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> # Quantumentomography.mw
> with(linalg) :
> # The Kronecker product of matrices A and B
kr:=proc(A,B)
  # This procedure computes the Kronecker product of two matrices
  local mm,nn,ans,ans1;
  ans:=[];
  for mm from 1 to rowdim(A) do
    for nn from 1 to coldim(A) do
      ans:=[op(ans), scalarmul(B, A[mm,nn] )];
    od;
  od;
  ans1:=blockmatrix(rowdim(A),coldim(A), ans );
  RETURN(evalm(ans1));
end:
> # Creation of Pauli spin operators
S||0:=diag(1,1):
S||1:=matrix(2,2,[0,1,1,0]):
S||2:=matrix(2,2,[0,-I,I,0]):
S||3:=matrix(2,2,[1,0,0,-1]):
#
> for p from 0 to 3 do
  for q from 0 to 3 do
    S||p||q:=kr(S||p,S||q)
  od;
od;
> # Creation of the projectors  $P_1 = P_{+1}, P_2 = P_{+2}, P_3 = P_{+3}$ 
P1 := evalm( ( ( 1/2 ) · matrix(2, 2, [1, 1, 1, 1]) );
P2 := evalm( ( ( 1/2 ) · matrix(2, 2, [1,-I, I, 1]) );
P3 := matrix(2, 2, [1, 0, 0, 0]);

      table( [(1, 1) = 1/2, (2, 2) = 1/2, (1, 2) = 1/2, (2, 1) = 1/2] )
      table( [(1, 1) = 1/2, (2, 2) = 1/2, (1, 2) = -1/2*I, (2, 1) = 1/2*I] )
      table( [(1, 1) = 1, (2, 2) = 0, (1, 2) = 0, (2, 1) = 0] )
> # Creation of the projectors  $P_{10} = P_{+10}, P_{01} = P_{+01}, P_{20} = P_{+20}, P_{02} = P_{+02}, P_{30} = P_{+30},$ 
      P03 = P_{+03}
for p from 1 to 3 do
  P||p||0 := kr(P||p,S0);
  P||0||p := kr(S0,P||p);

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(1)

od;

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table([(1, 4) = 0, (1, 1) = 1/2, (3, 4) = 0, (2, 2) = 1/2, (3, 3) = 1/2, (4, 2) = 1/2, (1, 2) = 0, (2, 4) = 1/2, (3, 2) = 0, (4, 1) = 0, (3, 1) = 1/2, (1, 3) = 1/2, (4, 4) = 1/2, (2, 3) = 0, (4, 3) = 0, (2, 1) = 0]) (2)
table([(1, 4) = 0, (1, 1) = 1/2, (3, 4) = 1/2, (2, 2) = 1/2, (3, 3) = 1/2, (4, 2) = 0, (1, 2) = 1/2, (2, 4) = 0, (3, 2) = 0, (4, 1) = 0, (3, 1) = 0, (1, 3) = 0, (4, 4) = 1/2, (2, 3) = 0, (4, 3) = 1/2, (2, 1) = 1/2])
table([(1, 4) = 0, (1, 1) = 1/2, (3, 4) = 0, (2, 2) = 1/2, (3, 3) = 1/2, (4, 2) = 1/2*I, (1, 2) = 0, (2, 4) = -1/2*I, (3, 2) = 0, (4, 1) = 0, (3, 1) = 1/2*I, (1, 3) = -1/2*I, (4, 4) = 1/2, (2, 3) = 0, (4, 3) = 0, (2, 1) = 0])
table([(1, 4) = 0, (1, 1) = 1/2, (3, 4) = -1/2*I, (2, 2) = 1/2, (3, 3) = 1/2, (4, 2) = 0, (1, 2) = -1/2*I, (2, 4) = 0, (3, 2) = 0, (4, 1) = 0, (3, 1) = 0, (1, 3) = 0, (4, 4) = 1/2, (2, 3) = 0, (4, 3) = 1/2*I, (2, 1) = 1/2*I])
table([(1, 4) = 0, (1, 1) = 1, (3, 4) = 0, (2, 2) = 1, (3, 3) = 0, (4, 2) = 0, (1, 2) = 0, (2, 4) = 0, (3, 2) = 0, (4, 1) = 0, (3, 1) = 0, (1, 3) = 0, (4, 4) = 0, (2, 3) = 0, (4, 3) = 0, (2, 1) = 0])
table([(1, 4) = 0, (1, 1) = 1, (3, 4) = 0, (2, 2) = 0, (3, 3) = 1, (4, 2) = 0, (1, 2) = 0, (2, 4) = 0, (3, 2) = 0, (4, 1) = 0, (3, 1) = 0, (1, 3) = 0, (4, 4) = 0, (2, 3) = 0, (4, 3) = 0, (2, 1) = 0])
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> assume(a00, 'real');
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assume(a01, 'real'); assume(b01, 'real');
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assume(a10, 'real'); assume(b10, 'real');
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assume(a11, 'real'); assume(b11, 'real');
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> # Creation of the state  $\psi$  to be measured
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 $\psi := \text{matrix}(4, 1, [a00, a01 + I \cdot b01, a10 + I \cdot b10, a11 + I \cdot b11]);$ 
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 $\text{psiAdj} := \text{matrix}(1, 4, [a00, a01 - I \cdot b01, a10 - I \cdot b10, a11 - I \cdot b11]);$ 
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table([(1, 1) = a00, (4, 1) = a11 + I * b11, (3, 1) = a10 + I * b10, (2, 1) = a01 + I * b01]) (3)
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table([(1, 4) = a11 - I * b11, (1, 1) = a00, (1, 2) = a01 - I * b01, (1, 3) = a10 - I * b10])
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> p10 := expand( (evalm(psiAdj & P || 1 || 0 & \cdot \psi)) [1, 1] );
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p01 := expand( (evalm(psiAdj & P || 0 || 1 & \cdot \psi)) [1, 1] );
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p20 := expand( (evalm(psiAdj & P || 2 || 0 & \cdot \psi)) [1, 1] );
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p02 := expand( (evalm(psiAdj & P || 0 || 2 & \cdot \psi)) [1, 1] );
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p30 := expand( (evalm(psiAdj & P || 3 || 0 & \cdot \psi)) [1, 1] );
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p03 := expand( (evalm(psiAdj & P || 0 || 3 & \cdot \psi)) [1, 1] );
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```
 $\frac{1}{2} a00^2 + a00 \cdot a10 + \frac{1}{2} a01^2 + \frac{1}{2} b01^2 + a11 \cdot a01 + b11 \cdot b01 + \frac{1}{2} a10^2$  (4)
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 $+ \frac{1}{2} b10^2 + \frac{1}{2} a11^2 + \frac{1}{2} b11^2$ 
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 $\frac{1}{2} a00^2 + a00 \cdot a01 + \frac{1}{2} a01^2 + \frac{1}{2} b01^2 + \frac{1}{2} a10^2 + \frac{1}{2} b10^2 + a11 \cdot a10$ 
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 $+ b11 \cdot b10 + \frac{1}{2} a11^2 + \frac{1}{2} b11^2$ 
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 $\frac{1}{2} a00^2 + a00 \cdot b10 + \frac{1}{2} a01^2 + \frac{1}{2} b01^2 - a11 \cdot b01 + b11 \cdot a01 + \frac{1}{2} a10^2$ 
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```
 $+ \frac{1}{2} b10^2 + \frac{1}{2} a11^2 + \frac{1}{2} b11^2$ 
```

$$\begin{aligned} & \frac{1}{2} a_{00}^2 + a_{00} b_{01} + \frac{1}{2} a_{01}^2 + \frac{1}{2} b_{01}^2 + \frac{1}{2} a_{10}^2 + \frac{1}{2} b_{10}^2 - a_{11} b_{10} \\ & + b_{11} a_{10} + \frac{1}{2} a_{11}^2 + \frac{1}{2} b_{11}^2 \\ & \qquad \qquad \qquad a_{00}^2 + a_{01}^2 + b_{01}^2 \\ & \qquad \qquad \qquad a_{00}^2 + a_{10}^2 + b_{10}^2 \end{aligned}$$

> eq10 := p10-x10=0; eq01 := p01-x01;

eq20 := p20-x20=0; eq02 := p02-x02=0;

eq30 := p30-x30=0; eq03 := p03-x03=0;

eq00 := (a00)<sup>2</sup> + (a01)<sup>2</sup> + (b01)<sup>2</sup> + (a10)<sup>2</sup> + (b10)<sup>2</sup> + (a11)<sup>2</sup> + (b11)<sup>2</sup> - 1 = 0;

$$\begin{aligned} & \frac{1}{2} a_{00}^2 + a_{00} a_{10} + \frac{1}{2} a_{01}^2 + \frac{1}{2} b_{01}^2 + a_{11} a_{01} + b_{11} b_{01} + \frac{1}{2} a_{10}^2 \\ & + \frac{1}{2} b_{10}^2 + \frac{1}{2} a_{11}^2 + \frac{1}{2} b_{11}^2 - x_{10} = 0 \end{aligned} \tag{5}$$

$$\begin{aligned} & \frac{1}{2} a_{00}^2 + a_{00} a_{01} + \frac{1}{2} a_{01}^2 + \frac{1}{2} b_{01}^2 + \frac{1}{2} a_{10}^2 + \frac{1}{2} b_{10}^2 + a_{11} a_{10} \\ & + b_{11} b_{10} + \frac{1}{2} a_{11}^2 + \frac{1}{2} b_{11}^2 - x_{01} \end{aligned}$$

$$\begin{aligned} & \frac{1}{2} a_{00}^2 + a_{00} b_{10} + \frac{1}{2} a_{01}^2 + \frac{1}{2} b_{01}^2 - a_{11} b_{01} + b_{11} a_{01} + \frac{1}{2} a_{10}^2 \\ & + \frac{1}{2} b_{10}^2 + \frac{1}{2} a_{11}^2 + \frac{1}{2} b_{11}^2 - x_{20} = 0 \end{aligned}$$

$$\begin{aligned} & \frac{1}{2} a_{00}^2 + a_{00} b_{01} + \frac{1}{2} a_{01}^2 + \frac{1}{2} b_{01}^2 + \frac{1}{2} a_{10}^2 + \frac{1}{2} b_{10}^2 - a_{11} b_{10} \\ & + b_{11} a_{10} + \frac{1}{2} a_{11}^2 + \frac{1}{2} b_{11}^2 - x_{02} = 0 \end{aligned}$$

$$a_{00}^2 + a_{01}^2 + b_{01}^2 - x_{30} = 0$$

$$a_{00}^2 + a_{10}^2 + b_{10}^2 - x_{03} = 0$$

$$a_{00}^2 + a_{01}^2 + b_{01}^2 + a_{10}^2 + b_{10}^2 + a_{11}^2 + b_{11}^2 - 1 = 0$$

> eqList := [eq10, eq01, eq20, eq02, eq30, eq03, eq00];

varList := [a00, a01, b01, a10, b10, a11, b11];

$$\begin{aligned} & \left[ \frac{1}{2} a_{00}^2 + a_{00} a_{10} + \frac{1}{2} a_{01}^2 + \frac{1}{2} b_{01}^2 + a_{11} a_{01} + b_{11} b_{01} + \frac{1}{2} a_{10}^2 \right. \\ & \left. + \frac{1}{2} b_{10}^2 + \frac{1}{2} a_{11}^2 + \frac{1}{2} b_{11}^2 - x_{10} = 0, \right. \end{aligned} \tag{6}$$

$$\begin{aligned} & \frac{1}{2} a_{00}^2 + a_{00} a_{01} + \frac{1}{2} a_{01}^2 + \frac{1}{2} b_{01}^2 + \frac{1}{2} a_{10}^2 + \frac{1}{2} b_{10}^2 + a_{11} a_{10} \\ & + b_{11} b_{10} + \frac{1}{2} a_{11}^2 + \frac{1}{2} b_{11}^2 - x_{01}, \end{aligned}$$

$$\begin{aligned} & \frac{1}{2} a_{00}^2 + a_{00} b_{10} + \frac{1}{2} a_{01}^2 + \frac{1}{2} b_{01}^2 - a_{11} b_{01} + b_{11} a_{01} + \frac{1}{2} a_{10}^2 \\ & + \frac{1}{2} b_{10}^2 + \frac{1}{2} a_{11}^2 + \frac{1}{2} b_{11}^2 - x_{20} = 0, \end{aligned}$$

$$\frac{1}{2} a_{00}^2 + a_{00} b_{01} + \frac{1}{2} a_{01}^2 + \frac{1}{2} b_{01}^2 + \frac{1}{2} a_{10}^2 + \frac{1}{2} b_{10}^2 - a_{11} b_{10}$$

$$\begin{aligned}
&+ b_{11} \sim a_{10} \sim + \frac{1}{2} a_{11} \sim^2 + \frac{1}{2} b_{11} \sim^2 - x_{02} = 0, a_{00} \sim^2 + a_{01} \sim^2 + b_{01} \sim^2 - x_{30} = 0, \\
&a_{00} \sim^2 + a_{10} \sim^2 + b_{10} \sim^2 - x_{03} = 0, \\
&a_{00} \sim^2 + a_{01} \sim^2 + b_{01} \sim^2 + a_{10} \sim^2 + b_{10} \sim^2 + a_{11} \sim^2 + b_{11} \sim^2 - 1 = 0 \Big] \\
&\quad [a_{00} \sim, a_{01} \sim, b_{01} \sim, a_{10} \sim, b_{10} \sim, a_{11} \sim, b_{11} \sim]
\end{aligned}$$

> solve(eqList, varList);

Warning, computation interrupted

> p10-p01; p20-p02; p30-p03;

$$\begin{aligned}
&a_{00} \sim a_{10} \sim + a_{11} \sim a_{01} \sim + b_{11} \sim b_{01} \sim - a_{00} \sim a_{01} \sim - a_{11} \sim a_{10} \sim - b_{11} \sim b_{10} \sim \\
&a_{00} \sim b_{10} \sim - a_{11} \sim b_{01} \sim + b_{11} \sim a_{01} \sim - a_{00} \sim b_{01} \sim + a_{11} \sim b_{10} \sim - b_{11} \sim a_{10} \sim \\
&\quad a_{01} \sim^2 + b_{01} \sim^2 - a_{10} \sim^2 - b_{10} \sim^2
\end{aligned}$$

(7)

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