are the answers:

- A. Answer A
- B. Answer A or Answer B
- C. Answer B or Answer C



We may determine the correct answer using discrete math

- $R(x)$ : Answer  $x$  is right
- The correct answer must be the only one

Suppose A correct (  $R(A) = \text{True}$  ), we have the following answers:

- $R(A) \Leftrightarrow T \Leftrightarrow \text{True}$
  - $R(A) \vee \neg R(B) \Leftrightarrow T \vee F \Leftrightarrow \text{True}$
  - $\neg R(B) \vee \neg R(C) \Leftrightarrow F \vee F \Leftrightarrow \text{False}$
- } **False**

Knowing this may only have one correct answer, we can determine that this answer is not right.

# Multiple Choice Help

Suppose  $R(B) = \text{True}$

- $\neg R(A) \Leftrightarrow F \Leftrightarrow \text{False}$
  - $\neg R(A) \vee R(B) \Leftrightarrow F \vee T \Leftrightarrow \text{True}$
  - $R(B) \vee \neg R(C) \Leftrightarrow T \vee F \Leftrightarrow \text{True}$
- } **False**

Suppose  $R(C) = \text{True}$

- $\neg R(A) \Leftrightarrow F \Leftrightarrow \text{False}$
  - $\neg R(A) \vee \neg R(B) \Leftrightarrow F \vee F \Leftrightarrow \text{False}$
  - $\neg R(B) \vee R(C) \Leftrightarrow F \vee T \Leftrightarrow \text{True}$
- } **True**

Comparing each solution, we know that the correct answer must be C. We didn't have to look at the question!



# False Statement

Which statement is false (assuming only one is false)?

- A. Statement D is true
- B. Statement A is false
- C. Statement B is false
- D. Statement C is true

When statement B is true, it results in statement A being false, which results in statement D being false also. This results in more than one false statement, so statement B is the false one.

# Conclusion

- **What are logic puzzles?**
- **Who started logic puzzles?**
- **The master of logic puzzles**
  - **Knights and Knaves**
- **Method of thinking for logic puzzles**

**Questions?**

# Pop Quiz!

1. The next question with the same answer as this one is:  
(A) 2 (B) 3 (C) 4 (D) 5
2. The first question with answer C is:  
(A) 1 (B) 2 (C) 3 (D) 4
3. The last question with answer A is:  
(A) 5 (B) 6 (C) 7 (D) 8
4. The number of questions with answer D is:  
(A) 1 (B) 2 (C) 3 (D) 4
5. The answer occurring the most is (if tied, first alphabetically):  
(A) A (B) B (C) C (D) D
6. The first question with the same answer as the question following it is:  
(A) 2 (B) 3 (C) 4 (D) 5
7. The answer occurring the least is (if tied, last alphabetically):  
(A) A (B) B (C) C (D) D
8. The highest possible score on this test is:  
(A) 5 (B) 7 (C) 6 (D) 8

