

Homework 3, Due June 23rd

You are required to solve TEN problems.

Problems 1, 2, 5, 9, 11 are not optional.

Solve any 5 problems out of the rest.

Each problem carries 4 points.

Extra credit for extra problems solved.

1. Let G be the grammar

$$S \rightarrow ASB \mid \epsilon$$

$$A \rightarrow aAb \mid \epsilon$$

$$B \rightarrow bBa \mid ba$$

(a) Give a leftmost derivation of $aabbba$.

(b) Give a rightmost derivation of $abaabbbabbaa$.

(c) Build the derivation tree for the derivations in parts (a) and (b).

For each of the following context-free grammars, use set notation to define the language generated by the grammar.

2.

$$S \rightarrow aaSB \mid \epsilon$$

$$B \rightarrow bB \mid b$$

3.

$$S \rightarrow aSb \mid A$$

$$A \rightarrow cAd \mid cBd$$

$$B \rightarrow aBb \mid ab$$

4.

$$S \rightarrow aSB \mid aB$$

$$B \rightarrow bb \mid b$$

Construct CFGs for the following languages.

5. $\{w \mid w \in \{a, b, c\}^*, \text{ and } w \text{ starts and ends with the same symbol}\}$
6. $\{a^m b^n \mid m \geq n\}$
7. $\{a^m b^i a^n \mid i = m + n\}$.
8. $\{a^m b^n c^p d^q \mid m + n = p + q\}$

Convert the following grammars into Chomsky normal form.

9.

$$\begin{aligned} S &\rightarrow ABC \mid aBC \\ A &\rightarrow aA \mid BC \\ B &\rightarrow bB \mid \epsilon \\ C &\rightarrow cC \mid \epsilon \end{aligned}$$

10.

$$\begin{aligned} S &\rightarrow A \mid ABa \mid AbA \\ A &\rightarrow Aa \mid \epsilon \\ B &\rightarrow Bb \mid BC \\ C &\rightarrow CB \mid CA \mid bB \end{aligned}$$

11. Show that the following CFG is ambiguous.

$$\begin{aligned} S &\rightarrow aSA \mid \epsilon \\ A &\rightarrow bA \mid \epsilon \end{aligned}$$

12. Prove that the language $\{w \mid w \in \{a, b\}^* \text{ and } w \text{ is a palindrome}\}$ is non regular. Also come up with a string z in the language on which pumping lemma holds good.