Homework 2, Due June 14th

You are required to solve TEN problems.
Problems 2, 3, 4, 5, 6 are not optional.
Solve any FIVE problems out of the rest.
Each problem carries 4 points.
Extra credit for extra problems solved.

1. Let $L \subseteq \Sigma^*$ be a regular language. Define the language Prefix(L) as follows:
   Prefix(L)$= \{w \in \Sigma^* : x = wy$ for some $x \in L$, $y \in \Sigma^*\}$.
   Show that the language Prefix(L) is also regular.

2. Construct DFAs equivalent to the corresponding NFAs given in the Figure 1.

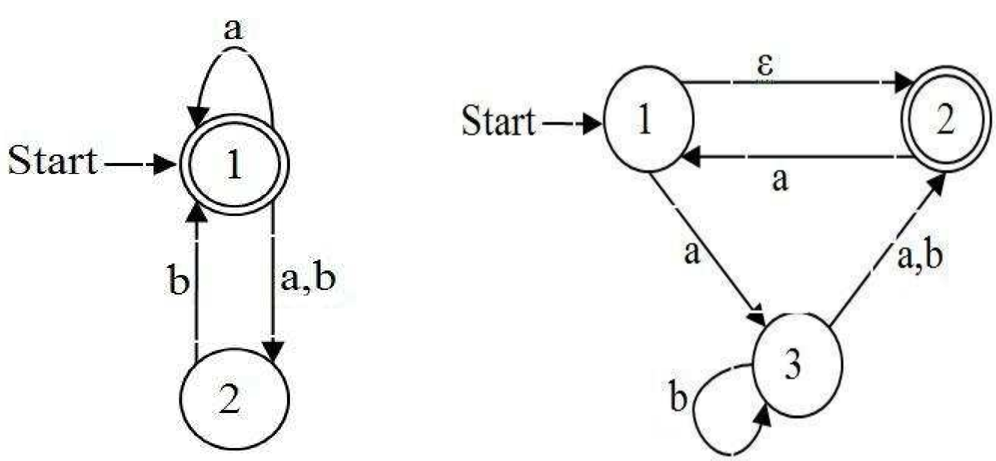


Figure 1.

3. Describe in English the sets denoted by the following regular expressions.

   (a) $(a \cup ba \cup bb)(a \cup b)^*$

   (b) $(a \cup b)^* a(a \cup b)(a \cup b)(a \cup b)$

4. Write regular expressions for each of the following languages over the alphabet $\{0, 1\}$.

   (a) $L_1 = \{w | w$ starts with 0 and has odd length, or w starts with 1 and has even length $\}$

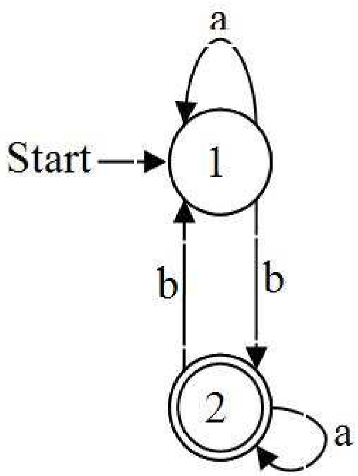
   (b) $L_2 = \{w |$ every odd position of w is a $1\}$

5. Construct finite automata equivalent to the following regular expressions.

   (a) $(11 \cup 0^*)(00 \cup 1)^*$

   (b) $(((00)^*(11)) \cup 01)^*$

6. Construct regular expressions corresponding to the state diagrams given in the Figure 2.



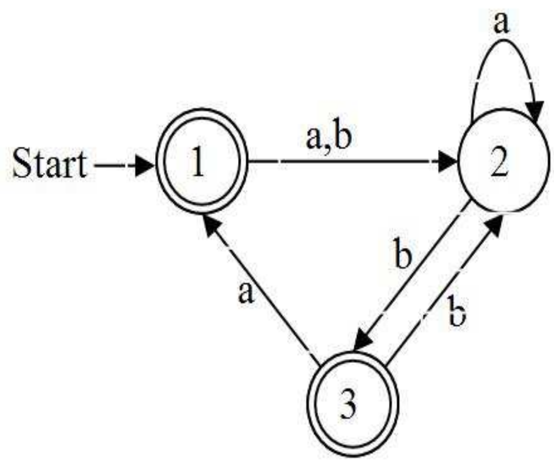


Figure 2.

Prove that the following languages are not regular using pumping lemma.

7. $A_1 = \{0^n1^n2^n | n \geq 0\}$

8. $A_2 = \{x \in \{0,1,2\}^* | x = w2w$, with $w \in \{0,1\}^*\}$

9. $A_3 = \{a^nba^mba^{m+n} : n, m \geq 1\}$.

10. $A_4 = \{ww | w \in \{0,1\}^*\}$

11. $A_5 =$ Set of strings over $\{(,)\}$ in which the parentheses are paired.
    Some examples of strings in A_5 are $(), ()(), (()), (())()$.

12. $A_6 = \{ww^R | w \in \{0,1\}^*\}$. w^R is w written backwards. For example, $(011)^R = 110$.