CMSC 203 Discrete Structures

April 20 Name_____

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induction

1. Show by using mathematical induction that

$2(n + 2) <= (n + 2)^2$ where ^ denotes exponentiation
assume here $n >= 0$

Make sure that your proof contains a base case and a clear statement of the induction hypothesis.

base case $n = 0$ $2 \cdot 2 \leq 2^2$ ok

induction hyp. $2(k+2) \leq (k+2)^2$

$n = k+1$

$2(k+1+2) = 2(k+2)+2 \overset{\text{I.H.}}{\leq} (k+2)^2 + 2$

$= k^2 + 4k + 4 + 2 = k^2 + 4k + 6$

$\leq k^2 + 6k + 9 = ((k+1)+2)^2$

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Counting....

1. A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.
   a) How many socks must he take out to be sure that he has at least two socks of the same color? Choose the correct answer.

a) 2
b) 3
c) 12
d) 13

e) 14

b) How many socks must he take out, at random, to be sure that he has two black socks.

a) 2
b) 3
c) 12
d) 13
e) 14

note- first 12 may be ~~white~~ brown socks

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2. The pidgeonhole principle says if we have n + 1 items and n places for these items then some place must contain at least two items. Use this principle to solve the problem below

A computer network consists of eight computers. Each computer is directly connected to at least one of the other computers. Show that there are at least two computers that are directly connected to the same number of other computers. Give a clear explanation of your reasoning.

8 computers but seven categories of connectivity (0 is omitted)

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3. How many bit strings of length 10 contain five consecutive 0s.

Note- these consecutive 0s or 1s form a block. Any bits adjacent to this block but be 1s. Other positions in the string can be either 0 or 1. For example,

0100000110 is a possible string

circle correct answer

a) 6
b) c(10,5)
c) 32
d) 64
e) none of the above

note two different types of configurations

type ① $2 \cdot 2^4 = 32$

type ② $4 \cdot 2^3 = 32$

total = 64

① 0 0 0 0 0 1 X X X X

or X X X X 1 0 0 0 0 0

② block of 0s is internal

X = don't care

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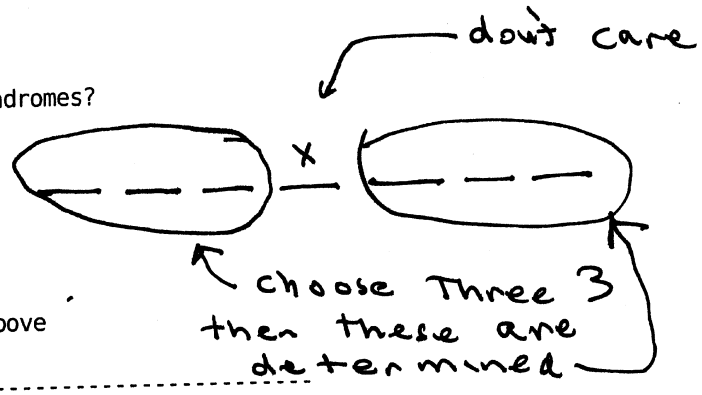
4. A palindrome is a string whose reversal is identical to the string.

For example 11011011011 is a palindrome

How many bit strings of length 7 are palindromes?



multiplication rule

$2^4 = 16$

a) 10
b) 8
c) 16
d) 128
e) none of the above

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5. What is the coefficient of x^4 (x to the fourth) in the expansion of (1 + x)^6? (1 plus x to the sixth).

a) 1
b) 6
c) 20
d) 15
e) none of the above

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6. How many solutions are there to the equation

x + y + z + w = 17 ?

where x,y,z, and w are non-negative integers.

a) C(20,17)
b) C(17,4)
c) 17
d) 1032
e) none of the above

n = number of items (1's)

r = categories (variables)

$\binom{n+r-1}{n}$ $n = 17$ $r = 4$

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7. How any ways can eight books be placed on 3 distinguishable shelves

a) if the books are indistinguishable copies of the same title?

a) 33
b) 40
c) 45
d) C(8,3)

$n = 8$, $r = 3$

$\binom{10}{8} = \frac{10 \cdot 9}{2} = 45$

b) if no two books are the same, and the positions of the books on the shelves matter?

a) C(10,8)*8!
b) C(8,3)*3!
c) C(8,3)*8!
d) C(10,7)*8
e) none of the above

1. Choose book positions (see above)
2. choose individual books

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8. A group consists of three men and six women.
How many teams of five are there total? (order of selection not important)

a) 64
b) 9
c) 9!
d) 126
e) none of the above

9 - people

$\binom{9}{5}$ committees

$= \frac{9 \cdot 8 \cdot 7 \cdot 6}{24} = 126$

how many groups of five contain at least one man?

a) 58
b) 3
c) 9! - 6
d) 120
e) none of the above

Note = total

- number of committees with no men

$126 - \binom{6}{5} = 120$
