

# ENE 610/Spring 07/Project III: A general process for Linear-Phase filter design

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## Abstract

In class we have discussed the importance of linear-phase. Additionally our lectures have focused on the various types of linear-phase filters (Type I, II, III and IV). In this project students will focus on programming a general purpose filter design technique using least-squares methods. If the students understand the present project, they will be able to easily design standard frequency selective filters (low-pass, high-pass, band-pass and band-stop), differentiators, Hilbert Transformers, maximally flat filters and interpolating filters.

In this project students will write a function in MATLAB to enable them to create different filters.

**Keywords:** Filter design, least-squares methods, linear-phase filters.

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## 1 Linear-Phase filters

LP filters with  $N$  taps having an impulse response  $h[n]$ ,  $n = 0, \dots, N - 1$ , can be divided into four types as described in Oppenheimer, Shafer and Buck (OSB). Type I have an odd number of taps while Type II have an even number and both are symmetric. The FRF of these filters is

$$H(e^{j\omega}) = M(\omega)e^{-j(N-1)\omega/2}$$

where,

$$M(\omega) = \begin{cases} \sum_{n=0}^{\frac{N-1}{2}} b[n] \cos(n\omega) & \text{Type I} \\ \sum_{n=0}^{\frac{N}{2}} b[n] \cos(n - \frac{1}{2})\omega & \text{Type II} \end{cases} \quad (1)$$

If  $N$  is odd,  $b[0] = h[(N - 1)/2]$  and  $b[n] = 2h[(N - 1)/2 - n]$  for  $1 \leq n \leq (N - 1)/2$ . If  $N$  is even,  $b[n] = 2h[N/2 - n]$  for  $1 \leq n \leq N/2$ .

On a similar note, Type III and IV filters have an antisymmetrical impulse response  $h[n] = -h[N - 1 - n]$ . Type III filters have an odd number taps and Type IV even. The FRF of these types of filters is

$$H(e^{j\omega}) = jM(\omega)e^{-j(N-1)\omega/2}$$

$$M(\omega) = \begin{cases} \sum_{n=0}^{\frac{N-1}{2}} b[n] \sin(n\omega) & \text{Type III} \\ \sum_{n=0}^{\frac{N}{2}} b[n] \sin(n - \frac{1}{2})\omega & \text{Type IV} \end{cases} \quad (2)$$

If  $N$  is odd,  $b[n] = 2h[(N - 1)/2 - n]$  for  $1 \leq n \leq (N - 1)/2$ . If  $N$  is even,  $b[n] = 2h[N/2 - n]$  for  $1 \leq n \leq N/2$ .

## 2 Going from ideal to non-ideal filters

Pages 43 and 44 of OSB show ideal filters. We have also discussed that ideal filters are not realizable. We therefore need to know how close our actual filter design is from ideal. While there are many ways to do this, we use a weighted mean-square difference between the ideal amplitude response  $D(\omega)$  and the amplitude response of the filter  $M(\omega)$ . This is also known as an objective function and is written as:

$$E_{mse} = \frac{\alpha}{\pi} \int_P W_P(\omega) [D(\omega) - M(\omega)]^2 d\omega + \frac{\beta}{\pi} \int_S W_S(\omega) M(\omega)^2 d\omega, \quad (3)$$

where  $P$  is the passband and  $S$  is the stopband.  $\alpha$  and  $\beta$  represent the weights placed on the entire passband and stopband respectively.  $W_P(\omega)$  and  $W_S(\omega)$  are also weighting functions for the passband and stopband but are used to emphasize certain frequencies in contrast to other frequencies.

Consider Type I and II filters. We can write

$$M(\omega) = \mathbf{b}^T \mathbf{c}(\omega)$$

with

$$\mathbf{b} = \begin{cases} [b[0] \ b[1] \ \dots \ b[(N-1)/2]]^T & \text{Type I} \\ [b[1] \ b[2] \ \dots \ b[N/2]]^T & \text{Type II} \end{cases} \quad (4)$$

and

$$\mathbf{c}(\omega) = \begin{cases} [1 \ \cos \omega \ \dots \ \cos(\frac{1}{2}(N-1)/2)]^T & \text{Type I} \\ [\cos \frac{1}{2}\omega \ \cos \frac{3}{2}\omega \ \dots \ \cos(\frac{1}{2}(N-1)/2)]^T & \text{Type II} \end{cases} \quad (5)$$

Modification of Equation (3) leads to

$$E_{mse} = \frac{\alpha}{\pi} \int_P W_P(\omega) [D(\omega)^2 - 2D(\omega)\mathbf{b}^T \mathbf{c}(\omega) + \mathbf{x}] d\omega + \frac{\beta}{\pi} \int_S W_S(\omega) \mathbf{x} d\omega. \quad (6)$$

In equation (6) the student should obtain  $\mathbf{x}$  in terms of  $\mathbf{b}$ ,  $\mathbf{c}(\omega)$ .

Students should know that an objective function can be minimized or maximized. In this case we wish to minimize  $E_{mse}$  because that leads to filter coefficients that are optimal using the weighted least-squares criteria. Hence, setting the following partial derivatives,

$$\frac{\partial E_{mse}}{\partial b[i]}$$

leads to a set of linear equations

$$[\alpha \mathbf{Q} + \beta \mathbf{R}] \mathbf{b} = \alpha \mathbf{d}. \quad (7)$$

The student should obtain explicit expressions for  $\mathbf{Q}$ ,  $\mathbf{R}$  and  $\mathbf{d}$  in terms of  $W_P(\omega)$ ,  $W_S(\omega)$ ,  $\mathbf{c}(\omega)$  and  $D(\omega)$ . Comment on an important property of  $\mathbf{Q}$  and  $\mathbf{R}$ .

Following a similar derivation for the Type III and IV filters, we get

$$[\alpha \mathbf{Q}_1 + \beta \mathbf{R}_1] \mathbf{b} = \alpha \mathbf{d}_1. \quad (8)$$

The student is asked to obtain expressions for  $\mathbf{Q}_1$ ,  $\mathbf{R}_1$  and  $\mathbf{d}_1$ . Comment on an important property of  $\mathbf{Q}_1$  and  $\mathbf{R}_1$ .

The set of linear equations defined by equations (8), (7) can now be solved using a variety of algorithmic techniques most of which are available in MATLAB. We recommend a Cholesky decomposition method.

## 3 What to do?

For frequency-selective filters,  $D(\omega) = 1$  in  $P$  and is 0 in  $S$ . For a low pass filter  $P = [0, \omega_p]$  and  $S = [\omega_s, \pi]$ . For a bandpass filter  $P = [\omega_{p1}, \omega_{p2}]$  and  $S = [0, \omega_{s1}] \cup [\omega_{s2}, \pi]$ .

1. Write a MATLAB script that obtains the coefficients of any frequency-selective filter and plots the magnitude response for it. Your code will print your filter coefficients, i.e., your code will print out the coefficients on the screen. In order to make your code simple, it will only accept  $W_P(\omega) = W_S(\omega) = 1$ . Input parameters will be the number of taps in the filter  $N$ , whether it is type I or II (we will ignore III and IV),  $\alpha$  and  $\beta$  and  $P$  and  $S$ .
2. Use your code to solve problems 1 and 2 below.

Solve the following two problems with your code:

**Problem 1:** Design an LP filter with  $N = 29$ ,  $\omega_p = 0.3\pi$ ,  $\omega_s = 0.4\pi$ ,  $\alpha = \beta = 0.5$  and  $W_P(\omega) = W_S(\omega) = 1$ .

**Problem 2:** Design a bandpass filter with  $N = 51$ ,  $\omega_{p1} = 0.35\pi$ ,  $\omega_{p2} = 0.7\pi$ ,  $\omega_{s1} = 0.3\pi$ ,  $\omega_{s2} = 0.8\pi$ ,  $\alpha = 2/3$ ,  $\beta = 1/3$  and  $W_P(\omega) = W_S(\omega) = 1$ .

**Due date I:** 11:59pm on 7 May 2007 by email (derivations). You can scan in your derivations and send it to Haleh Safavi.

**Due date II:** 11:59pm on 16 May 2007 by email (master program that solves problem 1 and problem 2) only to Haleh Safavi.

**Start immediately. Plan for the inevitable rush that occurs at the end of the semester. Projects will not be accepted late.**