Homework 4, Due June 30th

Solve any TEN problems.
Each problem carries 4 points.
Extra credit for extra problems solved.

1. Show that, if G is a CFG in Chomsky normal form, then for any string $w \in L(G)$ of length $n \geq 1$, exactly $2n - 1$ steps are required for any derivation of w.

2. Suppose that L is context–free and R is regular. Is $L - R$ necessarily context–free? What about $R - L$? Justify your answers.

Use pumping lemma to show that the following languages are not context–free.

3. $\{0^n 1^n 0^n 1^n \mid n \geq 0\}$

4. $\{0^n \# 0^{2n} \# 0^{3n} \mid n \geq 0\}$

5. $\{w \mid w \in \{0, 1, 2\}^*$ and w contains equal number of 0's, 1's and 2's$\}$.
   Give an example of a string in the language on which pumping lemma holds good.

6. $\{a^n b^n c^m \mid n \leq m \leq 2n\}$

Construct PDAs for the following context–free languages.

7. $\{a^n b^{2n} \mid n \geq 0\}$
8. $\{a^i b^j c^k \mid i = j$ or $j = k\}$.
9. $\{a^m b^n c^p d^q \mid m + n = p + q\}$
10. $\{w \mid w \in \{a, b\}^*$ and w has the same number of a's and b's$\}$

11. Let $T = \{(i, j, k) \mid i, j, k \in \mathcal{N}\}$. Show that T is countable.
12. Let $\mathcal{B}$ be the set of all infinite sequences over $\{0, 1\}$. Show that $\mathcal{B}$ is uncountable using a proof by diagonalization.