GPU Shading and Rendering: Introduction & Graphics Hardware

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Schedule

Shading Technolgy	GPU Rendering
8:30 Intro/Hardware (Olano)	1:45 Rendering
9:25 Compilers (Bleiweiss)	Algorithms (Hart)
Shading Languages	2:35 GPU Production Rendering (Gritz)
10:30 GLSL (Olano)	Hardware Systems
10:55 Cg (Kilgard)	3:45 ATI (Sander)
11:20 HLSL (Sander)	4:25 NVIDIA (Kilgard)
11:45 Sh (McCool)	5:05 Panel Q&A (all)

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Part I

Introdution



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• Graphics Processing Unit

- Graphics accelerator
- Parallel processing unit
- We're doing graphics, what is it good for?
 - Better real-time graphics
 - Faster non-real-time graphics

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- Programmable
 - Flexible Appearance
 - Arbitrary computation

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- More realistic appearance
 - Bump mapping, Anisotropic, Precomputed radiance transfer, ...

- Non-realistic appearance
 - Cartoon, Sketch, Illustration, ...
- Animated appearance
 - Skinning, Water, Clouds, ...
- Visualization
 - Data on surfaces, Volume rendering, ...

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What is Rendering?

- The rest of the problem!
- In our case, using GPU for other than polygon rendering

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- Curved surfaces
- Ray tracing
- Point based rendering
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Real-Time

Tens of frames per second Thousand instruction shaders Limited computation, texture, memory, ...

Non-Real-Time

Seconds to hours per frame Thousands line shaders "Unlimited" computation, texture, memory, ...

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How is this possible?

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- GPUs are programmable!
 - Per-vertex programs
 - Per-fragment programs

- Pixel-Planes 5 [Rhoades et al., 1992]
- PixelFlow/pfman [Olano and Lastra, 1998]
- RTSL [Proudfoot et al., 2001]
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Ignoring Hardware Differences

Part II

Graphics Hardware

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Outline

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Ignoring Hardware Differences

Simplified Models RenderMan Hardware

Machine Complexity

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• Graphics machines are complex

- User does not want to know
 - How machine does what it does
 - Tons of machine-specific differences
- Answer:
 - Simple model of machine

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Simplified Machine

• User's mental model

- Hide details
- Device independent
- Procedural stages

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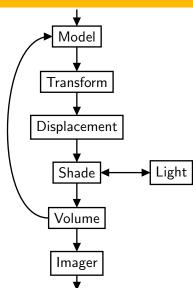
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RenderMan Model

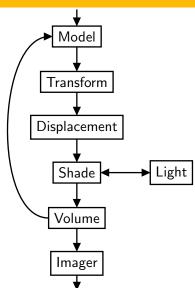


- "Abstract" interface
 - Blocks = procedures
 - Block interfaces well defined
- Connections
 - Inputs & outputs don't have to match

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 System handles conversion

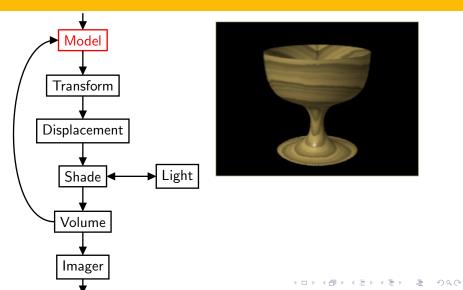
RenderMan Model

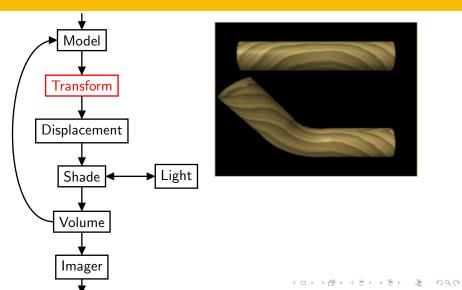


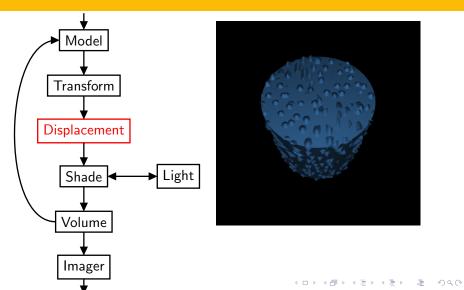
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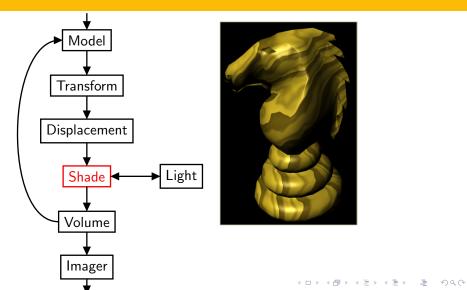
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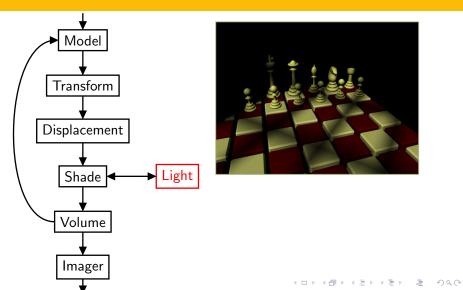
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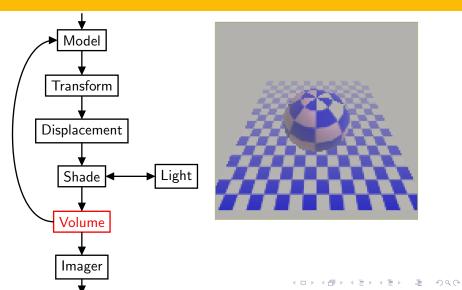


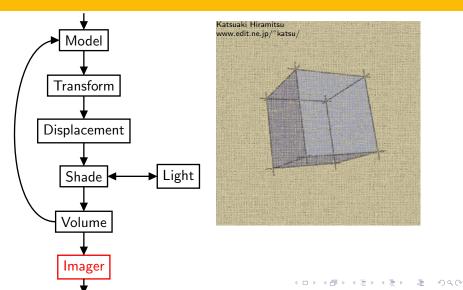




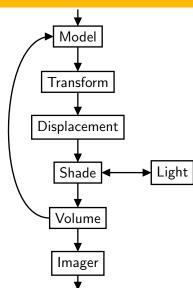








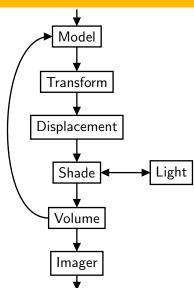
RenderMan Model



- What it says:
 - Input and output of each block
 - What each block should do

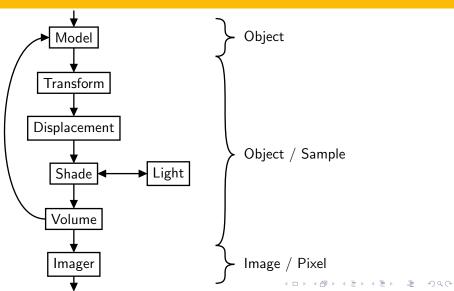
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RenderMan Model

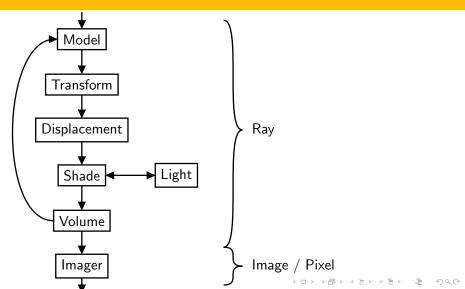


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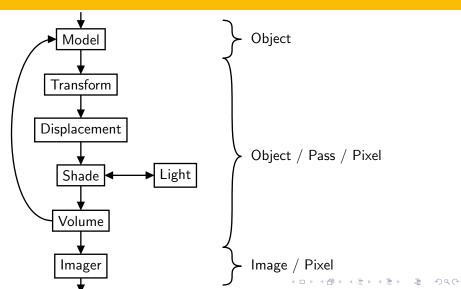
RenderMan REYES



RenderMan Ray Tracing



RenderMan SGI Multi-pass RenderMan

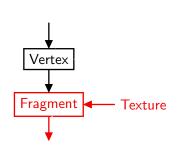


- Vertex shading
 - Transform
 - Procedural transformation

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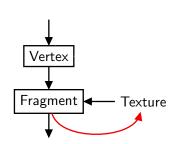
- Skinning
- Shade
 - Per-vertex shading
 - Computed texture coordinates



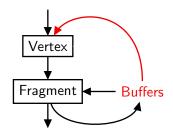


- Fragment shading
 - Per-fragment shading
 - Computed and dependent texture

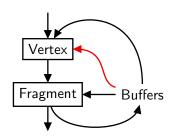
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- Render to texture
 - Rendered shadow & environment maps
 - Multi-pass fragment shading [Proudfoot et al., 2001]

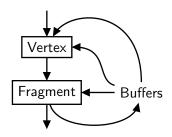


- Render to vertex array / buffer objects
 - Geometry images [Gu et al., 2002]
 - Multi-pass vertex shading
 - Merge vertex & fragment capabilities



- Vertex texture
 - Texture-based vertex displacement
 - Tabulated functions

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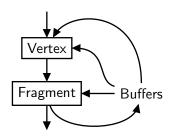


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 - Vertex processing order
 Fragment processing order
 Interleaving of vertex and fragment

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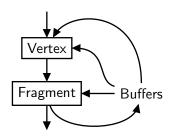
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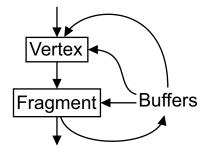
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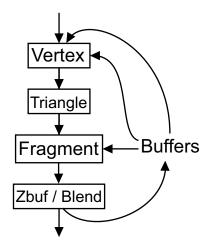
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- And more parallelism

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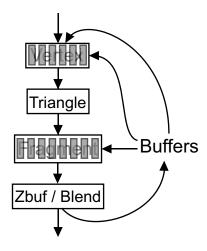


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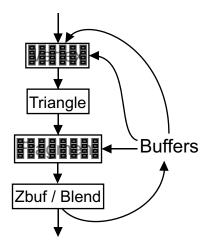
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Modifications 0000000000

Part III

Noise



Outline

What is this Noise?

Perlin noise

Modifications

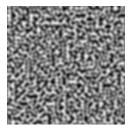
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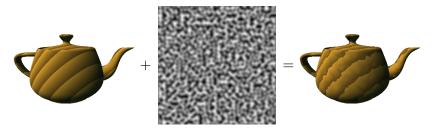
Why Noise?

- Introduced by [Perlin, 1985]
 - Heavily used in production animation
 - Technical Achievement Oscar in 1997
- "Salt," adds spice to shaders



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Noise Characteristics

• Random

- No correlation between distant values
- Repeatable/deterministic
 - Same argument always produces same value
- Band-limited
 - Most energy in one octave (e.g. between f & 2f)



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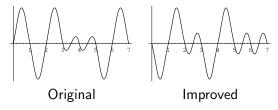


What is this Noise?

Perlin noise

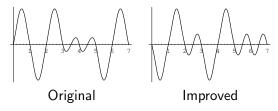
Gradient Noise

- Original Perlin noise [Perlin, 1985]
- Perlin Improved noise [Perlin, 2002]
- Lattice based
 - Value=0 at integer lattice points
 - Gradient defined at integer lattice
 - Interpolate between
- 1/2 to 1 cycle each unit



Gradient Noise

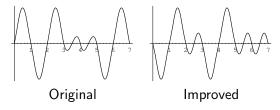
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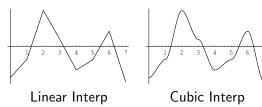
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Value Noise

- Lattice based
 - Value defined at integer lattice points
 - Interpolate between
- At most 1/2 cycle each unit

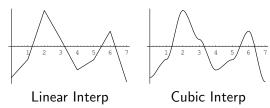
Significant low-frequency content

Easy hardware implementation with lower quality



Value Noise

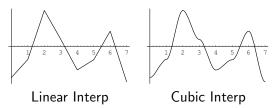
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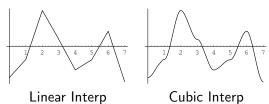
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Hardware Noise

- Value noise
 - PixelFlow [Lastra et al., 1995]
 - Perlin Noise Pixel Shaders [Hart, 2001]
 - Noise textures
- Gradient noise
 - Hardware [Perlin, 2001]
 - Complex composition [Perlin, 2004]
 - Shader implementation [Green, 2005]

Outline

What is this Noise?

Perlin noise

Modifications

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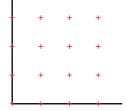
Noise Details

- Subclass of gradient noise
 - Original Perlin
 - Perlin Improved
 - All of our proposed modifications

Modifications

Find the Lattice

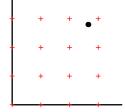
- Lattice-based noise: must find nearest lattice points
- Point $\vec{p} = (\vec{p}^x, \vec{p}^y, \vec{p}^z)$
- has integer lattice location $\vec{p}_i = (\lfloor \vec{p}^x \rfloor, \lfloor \vec{p}^y \rfloor, \lfloor \vec{p}^z \rfloor) = (X, Y, Z)$
- and fractional location in cell $\vec{p}_f = \vec{p} - \vec{p}_i = (x, y, z)$



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Find the Lattice

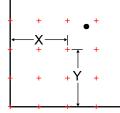
- Lattice-based noise: must find nearest lattice points
- Point $\vec{p} = (\vec{p}^x, \vec{p}^y, \vec{p}^z)$
- has integer lattice location $\vec{p}_i = (\lfloor \vec{p}^x \rfloor, \lfloor \vec{p}^y \rfloor, \lfloor \vec{p}^z \rfloor) = (X, Y, Z)$
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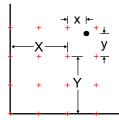


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Gradient

• Random vector at each lattice point is a function of \vec{p}_i

$g(\vec{p}_i)$

• A function with that gradient

$$\begin{aligned} \mathsf{grad}(\vec{p}) &= \mathsf{g}(\vec{p}_i) \bullet \vec{p}_f \\ &= \mathsf{g}^{\times}(\vec{p}_i) \ast \times + \mathsf{g}^{\vee}(\vec{p}_i) \ast \times + \mathsf{g}^{\mathbb{Z}}(\vec{p}_i) \ast \end{aligned}$$

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$$grad(\vec{p}) = g(\vec{p}_i) \bullet \vec{p}_f$$
$$= g^{\times}(\vec{p}_i) * \times + g^{\vee}(\vec{p}_i) * y + g^{z}(\vec{p}_i) * z$$

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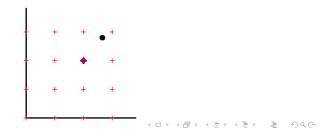
= $g^x(\vec{p}_i) * x + g^y(\vec{p}_i) * y + g^z(\vec{p}_i) * z$

- Interpolate nearest 2^n gradient functions
- 2D $noise(\vec{p})$ is influenced by
- $ec{
 ho}_i + (0,0)$; $ec{
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- Linear interpolation
 - lerp(t, a, b) = (1 t) a + t b
- Smooth interpolation

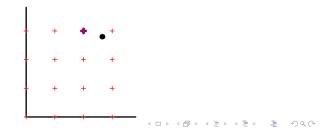


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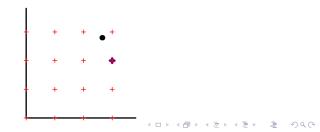
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•
$$lerp(t, a, b) = (1 - t) a + t b$$

Smooth interpolation



- Interpolate nearest 2ⁿ gradient functions
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- Linear interpolation
 - lerp(t, a, b) = (1 t) a + t b
- Smooth interpolation
 - fade(t) = $\begin{cases} 3t^2 2t^2 & \text{for original} \end{cases}$
 - flerp(t) = lerp(fade(t), a, b)



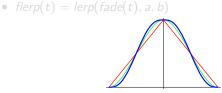
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 - $fade(t) = \begin{cases} 3t^2 2t^3 & \text{for original noise} \\ 10t^3 15t^4 + 6t^5 & \text{for improved noise} \end{cases}$ • flerp(t) = lerp(fade(t), a, b)



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for original noise for improved noise

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•
$$flerp(t) = lerp(fade(t), a, b)$$

Hash

• n-D gradient function built from 1D components

$g(\vec{p}_i)$

- Both original and improved use a permutation table hash
- Original: g is a table of unit vectors
- Improved: g is derived from bits of final hash

Hash

• n-D gradient function built from 1D components

g(hash(X, Y, Z))

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Hash

• n-D gradient function built from 1D components

g(hash(Z + hash(X, Y)))

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Outline

What is this Noise?

Perlin noise

Modifications Corner Gradients Factorization Hash

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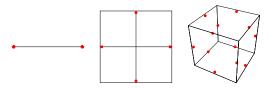
Gradient Vectors of n-D Noise

- Original: on the surface of a n-sphere
 - Found by hash of \vec{p}_i into gradient table
- Improved: at the edges of an n-cube
 - Found by decoding bits of hash of \vec{p}_i



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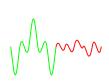


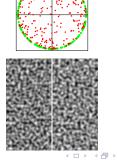
Gradients of noise(x,y,0) or noise(x,0)

- Why?
 - Cheaper low-D noise matches slice of higher-D
 - Reuse textures (for full noise or partial computation)
- Original: new short gradient vectors
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 - Possibly including 0 gradient vector!

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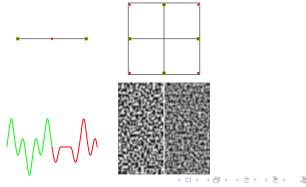
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Solution?

• Observe: use gradient function, not vector alone

$$grad = g^x x + g^y y + g^z z$$

• In any integer plane, fractional z = 0

$$grad = g^x x + g^y y + 0$$

• Any choice keeping projection of vectors the same will work

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 - Instead use cube corners!

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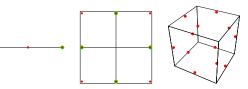
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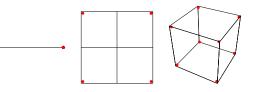
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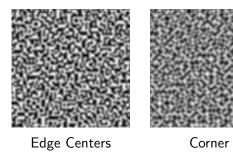
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Corner Gradients

- Simple binary selection from hash bits $\pm x, \pm y, \pm z$
- Perlin mentions "clumping" for corner gradient selection
 - Not very noticeable in practice
 - Already happens in any integer plane of improved noise

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Separable Computation

• Like to store computation in texture

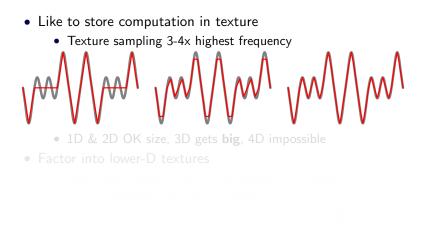
• Texture sampling 3-4x highest frequency

1D & 2D OK size, 3D gets big, 4D impossible Factor into lower-D textures

Perlin noise

Modifications

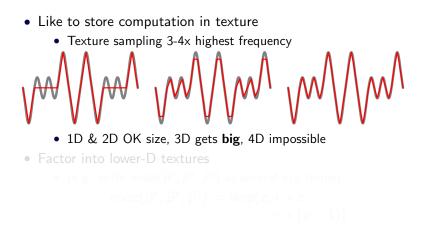
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Perlin noise

Modifications

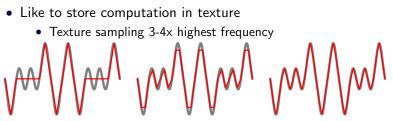
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Perlin noise

Modifications

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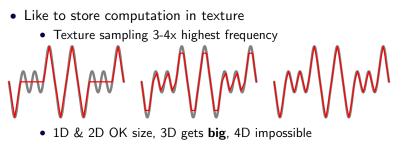


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Perlin noise

Modifications

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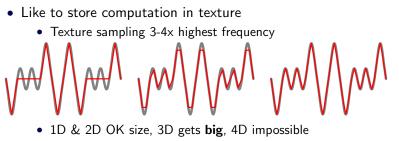


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Perlin noise

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Separable Computation



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Factorization Details

$\begin{array}{l} \textit{noise}(\vec{p}) = \textit{flerp}(z, \textit{zconst}(\vec{p}^{x}, \vec{p}^{y}, Z_{0}) + \textit{zgrad}(\vec{p}^{x}, \vec{p}^{y}, Z_{0}) * z, \\ \textit{zconst}(\vec{p}^{x}, \vec{p}^{y}, Z_{1}) + \textit{zgrad}(\vec{p}^{x}, \vec{p}^{y}, Z_{1}) * (z-1)) \end{array}$

• With nested hash,

 $zconst(\vec{p}^{x}, \vec{p}^{y}, Z_{0}) = zconst(\vec{p}^{x}, \vec{p}^{y} + hash(Z_{0}))$ $zgrad (\vec{p}^{x}, \vec{p}^{y}, Z_{0}) = zgrad (\vec{p}^{x}, \vec{p}^{y} + hash(Z_{0}))$

• With corner gradients, *zconst* = *noise*!

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Perlin's Hash

- 256-element *permutation array*
 - Turns each integer 0-255 into a different integer 0-255
- Chained lookups

- Must compute for each lattice point around \vec{p}
- Even with a 2D hash(Y + hash(X)) texture, that's

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2 hash lookups for 1D noise

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What is this Noise?

Perlin noise

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Alternative Hash

- Many choices; I kept 1D chaining
- Desired features
 - Low correlation of hash output for nearby inputs
 - Computable without lookup
- Use a random number generator?
 - Seed
 - Successive calls give uncorrelated values

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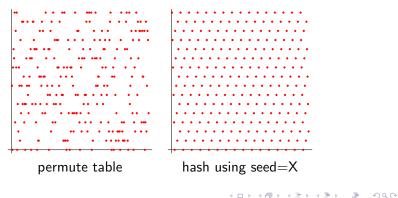
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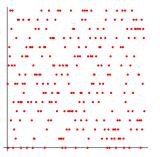
Random Number Generator Hash

- Hash argument is seed
 - Most RNG are highly correlated for nearby seeds
- Hash argument is number of times to call
 - Most RNG are expensive (or require n calls) to get nth number
 - Should noise(30) be 30 times slower than noise(1)?

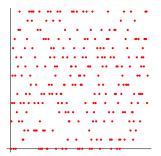


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 - Most RNG are highly correlated for nearby seeds
- Hash argument is number of times to call
 - Most RNG are expensive (or require n calls) to get nth number
 - Should noise(30) be 30 times slower than noise(1)?



permute table



hash using X^{th} random number

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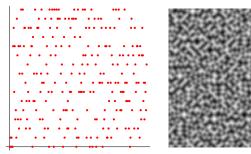
Modifications

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Blum-Blum Shub

 $x_{n+1} = x_i^2 \mod M$ M =product of two large primes

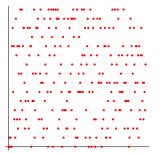
- Uncorrelated for nearby seeds...
- But large M is bad for hardware...
- But reasonable results for smaller M...
- And square and mod is simple to compute!

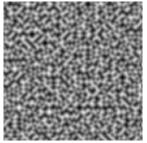


Blum-Blum Shub

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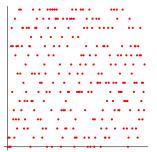


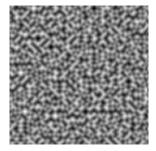


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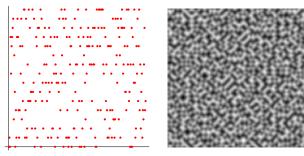


Blum-Blum Shub

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 $M =$ product of two large primes

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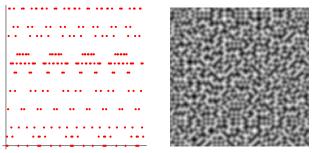
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Blum-Blum Shub

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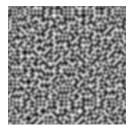


61

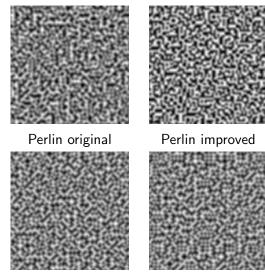
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Modified Noise

- Square and mod hash
 - *M* = 61
- Corner gradient selection
 - One 2D texture for both 1D and 2D
- Factor
 - Construct 3D and 4D from 2 or 4 2D texture lookups



Comparison



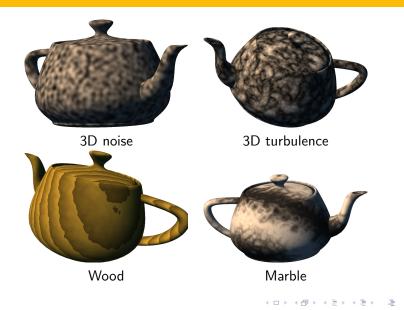
Corner gradients

 $Corner + Hash \rightarrow (E) (E) = (\circ) (\circ)$

Modifications

590

Using Noise



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