

Homework 7, Due Tuesday October 30, 72 points total

Problem 1. Exercise 16.1-4, page 379.

Problem 2. Exercise 16.2-4, page 384.

Problem 3. In this question, we consider a problem in car ownership. As a car gets older, the maintenance costs (fuel costs, repair costs, insurance costs, etc) for the car may increase to the extent that it would be advantageous to sell the current car and buy a new car. The difficulty in this problem is that the prices of new cars change from year to year, the maintenance costs of cars purchased in different years may be different and the resale value of a car can change from year to year as well.

For this question, you are given the following information:

- $p(i)$ = the price of a new car in year i , for $1 \leq i \leq n$
- $v(i, k)$ = the resale value of a car purchased in year i and sold in year k , for $1 \leq i < k \leq n + 1$.
- $m(i, k)$ = the maintenance cost during year k of a car purchased in year i , for $1 \leq i \leq k \leq n$.

The problem is to determine the years y_1, y_2, \dots, y_r when you would purchase new cars such that the total cost of car ownership from years 1 through n is minimized. The total cost is the sum of the maintenance costs for years 1 through n plus the price of each car purchased minus the resale value of each car sold.

For example, if $n = 10$, $y_1 = 1$, $y_2 = 5$ and $y_3 = 7$, then this solution states that you should purchase a new car in year 1, buy the second car in year 5 and buy the third car in year 7. (You would also sell the first car in year 5, the second car in year 7 and the third car at the beginning of year 11.) In addition, you should make the following assumptions.

- you don't have a car before year 1. (Since you have to buy a new car in year 1, $y_1 = 1$.)
- you only want to purchase new cars.
- you are always able to sell your old car for $v(i, k)$ dollars.
- at the end of n years, you sell your last car for $v(y_r, n + 1)$ dollars.

Answer the following questions about this problem:

1. Provide a concrete example with $n = 4$ where the lowest cost solution is to keep the first car for all n years.
2. Provide a concrete example with $n = 4$ where the lowest cost solution is to buy a new car every year.
3. One greedy approach to solve this problem is to buy a new car in year k if the maintenance cost of the current car during year k is greater than the price of a new car in year k plus the cost for maintaining the new car during year k minus the resale value of the current car in year k .
Provide a concrete example with $n = 4$ which shows that this greedy approach does not always produce an optimal solution.
4. Describe in detail a dynamic programming algorithm which finds the optimal solution to this problem. Be sure to include a description of the content and size of the memoization table. Analyze and justify the running time of your algorithm.