Homework 5, Due July 7th

Solve any TEN problems.
Extra credit for extra problems solved. You can give informal descriptions for the TMs you constructed wherever applicable except for problems 3 and 4 where you need to give the precise state diagrams.

1. Show that if L is regular, then $L^R = \{x \mid x^R$ is in L $\}$ is also regular.
   Hint: Try to construct an NFA for L^R.

2. Construct a Turing machine computing the function $m - n$, where m, n are integers and $m \geq n \geq 0$. You can assume that the input to the TM is always given in a suitable encoding of your choice without any errors.

3. Construct a Turing machine recognizing the language $\{0^n 1^m 0^n \mid n \geq 0, m > 1\}$.

4. Construct a Turing machine recognizing the language $\{w \mid w \in \{0,1\}^*$ and w does not contain twice as many 0s as 1s $\}$.

5. Show that the collection of decidable languages is closed under the operations of

   (a) union

   (b) complementation

   (c) intersection

   Hint: Try to construct suitable TMs recognizing the union, complementation and intersection of the corresponding decidable languages.

6. Show that the collection of decidable languages is closed under the operations of

   (a) concatenation

   (b) star

   Hint: Try to construct nondeterministic TMs recognizing the concatenation, star of the languages. Nondeterminism helps in guessing how to split a given input string. Once you have a nondeterministic TM deciding the required languages, since we know that deterministic TMs and nondeterministic TMs are equivalent in power, we are done.

7. Show that the collection of Turing–recognizable languages is closed under the operations of

   (a) union

   (b) intersection

8. A **Turing machine with left reset** is similar to an ordinary TM except that the transition function has the form

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{R, RESET\}$$

   If $\delta(q, a) = (r, b, RESET)$, when the machine is in state q reading an a, the machine's head jumps to the left–hand end of the tape after it writes b in the tape and enters state r. Note that these machines do not have the usual ability to move the head one symbol left. Show that Turing machines with left reset recognize the class of Turing-recognizable languages.

9. Show that all Turing–recognizable problems mapping reduce to A_{TM}.

10. Show that if A is Turing–recognizable and $A \leq_m \overline{A}$, then A is decidable.

11. Consider the problem of testing whether a Turing Machine M on an input w ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.

12. Let $J = \{w \mid w = 0x \text{ for some } x \in A_{TM} \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$. Show that neither J nor $\overline{J}$ is Turing–recognizable.