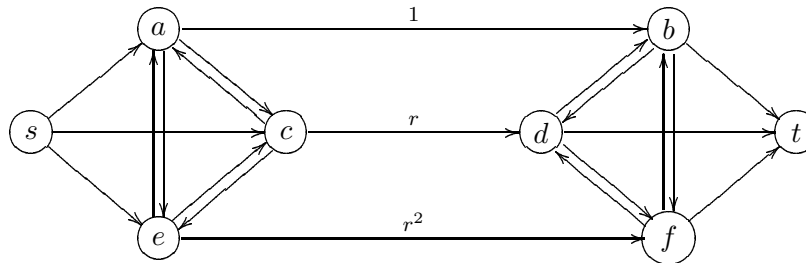


1. Exercise 25.2-6, page 635

2. Problem 26-1, page 692

3. Maximum Flow: Irrational Capacities

The proof that the Ford-Fulkerson method produces the maximum flow depends on the assumption that the edge capacities are integers. In this question, we will use the following network where $r = (\sqrt{5} - 1)/2 \approx 0.618033988$ and the capacities of the unlabeled edges are 100. (The value 100 is much larger than any flow that could be sent through the network.)



The value r is one of the roots of the quadratic equation $r^2 + r - 1 = 0$. This equation also gives us several convenient equalities:

$$r^2 = 1 - r, \quad r^3 = r - r^2, \quad r^4 = r^2 - r^3, \quad \dots$$

$$r = 1 - r^2, \quad r^2 = r - r^3, \quad r^3 = r^2 - r^4, \quad \dots$$

Answer the following questions:

1. What is the maximum flow in the network above? How do you know this flow is maximum? Simplify your answer for the flow value of the maximum flow.
2. Construct a series of augmenting paths with path capacities $1, r^2, r^3$ and r^4 . Draw the residual graph after each of these augmentations. (Do not bother with the capacities of the unlabeled edges. The value 100 is so big for this problem, it is essentially $+\infty$.)
 (*Hint:* there is more than one augmenting path with capacity r^2 .)
3. Argue that you can continue in this pattern and produce an infinite sequence of augmenting paths with path capacities $1, r^2, r^3, r^4, r^5, r^6, \dots$
4. What value does the flows constructed from this sequence of augmenting paths converge to? (Show your work.)
5. Give an example of a flow network with some irrational edge capacities where you can have an infinite sequence of augmenting paths and the flows constructed from these paths do not converge to a maximum flow.